Optimizing analysis of reasonable limit for depth of deflection point with three-dimensional wellbore trajectory

The optimization of reasonable limit for depth of deflection point has a fundamental significance to guide the design of wellbore trajectory. Considering energy saving and optimum scheme for mechanical oil production, the section plane type of wellbore trajectory is selected. The reasonable limit for depth of deflection point is optimized with different vertical depths and horizontal displacements. The results show that: if the evaluation life of oil well is short, take ten years for example, the optimal depth of deflection point is influenced by the cost of increasing drilling footage. And the optimal depth of deflection point is small. When evaluation life of oil well is long, take fifty years for example, the optimal depth of deflection point is influenced by cost of increase in energy consumption and centralizer. And the optimal depth of deflection point is large. With increasing the horizontal displacement, the optimal depth of deflection point is restrained by minimum curvature radius in the process of drilling. The above results have great theoretical and practical significance on allocation optimization of multiple well platform and rational evaluation of energy-saving potential.

Keywords: Wellbore Trajectory, depth of deflection point, reasonable limit, optimizing analysis

1.0 Introduction

In China, the directional drilling technology is used for developing low-permeability reservoir well, and good effect is obtained. The drilling technology of complex structure well of high-inclination or large displacement for Shengli and Zhongyuan oilfield is in the domestic leading position in China. However, the optimization of wellbore trajectory and reasonable distance between wellhead and target layer is foundation of drilling optimization technology.

Therefore, the study of wellbore trajectory is paid attention by domestic and international specialist's scholar [1-4]. The major factors which effect wellbore trajectory are bottom hole assembly, drilling parameters, borehole geometry and formation characteristics. Millheim studies the effect of bottomhole assembly dynamics on the trajectory of a bit [5]. Brakel presents a numerical dynamical model to predict wellbore trajectory in both the vertical and horizontal planes [6]. Combined with the BHA analysis, the models are used to realize the prediction of drilling trajectory and to back-calculate the related coefficients from the well history information [7]. Considering the nonlinear programming method under nonlinear constraints to well trajectory design while drilling, an optimization method for well trajectory design is put forward by Zhang [8]. Considering the relationships between sandstone reservoir with horizontal well track and carbonate reservoir with the well track, a model of denote the relationships between the sandstone reservoir and the well track is built [9]. In order to observe well trajectory directly, design reasonably and control effectively, a series of parametric equations are derived [10-12]. Aslannezhad has studied a safe mud window and a safe drilling direction based on well data such as situ stresses, pore pressure, and rock mechanical properties [13]. The effect of well trajectory on liquid removal in horizontal gas wells is evaluated by Brito [14].

At present, the design principle of wellbore trajectory in the directional well (horizontal well) is to save drilling cost. Based on the position coordinate of oil well, vertical depth between target point and target layer, the parameters of horizontal displacement, deflection point and deviation angle are computed with coordinate transform. However, the most of aforementioned approaches and problems have not been studied sufficiently. But there are still many problems to solve in this field. In Daqing oilfield in China, the deflection point ranges from 100 to 700m, the deviation angle of building up section ranges from 1 to 20°, the deviation angle of hold section ranges from 24 to 19°, the deviation angle of drop off section ranges from 18 to 11°.

Messrs. Mingming Xing and Kaifeng Xue, School of Mechanical & Vehicle Engineering, Linyi University, Linyi, Shandong, 276000, China and also Municipal Modern Agriculture UAV Engineering and Technological Research Centers, Linyi, Shandong, 276000, China, Ms. Lili Zhou, School of Chemistry & Chemical Engineering, Linyi University, Linyi, Shandong, 276000, China

And the horizontal displacement ranges from 200 to 600m. With above parameters, the deflection point and horizontal displacement is larger than parameters of other oilfields. In this case, the eccentric wear of rod string problem is more and more serious with increasing horizontal displacement and deflection point. And the polished rod load and energy consumption increase. When the deviation angle is greater than 20°, it needs to be advance technique of well repairing. However, the technique of displacement directional well has not been mature so far. Based on the normal operation of drill work, the characteristic of lifting system need to be discussed with different horizontal displacements. Therefore, the simulation and structure optimization of wellbore trajectory need to be done.

2.0 Mathematical model and simulation method

As we know, the well structure of directional well (horizontal well) is composed of vertical section, building up section, hold section and drop off section. The schematic of well structure is given in Fig.1.



Fig.1: Schematic of well structure

In the design of well structure, the design principles are usually used. Firstly, the three-dimensional S-shaped is designed in order to avoid obstacle and well intersection. Secondly, the formation natural deflecting law is used to reduce the workload and difficult. Thirdly, considering the demand of oil production process, the curvature of wellbore should not be too large, generally, the vertical section is used. The environment of sucker rod string will be improved. Finally, the security and efficiency are considered, an improved optimization model of well structure need to be built based on the curvature of wellbore, deflection point and type of well profile.

The well structure of five sections is selected, and the geometrical model is given in Fig.2.

Based on the Fig.2, the simulation model of maximum deviation angle is derived, as follows.

$$\begin{cases} \left(2R_{0} - L_{A0}\right)\tan^{2}\left(\frac{\alpha_{m}}{2}\right) - 2L_{H0}\tan\frac{\alpha_{m}}{2} + A_{0} = 0\\ \tan\frac{\alpha_{m}}{2} = \frac{L_{H0} - \sqrt{L_{H0}^{2} + L_{A0}^{2} - 2R_{0}L_{A0}}}{2R_{0} - L_{A0}} \end{cases}$$
(1)

Where,

$$\begin{aligned} L_{H0} &= L_H - L_{Hz} - \Delta L_{Hxz} + R_2 \sin \alpha'' \\ L_{A0} &= L_A - \Delta L_{Axz} + R_2 (1 - \cos \alpha'') \\ R_0 &= \frac{5729.578}{K_1} + \frac{5729.578}{K_2} \end{aligned}$$
(2)

Where, $L_{\rm H}$ is the total vertical depth, m. $L_{\rm A}$ is the total horizontal displacement, m. $L_{\rm Hz}$ is the depth of deflection point, m. *a* is the deviation angle at bottom hole, °. $a_{\rm m}$ is the deviation angle, °. K_1, K_2 are the slopes of building up section and drop off section, °/100m. R_1, R_2 are the curvature radius of building up section and drop off section, m.



Fig.2: Geometrical model of well

Based on the simulation model of maximum deviation angle, the well structure is designed well. And the simulation model of total depth is given, as follow.

$$L_{t} = L_{HZ} + \Delta L_{1} + \Delta L_{2} + \Delta L_{3} + \Delta L_{xz}$$
(3)
Where,

$$\begin{split} \Delta L_{H1} &= R_1 \sin \alpha_m \\ \Delta L_{A1} &= R_1 \left(1 - \cos \alpha_m \right) \\ \Delta L_1 &= R_1 \frac{\alpha_m}{57.29578} \\ \Delta L_{H2} &= \Delta L_2 \cos \alpha_m \\ \Delta L_{A2} &= \Delta L_2 \sin \alpha_m \\ \Delta L_2 &= \sqrt{L_{H0}^2 + L_{A0}^2 - 2R_0 L_{A0}} \\ \Delta L_{H3} &= R_2 \left(\sin \alpha_m - \sin \alpha'' \right) \\ \Delta L_{A3} &= R_2 \left(\cos \alpha'' - \cos \alpha_m \right) \\ \Delta L_3 &= R_2 \frac{\alpha_m - \alpha''}{57.29578} \end{split}$$
(4)

Where, ΔL_{A1} , ΔL_{A2} , ΔL_{A3} are the horizontal displacement of building up section, hold section and drop off section, m. ΔL_1 , ΔL_2 , ΔL_3 are the total length of building up section, hold section and drop off section, m. Lt is the total length of oil well, m.

The curve of wellbore trajectory is usually described by the basic parameters of depth of deflection point, deviation angle and azimuth. The equation of profile curve of well is given, as follow.

$$y = f(x) \tag{5}$$

The formulas of three parameters are given.

$$\begin{cases} \alpha = tg^{-1}\left(\frac{dy}{dS}\right) \\ \varphi = \varphi(S) \\ S(x,y) = \int_0^x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \end{cases}$$
(6)

Where,

$$\begin{cases}
K_{\alpha} = \frac{d\alpha}{dS} \\
K_{\varphi} = \frac{d\varphi}{dS} \\
K = \sqrt{K_{\alpha}^{2} + K_{\varphi}^{2}} \sin(\alpha) \\
R = \frac{1}{|K|}
\end{cases}$$
(7)

Where, K_{α} is rate of deviation angle. K_{ϕ} is rate of azimuth. K is the curvature of wellbore.

The actual curve of wellbore trajectory in directional well is gained with measuring data of deviation angle, azimuth and depth of deflection point. With cubic spline interpolating function, the arrays of deviation angle, azimuth and depth of deflection point are gained. Based on above parameters, the governing equation of well profile for five sections is given as follows.

$$y = (l, L_{\rm H}, L_{\rm A}, L_{\rm Hz}, \alpha, K_1, K_2, \Delta L_{\rm H})$$
 (8)

The actual horizontal displacement of any point is defined with y_i^* (*i*=1, 2, ..., J), and the optimized horizontal displacement of any point is defined with y_i (*i*=1, 2, ..., J). Therefore, the objective function is given as follows

$$F(\vec{x}) = \sum_{i=1}^{J} |y_i - y_i^*|$$
(9)

Where,

$$\vec{x} = [x_1, x_2, x_3, x_4, x_5] = [L_{Hz}, K_1, K_2, \alpha, \Delta L_H]$$
(10)

Where, the optimized vector \vec{x} is governing parameter of profile curve. When the design method is different, the

dimension and constraint are different. In this paper, the type of five sections is selected, and the constraints are given as follows

$$\begin{cases} g_1(\vec{x}) = \alpha_m - x_4 \\ g_2(\vec{x}) = L_{H_0}^2 + L_{A0}^2 - 2L_{A0}R_0 \\ h_1(\vec{x}) = y_J - L_A \end{cases}$$
(11)

The control parameters were identified with the augmented Lagrange multiplier method. The mathematical model of constrained optimization problem is given

$$\begin{cases} \min F(\vec{x}) \\ \vec{x} \in \mathfrak{R} \subset R^n \\ \mathfrak{R} : \begin{cases} g_u(\vec{x}) \ge 0, u = 1, 2, \cdots, n_p \\ h_v(\vec{x}) = 0, v = 1, 2, \cdots, n_q \end{cases}$$
(12)

Where, n_p is number of inequality constrained. n_q is equality constrained.

The solve algorithm of optimization problem is divided into four steps. Firstly, the slack variable is introduced. And the inequality constrained is changed as follows

$$\begin{cases} g_u(\bar{x}) - z_u = 0, u = 1, 2, \cdots, n_p \\ h_v(\bar{x}) = 0, \quad v = 1, 2, \cdots, n_q \end{cases}$$
(13)

Secondly, the Lagrangian multiplier $u_v(v = 1, 2, ..., n_q)$, $\lambda_u(u = 1, 2, 3..., n_p)$ are introduced, and the Lagrangian function is given as follows

$$L\left(\vec{x},\vec{\mu},\vec{\lambda},\vec{z}\right) = F\left(\vec{x}\right) + \sum_{\nu=1}^{q} \mu_{\nu} h_{\nu}\left(\vec{x}\right) + \sum_{u=1}^{p} \lambda_{u} \left[g_{u}\left(\vec{x}\right) - z_{u}^{2}\right]$$
(14)

Thirdly, the penalty function is added, the augmented Lagrangian function is given

$$A(\bar{x},\bar{\mu},\bar{\lambda},\bar{z},r^{k}) = L(\bar{x},\bar{\mu},\bar{\lambda},\bar{z}) + r^{k} \left\{ \sum_{\nu=1}^{q} h_{\nu}^{2}(\bar{x}) + \sum_{u=1}^{p} \left[g_{u}(\bar{x}) - z_{u}^{2} \right]^{2} \right\} (15)$$

Finally, the optimal solutions $(x^*, \mu^*, \lambda^*, z^*)$ of non-constrained optimization problem are solved. And the optimal solutions of former constrained problem are gained.

$$\begin{cases} \vec{x}^* = \begin{bmatrix} x_1^*, x_2^*, \cdots, x_n^* \end{bmatrix}^T \\ F^* = F\left(\vec{x}^*\right) \end{cases}$$
(16)

Based on above recognition algorithm, when the value of objection function is minimal, the profile curve is similar to the curve of actual wellbore.

3.0 Optimization and result analysis

Based on the mathematical model proposed in this paper, the simulation and optimization of well profile are accomplished with Visual Basic 6.0. Besides, the basic data of wellbore trajectory is obtained with Daqing oilfield. The three-dimensional wellbore trajectory of number (000-00) is given in Fig.3.



Fig.3: Three-dimensional wellbore trajectory

3.1 Optimization of vertical depth of deflection point

When the vertical depth of oil well is 1800m, the horizontal displacement of directional well is 600m, the change curves of lifting performance versus vertical depth are given in Fig. 4. When design lifetime of oil well is different, the optimal results of vertical depth of deflection point versus vertical depth of oil well are given in Fig. 5.

With Figs.4 and 5, the maximum load of polished rod and input power of system are decreasing with increasing the depth of deflection point. Instead, some performance indicators are increasing with increasing of the depth of deflection point, such as minimum load of polished rod, maximum torque of crank, system efficiency. Therefore, the lifting performance of system is better with the increasing depth of deflection point. Comparing to vertical well, the cost of incremental drilling and rod-tube string are increasing with increasing the depth of deflection point. When the annual cost of incremental investment and the annual operating cost are considered, the optimal depth of deflection point is influenced by some factors, such as vertical depth of wellbore, horizontal displacement, cost of rod and tube, drilling cost, electricity and oil well life. In the detail, if the lifetime of oil well is ten years, the optimal depth of deflection point is influenced by the cost of increasing drilling footage, and the optimal depth of deflection point is smaller. If the lifetime of oil well is fifty years, the optimal depth of deflection point is influenced by the cost of incremental energy consumption and centralizer.

And the optimal depth of deflection point is bigger. When the horizontal displacement is becoming large, the optimal depth of deflection point is constrained by curvature radius of wellbore.



(c) The curve of rod or tube string mass





Fig.5: Optimal vertical depth of deflection point versus vertical depth of oil well

3.2 Reasonable limit of horizontal displacement of wellbore

When the vertical depth of oil well is 1500m, the depth of deflection point in directional well is 500m, the change curves of lifting performance versus horizontal displacement are given in Fig.6.

Based on the Fig.6, the lifting performance are poorer with increasing the horizontal displacement, such as maximum polished rod load, maximum crank torque, input power, system efficiency, drilling cost. Following the outcome, based on the minimum energy consumption and reasonable operation of oil exploration equipment, the optimal horizontal displacement changes as small as possible.







Fig.7: Power curves versus horizontal displacement and depth of deflection point

the pump diameter is 32mm, and the stroke length is 3m, the power curve of system versus vertical depth and horizontal displacement is given in Fig. 7.

With the Fig.7, when the depth of deflection point is from 200m to 800m, and the horizontal displacement is from 320m to 200m, the input power of system is reduced from 10.65kW to 4.92kW. Therefore, the difference of energy consumption between different combinations of horizontal displacement and depth of deflection point is large.

5.0 Conclusion

The following conclusions can be summarized from the theoretical formulation and the simulation studies in this paper:

Considering energy saving and optimum scheme for mechanical oil production, the section plane type of wellbore trajectory is selected. The recognition method of well profile is proposed with the mathematical model of five sections of well profile. In the detail, based on the cubic spline interpolation function method, the curve of wellbore trajectory is described by the basic parameters of depth of deflection point, deviation angle and azimuth. With the augmented Lagrange multiplier method, the nonlinear objective function and constants of well profile are derived. Based on the mathematical model proposed in this paper, the simulation and optimization of well profile are accomplished with Visual Basic 6.0. The reasonable limit for depth of deflection point is optimized with different vertical depths and horizontal displacements. The results show that: if the evaluation life of oil well is short, take ten years for example, the optimal depth of deflection point is influenced by the cost of increasing drilling footage. And the optimal depth of deflection point is small. When evaluation life of oil well is long, take fifty years for example, the optimal depth of deflection point is influenced by cost of increase in energy consumption and centralizer. And the optimal depth of deflection point is large. With increasing the horizontal

displacement, the optimal depth of deflection point is restrained by minimum curvature radius in the process of drilling. The above results have great theoretical and practical significance on allocation optimization of multiple well platform and rational evaluation of energy-saving potential.

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