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# Proposing a Characteristic Length Definition for Flow Characterization in Porous Media: A Methodology for Estimating Hydraulic Radius

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## Abstract

*This study explores the complex factors influencing fluid flow and associated head loss within porous media, focusing on particle size, shape, and packing porosity. The chosen characteristic length, hydraulic radius (denoted as "r"), integrates these factors, providing a comprehensive measure for characterizing flow behavior in specific packing configurations. Crushed stones and glass spheres of varying sizes are used as porous media. Porosity, size, and shape of the media are meticulously determined to understand their impact on flow characteristics. The study's findings offer valuable insights for researchers and designers in porous media applications, guiding the selection of appropriate characteristic length expressions. Additionally, this work contributes to a deeper understanding of porous media flow and provides a practical framework for characterizing and analyzing porous media properties, advancing the broader field of fluid dynamics in porous structures.*

**Keywords:** Characteristic Length, Flow through Porous Media, Hydraulic Radius, Porous Media.

## 1.0 Introduction

The exploration of fluid flow through porous media has remained a source of enduring fascination and challenge for scientists and engineers alike, persisting for nearly two centuries since Henry Darcy's groundbreaking work in 1856. At the heart of this challenge lies the need to comprehend and define a crucial parameter: the characteristic length, particularly in the context of porous media flow[1]. However, determining this characteristic length, even under controlled laboratory conditions, remains a formidable task due to the inherent randomness and variability in pore size and distribution within porous structures. Just as a precise understanding of characteristic length is indispensable in pipe flow, its significance is equally pronounced in porous media, where both flow transitions and pressure drop hinge upon its definition.

Researchers have determined that Darcy's linear relationship effectively describes flow through porous media within a certain velocity range[2-4]. Beyond this range, the relationship between velocity and the gradient of hydraulic head follows a binomial or power-law trend, marking a transition to what has been termed "high velocity flows," "non-Darcy flows," or "post-laminar flows"[5,6]. This transition in flow regimes carries profound implications across various applications, from understanding flow through rockfill dams[7-9] to estimating discharge in aquifers and managing oil and gas wells[10-12].

The key to understanding this transition lies in identifying the flow regime, and the Reynolds number

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(Re) has historically been employed for this purpose. However, a significant challenge arises in determining an appropriate characteristic length and characteristic velocity for porous media. While pipe flow relies on the pipe diameter as the characteristic length, channel flow employs the depth of flow, and boundary layer problems use distance from the leading edge. Similarly, the pore size must serve as the characteristic linear dimension in porous media flow. Yet, the complexity of pore geometry, influenced by factors like particle size[13-15], porosity[3], shape, surface roughness, and convergent angles[16-20], complicates the development of a straightforward mathematical model. Consequently, diverse expressions have emerged for computing Reynolds numbers[21] differing mainly due to their definitions of characteristic length, and can be categorized into three primary groups.

The first category employs media diameter as the characteristic length[22-32]. While this approach offers simplicity in calculation, it falls short in accurately representing pore size and, thus, has limited applicability. The second category seeks to incorporate pore size and geometry effects through hydraulic conductivity[33,34] or intrinsic permeability[21,35,36]. Several models have been developed to express permeability in terms of media and packing properties[37-43] Nevertheless, these models are limited in their ability to capture the full complexity of pore geometry.

The third category treats pore size as a function of measurable packing and media properties, such as media size, porosity, shape factor[21,44,45]. Various representations have been proposed, including hydraulic diameter[46], throat diameter, pore diameter[47] pore throat radius[48], and strut diameter, often accompanied by tortuosity considerations[49]. However, these definitions can be challenging to apply in real porous packing due to the inherent randomness in pore size distribution. However, recent studies by the authors have demonstrated that the use of hydraulic radius as characteristic length has produced some encouraging results[1,50,51]. Therefore, the present study attempts to demonstrate in detail the estimation of this parameter. The present method can be very much helpful aim to aid designers and researchers in selecting the most appropriate characteristic length expression for their specific porous media applications. Furthermore, these results can serve as a foundation for further research aimed at modeling and understanding flow characteristics within porous systems.

## 2.0 Determination of Media Properties

The velocity of fluid flow and the associated head loss within a porous medium are fundamentally influenced by various factors, including particle size, shape, and packing porosity. Consequently, the characteristic length chosen for this study must encompass all these factors that impact flow behavior in specific packing configurations.

In this study, the selected characteristic length is the hydraulic radius (denoted as "r")[20,52], as defined by Eq. 1.

$$r = \frac{e}{s_0} \tag{1}$$

This hydraulic radius integrates essential parameters such as the void ratio of the packing (denoted as *e* calculated as the ratio of porosity to specific surface area, represented by $s_0$). To conduct the study, crushed stones and glass spheres of varying sizes were employed as the porous media. These media were carefully sorted into different size categories using Indian Standard (IS) sieves, resulting in three distinct sizes for the crushed stones (29.80 mm, 34.78 mm, and 41.59 mm) and three for the glass spheres (17.51 mm, 25.46 mm, and 33.42 mm). The subsequent sections of this study describe the experimental techniques used to ascertain crucial media properties, including packing porosity, surface area, and media volume.

### 2.1 Determination of Porosity

Porosity in a packing material is typically defined as the ratio of the volume of void spaces to the total volume of the packing. This definition provides a global or average porosity value for larger porous packing structures. However, it's important to acknowledge that porosity can exhibit spatial variations within the packing. Detecting these variations can be challenging in large-scale setups, often leading to the assumption of uniform porosity.

In the present experimental setup, measuring porosity involves a systematic process. The permeameter section is first cleaned and filled with the porous medium. All piezometer tapings and outlet valves are securely closed, followed by the gradual introduction of fluid into the permeameter. The volume of the fluid contained within the permeameter is meticulously collected and measured using a measuring tank. This collected fluid volume is considered a representative estimate of the void volume within the packing. Porosity is then calculated as the ratio of the volume of fluid collected within the permeameter to the total volume of the permeameter.

For crushed stones of each size category, multiple repacking iterations are performed to achieve varying porosity levels. This approach helps in comprehending the influence of porosity variation on pressure loss characteristics within porous media. It is worth noting that regular-shaped media, by contrast, exhibit minimal porosity variation during repacking, leading to the recording of a single porosity value for such media.

## 2.2 Size of the Media

In the analysis, the size of regularly shaped particles, such as glass spheres, is characterized by their respective diameters. To determine these diameters with precision, a Vernier Caliper is employed, ensuring an accuracy of 0.01 mm in the measurements. The measurement process involves the selection of two hundred glass sphere samples randomly from each size category. Each individual particle's diameter is carefully measured using the Vernier Caliper. To minimize measurement errors, the glass spheres are deliberately placed on a flat surface and prevented from rolling during the size measurement procedure.

Subsequently, the average diameters of the collected samples are calculated and used as the representative measure of the media size for all further analyses and investigations. This meticulous approach ensures accuracy and reliability in characterizing the size of the regularly shaped particles in the study. Representing the size of irregularly shaped particles with a single dimension can be a formidable task due to their complex geometry. Therefore, in this study, the concept of a volume diameter, as defined by reference[20,53], is employed. The volume diameter is essentially the diameter of a spherical particle that has the same volume as the irregularly shaped particle under consideration. This approach provides a standardized measure of size for the crushed stones utilized in the present study, overcoming the challenges associated with representing irregular shapes with a single dimension.

Two hundred random samples are chosen from each size of crushed stones to minimize measurement errors in determining particle size. Each of these samples is placed into a graduated measuring cylinder filled with a predetermined volume of water. The change in water level before and after immersing the particle in the cylinder accurately reflects the volume of the irregularly shaped particle. Care is exercised throughout the process to prevent any splashing of water from the cylinder. Subsequently, the volume diameter (d) is computed using the Eq. 2.

$$d = \sqrt[3]{\frac{6 \times Volume\ of\ the\ crushed\ stone}{\pi}} \tag{2}$$

## 2.3 Shape of the Media

In this study, we consider the influence of media shape on flow characteristics, and we quantify this effect in terms of specific surface. Specific surface is defined as the surface area per unit volume. When dealing with glass spheres, the specific surface can be determined using the following formula:

$$s_0 = \frac{Surface\ Area}{Volume} = \frac{4\pi\left(\frac{d^2}{4}\right)}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \tag{3}$$

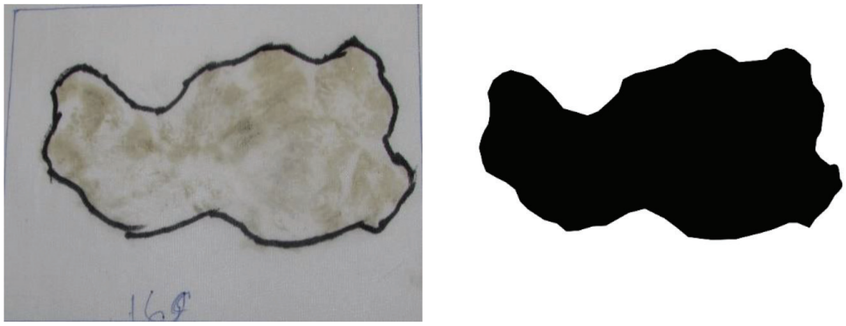
However, determining the specific surface of irregularly shaped media is considerably more intricate due to the inability to directly measure their surface area. To calculate the surface areas of the crushed stones, a methodology akin to that detailed by Thiruvengadam and Kumar (1997) is employed. The procedure for ascertaining surface area is as follows:

a. Randomly collected samples are immersed in a colored solution. Each sample is then enveloped in a rectangular white fabric with known dimensions, and impressions of all the surfaces are recorded (Figure 1(a)). The surface area of the rectangular section is measured and serves as the reference area.
b. The acquired impressions are photographed and processed digitally (Figure 1(b)) using suitable image processing tools.
c. Pixel areas for both the reference sections (rectangular background) and the media impressions are calculated from the processed images. Subsequently, the surface areas of the irregularly shaped media can be determined as follows:

$$\frac{\text{surface area of the rectangular background}}{\text{surface area of the media impression}} = \frac{\text{pixel area of the rectangular background}}{\text{pixel area of the media impression}}$$

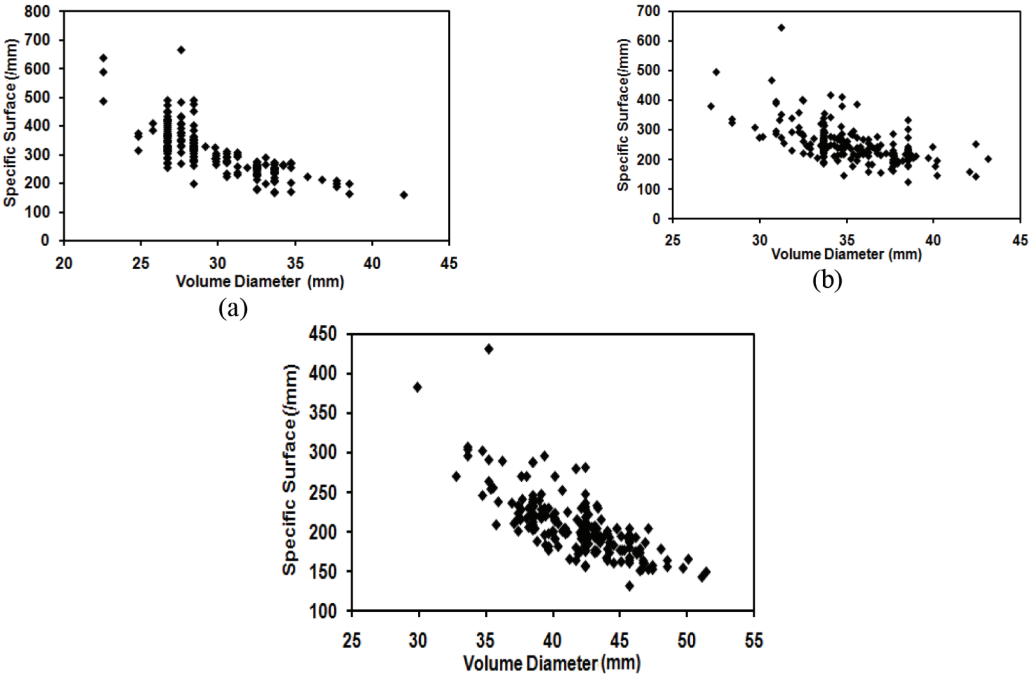
The specific surface of each sample and the hydraulic radius of each packing are then calculated from their definitions.

Figure 2 represent the variation between the specific surface and volume diameter for the media utilized in the present study.



(a) (b)

**Figure 1.** (**a**) Impression (**b**) Processed image of a crushed stone sample on a white background.



(a)

(b)

(c)

**Figure 2.** Variation of specific surface with volume diameter for crushed stones with (**a**) 29.80 mm (**b**) 34.78 mm (**c**) 41.59 mm average volume diameter.

## 3.0 Discussion and Conclusion

In conclusion, the exploration of fluid flow through porous media has endured as a captivating and challenging pursuit for scientists and engineers for nearly two centuries. This enduring challenge revolves around the crucial parameter of characteristic length, particularly in the context of porous media flow. The complexity of pore geometry, influenced by factors like particle size, porosity, shape, surface roughness, and convergent angles, has made the development of a precise mathematical model for characteristic length in porous media a formidable task.

This study illustrates the methodology that can be followed to estimate the characteristic length definitions named hydraulic radius. The chosen characteristic length for this study, hydraulic radius, integrates various factors affecting flow through porous media, including particle size, porosity, and specific surface area. The methodology proposed in the study can serve as valuable guidance for researchers and designers working with porous media applications, helping them choose the most suitable characteristic length expression for their specific needs. Additionally, these results lay the foundation for further research aimed at enhancing our understanding of flow characteristics within porous systems.

Through meticulous determination of media properties, including porosity, size, and shape, this study has contributed to a comprehensive understanding of the porous media used in the experiments. The use of hydraulic radius as the characteristic length provides a holistic representation of porous media properties, ensuring a more accurate assessment of flow characteristics.

In summary, the investigation presented here not only advances our knowledge of porous media flow but also provides a practical framework for characterizing and analyzing porous media properties. This work contributes to the broader scientific and engineering community's ongoing efforts to unlock the mysteries of fluid flow through porous structures.

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