

# Darcy Heat Transfer Flow of Ternary Hybrid Nanofluid over a Sinusoidal Wavy Surface

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## Abstract

*In this paper, heat transfer enhancement has been examined due to the ternary hybrid nanofluid over a vertical wavy surface. Darcy law is used to investigate the flow through porous medium. The non-uniform behavior of the vertical wall is defined by the sinusoidal nature. The governing equations for the conservation of mass, momentum along x and y directions and energy equations are derived and non-dimensionalized using appropriate transformations. Similarity transformations are used to convert the Partial Differential Equations (PDEs) to Ordinary Differential Equations (ODEs). The resultant ODEs are solved by employing spectral collocation method and the results are presented with various thermo-physical parameters of nanoparticles.*

**Keywords:** Darcy Porous Medium, Heat Transfer Enhancement, Sinusoidal Wavy Surface, Spectral Method, Ternary Hybrid Nanofluid

## 1.0 Introduction

In recent years, Nanofluids are significantly heard in the field of research due to their thermophysical and heat transfer features. Nanofluid is a fluid in which the nanoparticles whose size varies between 1 to 100 nanometers in diameter are dispersed. There are different types of nanoparticles based on various physical and chemical properties. They are Carbon-based nanoparticles (Fullerenes, Carbon nanotubes), Metal nanoparticles (Copper, Silver, Gold), Ceramic nanoparticles (Calcium oxide, Titanium oxide, Silicon oxide), Semiconductor nanoparticles (GaN, GaP, InP), Polymeric nanoparticles (Chitosan, alginate, cellulose) and Lipid based (Cisplatin, Irinotecan, paclitaxel) nanoparticles. When any 2 or more nanoparticles are dispersed in the base fluid it is termed as a hybrid nanofluid. A ternary hybrid nanofluid is a fluid in which three different nano particles are dispersed in a fluid.

Ternary hybrid nanofluids (TNFs) are extensively used in technical, scientific, and industrial applications. There are numerous applications of hybrid nanofluids which includes refrigeration, research based on solar energy, air conditioning, heat pipes, heat exchanger, electronic cooling, nuclear reactors, broadcasting, spacecraft, air purifiers, chemical process, cancer therapy, engineering process, electrical insulators, hair care products, dental products, green tires, fuel cells, optical chemical sensors, solar cells, biosensors, and automotive parts. TNFs have a broad range of thermal characteristics, including the capacity to solidify at very high temperatures.

The thermohydraulic performance and energy production in microchannel heat sinks that were treated using ternary/binary hybrid nanofluids were studied by<sup>1</sup>. For the purpose of cooling devices, they adopted rectangular heat exchangers sinks equipped with the hybrid nanofluids CuO/MgO/TiO<sub>2</sub>/water ternary and

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MgO/TiO<sub>2</sub>/ water binary<sup>2</sup> analyzed the performance of the ternary-nanoparticles in the base fluid through convergent-divergent nozzle. The properties of Distilled Water (DW) and Ethylene Glycol (EG) conventional fluids mixes in the ratios of (100:0), (60:40), (50:50), and (0:100) are employed in this study, together with silver (Ag), cobalt (Co), and zinc (Zn) nanoparticles. Using the “Galerkin finite element” model,<sup>3</sup> demonstrated the insertion of ternary hybrid-nanofluids to the Prandtl fluid model to enhance heat transit. They discovered that tri-hybrid nanoparticles exhibit higher fluid motion than pure fluid, nanofluid, and hybrid nanomaterials. They also found that tri-hybrid nanomaterials allow for the achievement of ultimate thermal performance. They used the nanoparticles (Al<sub>2</sub>O<sub>3</sub>, TiO<sub>2</sub> and SiO<sub>2</sub>) in base fluid ethylene glycol. Ebrahim *et al.*<sup>4</sup> investigated the energy and mass transmission across a stretched permeable surface through ternary hybrid nanofluid. They investigate the steady and incompressible trihybrid flow of nanofluid (TiO<sub>2</sub> + MgO + CoFe<sub>2</sub>O<sub>4</sub> / EG) along a horizontal stretching sheet<sup>5</sup> studied the heat and mass transfer characteristics of incompressible electro conductive ternary nanofluid over a stretching cylinder. They have designed a computational model to improve the conveying rate of mass and energy<sup>6</sup> studied the performance of thermal conductivity of a ternary hybrid nanomaterial subject to generation of entropy. They used Newton’s shooting technique to obtain the numerical solution. Here ferric oxide, titanium dioxide and silicon dioxide are used as nanoparticles are dissolved in the base fluid which is a mixture of water and ethylene glycol<sup>7</sup> investigated the heat and mass transfer characteristics of a 3D magnetohydrodynamic flow of trihybrid nanofluid through a porous media towards stretching surface<sup>8</sup> analyzed the thermal transfer and flow behavior of ternary nanofluid past a stretching sheet. They used the Runge-Kutta- Fehlberg (RKF45) method to obtain the numerical results. In this study, the nanoparticles titanium-aluminum-silicon dioxide were suspended into the base fluid water.

Porous medium is a medium that enables the movement of fluid (liquid or gas) through the pores. Porous medium plays a significant role in our day today lives. Due to the wide range of applications in modern technology, including packed bed heat exchangers, electronics cooling, heat pipes, biological, drying technology, tissue engineering, catalytic reactors,

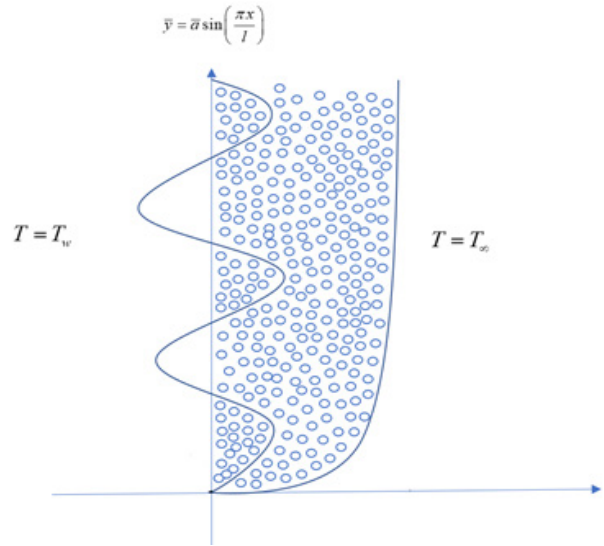
environmental engineering, drug delivery, nuclear waste repository, thermal insulation engineering, enhanced recovery of petroleum reservoirs, spreading of chemical waste, and grain sorghum, study on porous media has also been the subject of numerous studies. A porous media in which the fluid flow obeys Darcy’s law is known as Darcy porous media. Darcy’s law has been appreciated remarkably in the field of fluid flow and heat transfer through porous media. Such porous media has diverse applications in hydrogeology, petroleum engineering, coffee brewing, etc.,<sup>9</sup> analyzed the effects of Soret-Dufour parameters on the mixed convection in the presence of a vertical wavy surface in the Darcy porous medium. They have used Runge Kutta procedure with shooting-technique to obtain the numerical solution. The results obtained are verified with the literature. The effects of the thermophoresis phenomena on the mixed convection flow in conjunction with vertical wavy surface in Darcy porous media with changing thermophysical properties have indeed been studied by<sup>10</sup>. They used numerical method to compute the results and results were validated with the literature<sup>11</sup> presented the numerical results of the convective heat transfer flow in presence of the sinusoidal wavy surface in a fluid soaked sparsely packed Darcy porous medium with varying porosity, permeability and thermal conductivity. They used “local non-similarity method” to obtain the solution. Rees and Pop<sup>12</sup> investigated the natural convection prompted by the presence of the vertical sinusoidal surface in a Darcy porous medium with uniform heat flux. They used “Keller-box-method” obtain the numerical results<sup>13</sup> investigated the natural convective process along with vertical wavy surface in a Darcy porous medium that is saturated with a Newtonian fluid<sup>14</sup> presented the natural convection for the mass and heat transfer characteristics through a wavy surface placed in porous medium under the influence of varying viscosity and varying thermal conductivity in presence of cross diffusion<sup>12</sup> studied the heat transfer characteristics in Darcy free convective, forced convective and mixed convective processes induced by an vertical plate kept under isothermal condition in a porous media which is soaked with an elastic fluid of constant viscosity. He used approximate integral method to solve the governing equations of the flow and hence found the similarity solution.

Vertical wavy surfaces basically enhance the heat transfer effect that includes convective process. A process

like this is crucial for many engineering and industrial applications, including grain storage bins, heat transfer devices, refrigerator condensers, solar collectors, electrical and nuclear cooling components, building energy system design, chemical catalytic reactors, compact heat exchangers, etc. <sup>12</sup> analyzed the free convection in a porous medium for the first time which consisted of a vertically placed wavy surface maintained under isothermal heating. They used generalized similarity transformations to solve the boundary-layer equations<sup>15</sup> analyzed the streamline natural convection flow of a non-Newtonian fluid that obeys power law above a vertical wavy surface under isothermal condition<sup>16</sup> studied the influence of the varying thermal conductivity on the flow with convective mass and heat transfer along with the vertical plate in rotating system<sup>17</sup> analyzed numerically the free convection<sup>1</sup> within a porous-enclosure instigated by wavy surface with uniform heat flux using finite element method<sup>18</sup> studied the magneto hydro dynamic natural convection flow induced by a wavy surface under the viscous dissipation effect with heat generation. They used “Keller-box scheme” to derive the numerical solution<sup>19</sup> explored the bio-convective flow of a nanofluid with the base fluid as water that contained gyrotactic micro-organism through a wavy surface. They attained the numerical results with the aid of implicit FDM scheme<sup>20</sup> examined numerically the streamline natural convection in a porous medium saturated<sup>21</sup> studied the momentary heat transfer in a free convection of a nanofluid past a wavy surface. With the aid of thermal non-equilibrium method, the heat transfer effect is investigated. The authors planned to study the enhancement of heat transfer caused by the ternary hybrid nanofluid passing a wavy surface in light of the aforementioned literature and application.

## 2.0 Formulation of the Problem

Consider the laminar, 2D, steady incompressible flow of a nanofluid over the vertical wavy surface. The wavy surface is considered along the x-axis and its wavy ness is defined as  $\bar{y} = \bar{a} \sin\left(\frac{\pi \bar{x}}{l}\right)$ . The geometry is embedded in a porous medium. The surface is maintained with temperature  $T_w$  and the constant ambient temperature



**Figure 1.** Geometry of the problem.

$T_\infty$  is assumed to be far to the surface. Darcy’s law is used to describe the porous medium.

The governing equations are:

Momentum equations:

$$\begin{aligned} \frac{\mu_{Tnf}}{K} \bar{u} &= -\frac{\partial \bar{p}}{\partial \bar{x}} + \rho_{Tnf} g \beta_f (T - T_\infty) \\ \frac{\mu_{Tnf}}{K} \bar{v} &= -\frac{\partial \bar{p}}{\partial \bar{y}} \end{aligned} \tag{1}$$

Energy Equation:

$$\bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} = \alpha_{Tnf} \left( \frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2} \right) \tag{2}$$

with the boundary conditions

$$\bar{u} = 0; \bar{v} = 0; T = T_w \text{ at } \bar{y} = \bar{\sigma}(\bar{x}) = \bar{a} \sin\left(\frac{\pi \bar{x}}{l}\right) \text{ and}$$

$$\bar{u} \rightarrow 0; T \rightarrow T_\infty \text{ as } \bar{y} \rightarrow \infty \tag{3}$$

Thermo-physical properties:

$$\mu_{Tnf} = \frac{\mu_f}{(1 - \phi_1)^{2.5} (1 - \phi_2)^{2.5} (1 - \phi_3)^{2.5}}$$

$$K_{Tnf} = \left[ \frac{K_{np3} + 2K_{hnf} - 2\phi_3 (K_{hnf} - K_{np3})}{K_{np3} + 2K_{hnf} + \phi_3 (K_{hnf} - K_{np3})} \right]$$

$$\left[ \frac{K_{np2} + 2K_{nf} - 2\phi_2 (K_{nf} - K_{np2})}{K_{np2} + 2K_{nf} + \phi_2 (K_{nf} - K_{np2})} \right] \left[ \frac{K_{np1} + 2K_f - 2\phi_1 (K_f - K_{np1})}{K_{np1} + 2K_f + \phi_1 (K_f - K_{np1})} \right] K_f$$

$$\alpha_{Tnf} = \frac{K_{Tnf}}{(\rho C_p)_{Tnf}},$$

$$\rho_{Tnf} = (1-\phi_1)\{(1-\phi_2)[(1-\phi_3)\rho_f + \phi_3\rho_{np3}] + \phi_2\rho_{np2}\} + \phi_1\rho_{np1},$$

$$(\rho\beta)_{Tnf} = (1-\phi_1)\{(1-\phi_2)[(1-\phi_3)(\rho\beta)_f + \phi_3(\rho\beta)_{np3}] + \phi_2(\rho\beta)_{np2}\} + \phi_1(\rho\beta)_{np1},$$

$$(\rho C_p)_{Tnf} = (1-\phi_1)\{(1-\phi_2)[(1-\phi_3)(\rho C_p)_f + \phi_3(\rho C_p)_{np3}] + \phi_2(\rho C_p)_{np2}\} + \phi_1(\rho C_p)_{np1}$$

From the following non-dimensional variables,

$$x = \frac{\bar{x}}{l}; y = \frac{\bar{y}}{l}; \psi = \frac{\bar{\psi}}{\alpha_f}; a = \frac{\bar{a}}{l}; \theta = \frac{T - T_\infty}{T_w - T_\infty}; \frac{\partial \bar{\psi}}{\partial \bar{y}} = \bar{u}; -\frac{\partial \bar{\psi}}{\partial \bar{x}} = \bar{v}$$

we obtain the dimensionless equations,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = Ra \frac{B}{A} \frac{\partial \theta}{\partial y}$$

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = \frac{D}{C} \left( \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right)$$

where,

$$Ra = (\rho\theta^* g^* K^* \beta a^3 T_w - T_{inf}) / (\mu\alpha)$$

$$A = \frac{\mu_{Tnf}}{\mu_f}; B = \frac{(\rho\beta)_{Tnf}}{(\rho\beta)_f}; C = \frac{K_{Tnf}}{K_f}; D = \frac{(\rho C_p)_{Tnf}}{(\rho C_p)_f}$$

with the boundary conditions:

$$\psi = 0, \theta = 1 \text{ on } y = a \sin x \text{ and } \psi_y \rightarrow 0, \theta \rightarrow 0 \text{ as}$$

$$y \text{ tends to infinity} \tag{7}$$

Applying the following similarity transformation,

$$x = \xi; y = \xi^{1/2} Ra^{-1/2} \eta + a \sin x; \psi = Ra^{1/2} \hat{\psi}$$

equations (5) and (6) will be transformed to

$$(1 + a^2 \cos^2 \xi) \frac{\partial^2 \hat{\psi}}{\partial \eta^2} = \xi^{1/2} \frac{B}{A} \frac{\partial \theta}{\partial \eta} \tag{9}$$

$$(1 + a^2 \cos^2 \xi) \frac{\partial^2 \theta}{\partial \eta^2} = \xi^{1/2} \frac{D}{C} \left( \frac{\partial \hat{\psi}}{\partial \eta} \frac{\partial \theta}{\partial \xi} - \frac{\partial \theta}{\partial \eta} \frac{\partial \hat{\psi}}{\partial \xi} \right) \tag{10}$$

and the boundary conditions are

$$\hat{\psi} = 0, \theta = 1 \text{ on } \eta = 0 \text{ and } \hat{\psi}_y \rightarrow 0, \theta \rightarrow 0 \text{ as } \eta \rightarrow \infty \tag{11}$$

Performing similarity transformations using the stream function and temperature  $\hat{\psi} = \xi^{1/2} f(\hat{\eta}); \theta = \theta(\hat{\eta})$

$$\text{where the similarity variable } \hat{\eta} = \frac{\eta}{1 + a^2 \cos^2 \xi}$$

equations (9) and (10) will get the form,

$$\frac{\partial^2 f}{\partial \hat{\eta}^2} = \frac{B}{A} \frac{\partial \theta}{\partial \hat{\eta}} \tag{12}$$

$$\frac{\partial^2 \theta}{\partial \hat{\eta}^2} = -\frac{D}{2C} f(\hat{\eta}) \frac{\partial \theta}{\partial \hat{\eta}} \tag{13}$$

with the boundary conditions

$$f(0) = 0; \theta(0) = 1; f'(\infty) = 0, \theta(\infty) = 0 \tag{14}$$

The physical quantity of interest in this problem is to investigate on local Nusselt number. Local Nusselt number is given by,

$$Nu_x = \frac{qx}{K_f(T_w - T_\infty)}, \text{ where } q = -K_{Tnf} [\partial T / \partial y]_{y=0}$$

### 3.0 Numerical Procedure

Spectral Quasi-Linearization method<sup>22</sup> is used to compute the solutions of eqns. (12), (13) and (14). First eqns. (12) and (13) are linearized considering  $f = f_r$ , and  $\theta = \theta_r$ , we obtain,

$$a_{1,1}^{(2)} f_r'' + a_{1,2}^{(1)} \theta_r' - f_r'' + \frac{B}{A} \theta_r' = R_1 \tag{15}$$

$$a_{2,1}^{(0)} f_{r+1} + a_{2,2}^{(2)} \theta_{r+1}'' + a_{2,2}^{(1)} \theta_{r+1}' = R_2 \tag{16}$$

The boundary conditions are:

$$f_{r+1} = 0, f_{r+1}' = 0, \theta_{r+1} = 1, \theta_{r+1}' = 0 \text{ at } \eta = 0 \tag{17}$$

where,

$$a_{1,1}^{(2)} = 1; a_{1,2}^{(1)} = -\frac{B}{A}; a_{2,1}^{(0)} = \frac{D}{2C} \theta_r'; a_{2,2}^{(2)} = 1; a_{2,2}^{(1)} = \frac{D}{2C} f_r;$$

$$R_1 = 0; R_2 = \frac{D}{2C} \theta_r' f_r \tag{18}$$

“Chebyshev Gauss-Lobatto” points are considered as

$$z_j = \cos\left(\frac{\pi j}{n}\right), j = 0(1)n \tag{19}$$

where  $\eta$  is the collocation point. To approximate  $f_r$  and  $\theta_r$ , the Lagrange polynomials are used at  $N+1$  Gauss-Lobatto points, respectively.

$$f(\xi) = \sum_{j=0}^n \psi_j(\xi) f_j, \tag{20}$$

$$\theta(\xi) = \sum_{j=0}^n \psi_j(\xi) \theta_j. \tag{21}$$

The derivative at the collocation points with respect to the basic functions is:

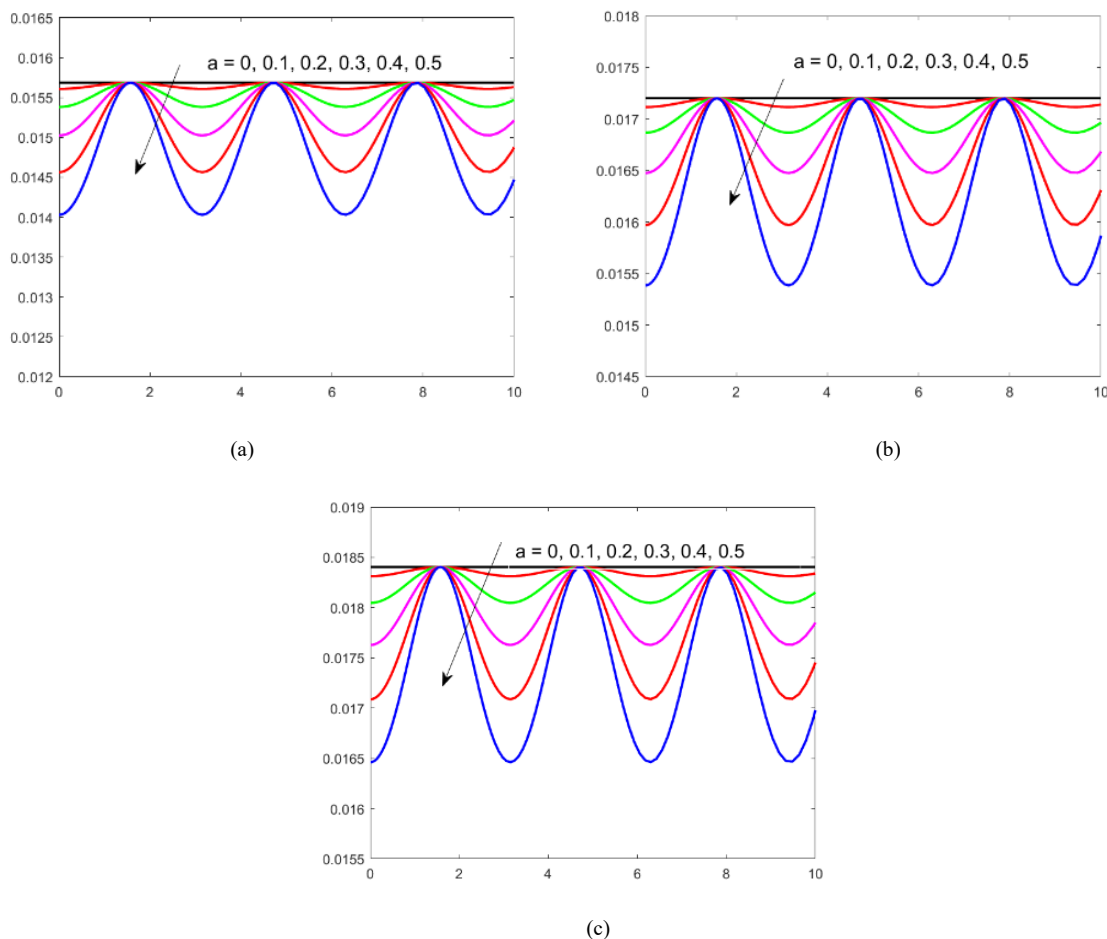
$$D(\ ) = 2 \sum_{j=0}^n \psi_j'(\xi_i) ( \ )_j. \tag{22}$$

**Table 1.** Physical quantities of the ternary hybrid nanofluid

	$\rho(\text{kg/m}^3)$	$\beta_f$	$c_f$	$K$
H <sub>2</sub> O	997.1	$21 \times 10^5$	4179	0.623
Cu	8933	$1.67 \times 10^5$	385	400
CuO	6500	$1.8 \times 10^5$	535.6	20
Fe <sub>2</sub> O <sub>3</sub>	5200	$1.3 \times 10^5$	670	6

**Table 2.** Comparison of Nusselt number with published results for Newtonian fluids and vertical flat surface

	Cheng and Minkowycz <sup>23</sup>	Present Results
Pure fluid	0.4440	0.4440425



**Figure 2.** (a) Cu. (b) Cu-CuO. (c) Cu-CuO-Fe<sub>2</sub>O<sub>3</sub>. Nusselt number for amplitude of the wavy surface.

To demonstrate the results of equations (15) and (16) at  $(r+1)^{th}$  iteration, field variables at the  $r^{th}$  position are considered as  $f_r = 0x$ , and  $\theta_r = 1+b/x$  Equations (15) and (16) with the boundary conditions are discretized. Therefore, matrix system of equations in the form  $AX = R$  (23)

where,

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}; \quad X = \begin{bmatrix} f_{r+1} \\ \theta_{r+1} \end{bmatrix}; \quad R = \begin{bmatrix} f_{r+1} \\ \theta_{r+1} \end{bmatrix};$$

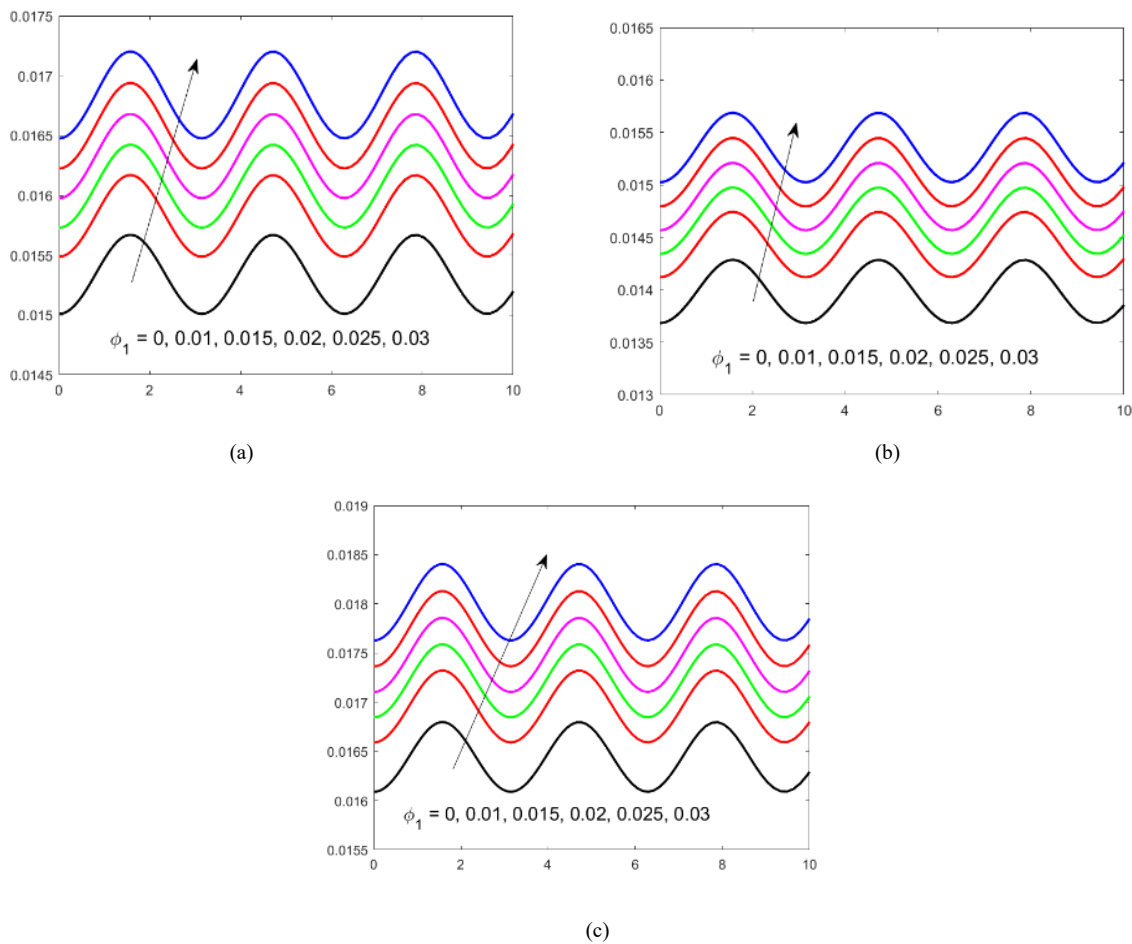
$$A_{11} = \text{diag}(a_{1,1}^{(2)})D^2; \quad A_{12} = \text{diag}(a_{1,2}^{(1)})D; \quad A_{21} = \text{diag}(a_{2,1}^{(0)})I;$$

$$A_{22} = \text{diag}(a_{2,2}^{(2)})D^2 + \text{diag}(a_{2,2}^{(1)})D$$

and  $I$  and  $D$  are identity and Chebyshev differentiation matrices having order  $N+1$ .

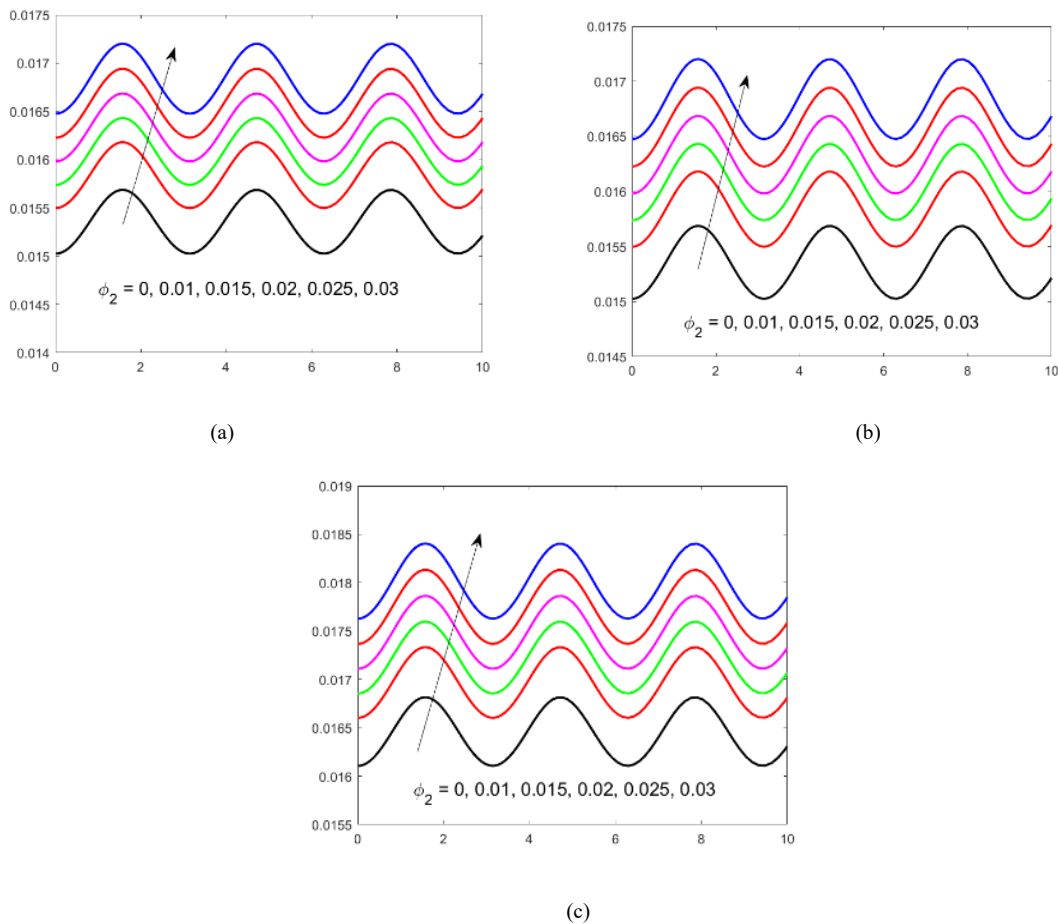
### 4.0 Results and Discussion

In this section, the results are presented on heat transfer rate and discussed for various types of fluids. Figure 2 instantiates the Nusselt number behavior for the various amplitudes of the wavy surface in three different cases namely, “nanofluid (Cu), hybrid-nanofluid (Cu-CuO) and ternary hybrid-nanofluid (Cu-CuO-Fe<sub>2</sub>O<sub>3</sub>)”. It is observed that as the amplitude increases, the Nusselt number decreases in all the three cases. One can understand that ternary hybrid nanofluids enhance the heat transfer more significantly than the hybrid nanofluids and nanofluids.



**Figure 3.** (a) Cu. (b) Cu-Cu. (c) Cu-CuO-F. Nusselt number for amplitude of the wavy surface  $\phi_r$ .





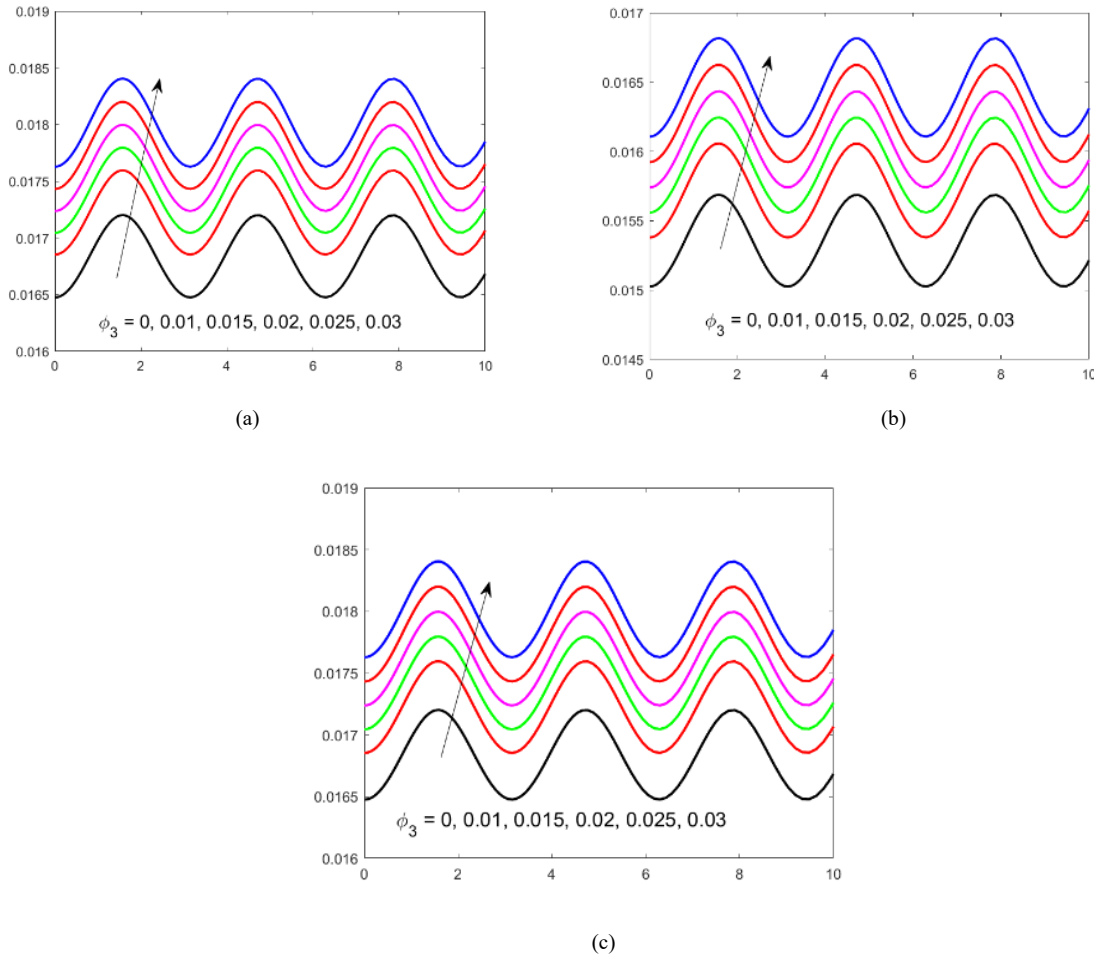
**Figure 4.** (a) Cu. (b) Cu-Cu. (c) Cu-CuO-Fe<sub>2</sub>O<sub>3</sub>. Nusselt number for amplitude of the wavy surface  $\phi_2$ .

Figures 3, 4 and 5 illustrate the thermal transfer amplification for various values of “volume fraction” in different types of fluids. It is noticed that the Nusselt number increases as the volume fraction of the fluid increases in all the three cases. It is also worth mentioning that ternary hybrid nanofluids enhance the heat transfer effects notably than the other two cases.

## 5.0 Conclusion

In this paper, mathematical model has been developed to investigate heat transfer characteristics due to ternary hybrid nanofluid over wavy surface. Darcy law is used

to investigate the flow through porous medium. The nanoparticles Cu, CuO and Fe<sub>2</sub>O<sub>3</sub> are considered to investigate their influence in the enhancement of heat transfer. The governing equations for the conservation of mass, momentum and energy equations are derived and non-dimensionalized using appropriate transformations. Similarity transformation is used to obtain ODE and computed the results using SQLM. The obtained results are presented graphically with various thermo-physical parameters of nanoparticles. It is noticed from the results that ternary hybrid nanofluids enhance the heat transfer effects notably than the other two cases.



**Figure 5.** (a) Cu. (b) Cu-Cu. (c) Cu-CuO-F. Nusselt number for amplitude of the wavy surface  $\phi_3$ .

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