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Analytical Approach to Nonlinear Oscillatory Flow Through a Semi-Infinite Vertical Permeable Wall in a Porous Medium with MHD Mixed Convection

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Abstract

The present work illustrates mixed MHD convection flow of an unstable two-dimensional viscous incompressible fluid flow in a porous material under the following conditions: semi-infinite vertical permeable wall support (i) There is a constant mean oscillation in the suction velocity normal to the wall throughout time. (ii) A constant mean velocity of the free stream; (iii) a constant wall temperature; and (iv) a somewhat high temperature differential between the wall and the free stream, which generates free convection currents. Conservation of mass, energy and momentum are used to obtain the governing equations. These equations are non-linear in nature and connected with one another. The nonlinear differential equations are non-dimensionalized employing non-dimensionalized parameters such as Prandtl number Pr, Grashof number Gr, Eckert number Ek, Hartmann number Mh, and Viscous ratio. An analytical method is used to solve approximate solutions for nonlinear PDEs utilizing the double regular perturbation technique. The numerical results for temperature, skin friction, heat flow, velocity, and skin friction are computed for various parameters and are demonstrated to be in good agreement with previous findings.

Keywords: *Linear Stability Analysis, Magneto Hydrodynamics (MHD), Porous Media*

1.0 Introduction

Blood flowing via a tube, geothermal power storage tanks, wind energy, waves from the tides, and improved oil extraction, drying of porous solids, thermal insulation, underground energy transport, packed-bed catalytic reactors, nuclear reactor cooling and many more applications of mass and heat transfer problems have emerged in recent years. Shojaefard[1] studied numerically about the control of flow by injection and suction. The flow is studied on a subsonic airfoil provided with suction and injection, the impact of aerodynamic co-efficient has been discussed. Uwanta *et al.*,[2] investigated the effect of injection or suction on the unsteady hydromagnetic free convection flow of viscous reactive fluid between two porous plates maintained vertically in the presence of thermal diffusion and used the perturbation series approach to solve analytically for the stable case. Basant *et al.*,[3] considered an incompressible, viscous, thermal radiation and hydromagnetic transient free convection of electrically conducting fluid flow in a vertical channel in presence of magnetic field and studied numerically by implicit finite difference. Marco *et al.*,[4] studied numerically about an incompressible oscillatory flow

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near the wall provided with fixed rigid spheres. Faladea *et al.,*[5] investigated the effects of injection and suction on an oscillating, unstable flow that was traveling through a channel with uneven wall temperatures when a magnetic field that is transverse is present. Michael[6] investigated the thermal radiation, porosity and heat source on convective flow in an annulus which is packed by absorbent material assuming internal surface of outer cylinder to be heated. Further, velocity, temperature, Nusselt number, mass flow rate and skin-friction are obtained.

Upreti H *et al.,*[7] clearly have given the significances of the heat absorption and generation, injection and suction of Ag-water nanofluid MHD flow on a stretching flat plate in presence of Ohmic-viscous dissipation and porous medium. Runge-Kutta-Fehlberg scheme via shooting method have been implemented to solve governing equations and The impact of dimensionless factors on circulation and temperature field has been examined. Basant *et al.,*[8] examined the behaviour of thermal and hydrodynamic mixed convective fully developed flow between two parallel vertical plates in a micro porous channel with temperature jump and slip velocity at the boundaries in existence of injection/suction. Further, the pictorial representation of temperature, velocity, rate of heat transfer, volume flow rate and skin friction have been discussed. Basant *et al.,*[9] examined the consequences of creating or absorbing fluid heat on mixed convective completely established vertical flow in annulus in presence of temperature jump and velocity slip at the inner and outer surface of outer and inner cylinder respectively.

Suresh Babu *et al.,*[10] performed a combination of forced and natural convection numerical calculation, incompressible, steadyviscous liquid above an object caused by electrically conducting couple stress and the variable characteristics of the fluid in an absorbent media. Similarity transformation is used to convert PDE to ODE, non-dimensional parameters have been obtained and solved by shooting method and discussed graphically. The application of oscillatory flow is seen in drilling of oil wells and the same is investigated experimentally by considering the oscillatory flow in a U-shaped vertical circular pipe at atmospheric pressure and room temperature by Maduranga *et al.,*[11] and non-invasive instantaneous flow velocity profiles are obtained by Particle Image Velocimetry (PIV) technique. Ullah *et al.,*[12] considered stretching sheet in 2D to analyze Reiner-Philippoff fluid flow of thin films with changeable heat transmission and radiation and studied the Reiner-Philippoff fluid behavior on velocity, concentration and temperature profiles. Upreti *et al.,*[13] studied numerically the influence of viscous dissipation, Joule heating and non-linear thermal radiation on borderline layer flow of mixed convection MHD nanofluid stream on movable needle using shooting method.

Vo *et al.,*[14] examined numerically about the H_2O-based nanoparticle convective motion simulation in permeable space with Lorentz effect, control-volume finite-element method have been implemented to solve the governing equations and the impact of physical parameters have been studied. Maria Javad[15] investigated the second-grade free convection fluid in an oscillating vertical infinite cylinder. Non-dimensional have been introduced to obtain the temperature and velocity equations. Prandtl numbers and Grashof numbers on time have been discussed graphically. Himanshu Upreti *et al.,*[16] studied how injection and suction affect Ag-kerosene oil nano fluid flow on a cone in presence of thermophores, magnetic field, ohmic effect, Brownian diffusion and viscous dissipation to study the heat and mass transfer numerically and discussed about the shifting of mass Flux from injection domain to suction domain.

Abiodun *et al.,*[17] investigated the effects of suction or injection, viscid dissipation, permeability of porous materials, and magnetic field supplied with electrically guiding fluid that is incompressible over a vertical porous channel that is filled with porous materials. The governing equations have been obtained by applying the Homotopy Perturbation Method. Using visual aids, the profiles of temperature and velocity have been examined. Basavaraj *et al.,*[18] studied the impact of uniform longitudinal magnetic field stability on small disturbances created by Newtonian fluid which is electrically conducting between two parallel horizontal plates. Chebyshev collocation method have been employed to solve the governing equations. Waleed *et al*.,[19] investigated the shedding of a vortex and the velocity profile of an oscillatory flow across a parallel plate structure that is situated in a thermos-acoustic system using computational fluid dynamics of the SST k-x turbulence model. Sincomb *et al.,*[20] studied the pulsating motion of cerebrospinal fluid in a slender canal of aqueduct of sylvius connecting the fourth and third ventricles of brain. The boundary-layer approximation is used in the slender canal. The relation between the stroke length and the interventricular pressure is evaluated. Sowbhagya[21] studied the influence of the commencement of porous convection employing the cubic density connection in the Darcy-Forchheimer model. The stability eigenvalue challenge is explained by applying the Galerkin approach. Vijaya Kumara *et al.*,[22] the effects of different Rayleigh numbers were numerically investigated while taking into account air flow in a trapezoidal permeable container with a warming side at the bottom.

2.0 Mathematical Formulation

A 2D unstable, viscous Boussinesq fluid flowing over an absorbent media confined by infinite vertical porous plate is considered. Imagine that the sinusoidal suction velocities and the stream's free velocities both maintain an average constant magnitude in the horizontal direction and are faraway above the vibrating permeable plate. It is pushed through a permeable plate opposing gravity while being kept at a steady temperature. As the flow rate outspreads to indefinite in the x-direction, the flow parameters are solely dependent on t and y tension p. Assuming such estimations, the basic calculations of flow outlined below:

Equation of retention of mass:

$$v_y = 0 \tag{1}$$

Equation of retention of momentum:

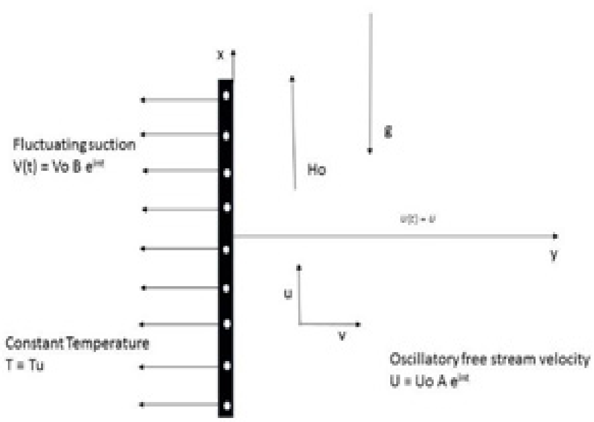

Figure 1. Schematic representation of the physical model.

$$u_t + v u_y = \upsilon\, u_y - \frac{\sigma B^2{}_0 u}{\rho_0} - \frac{1}{\rho_0} p_x + \beta \vec{g}(T - T_\infty) - \frac{\bar{\upsilon}}{\kappa} u \tag{2}$$

$$\rho_0 v_t = -p_y - \frac{\bar{\upsilon}}{\kappa} v \tag{3}$$

Equation of conservation of energy:

$$T_t + v\, T_y = -\frac{\kappa}{\rho_0 C_p} T_y + \frac{\mu}{\rho_0 C_p}(u_y)^2 + \frac{\mu_p}{\rho_0 C_p \kappa} u^2 - \frac{\sigma B^2{}_1}{\rho_0 C_p} u^2 \tag{4}$$

For the governing equations above, the following presumptions are assumed.

(i) X fluctuations among and for any physical quantity that is x-independent, pressure excluded.

(ii) The Boussiqness approximation, which states that anything other than body force, the

The density of the momentum equation is constant.

(iii) Fluid has a constant density and is incompressible.

(iv) $B_1 = H_0\ B_1 = \mu H_0$

(v) Since ρ density solely depends on temperature,

$$\rho = \rho_1[\beta(T_\infty - T) + 1] \tag{5}$$

Free-stream and suction velocities are assumed as below respectively.

$$U(t) = U_0 A\ e^{\text{int}} \tag{6}$$

$$v(t) = v_0 B\ e^{\text{int}} \tag{7}$$

Where all the physical quantities representations have been mentioned in nomenclature, suction is directed toward the porous plate when there is a negative sign. The physical structure depicted in Figure 1 serves as the basis for the boundary conditions of the system, which have the following form:

$$u = 0,\ T = T_\infty,\ y = 0,\ u \to U(t)\ \ T \to T_\infty,\ y \to \infty \tag{8}$$

By the assumptions of uniform temperature both at the plate and away from the plate, and of free stream velocity that varies over time.

The equation for free stream velocity is

$$U_t = -\frac{1}{\rho_0} p_x - \frac{\bar{\upsilon}}{\kappa} U - \frac{\sigma B^2{}_1 U}{\rho_0} \quad (9)$$

On solving (2) and (9) by eliminating pressure gradient, we obtain

$$u_t + v u_y = U_t + \upsilon u_y + \frac{\bar{\upsilon}}{\kappa}(U-u) + \frac{\sigma B^2{}_1 U}{\rho_0}(U-u) + \beta \vec{g}(T - T_\infty) \quad (10)$$

Making this equation dimensionless using

$$y^* = \frac{v_0 y}{v},\ u^* = \frac{u}{U_0},\ t^* = \frac{v^2{}_0 t}{v},\ n^* = \frac{vn}{v^2{}_0},$$
$$y^* = \frac{v}{v_0},\ \kappa = \frac{v^2 \kappa^*}{v^2{}_0},\ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \Pr = \frac{v}{\kappa},$$
$$Gr = \frac{\beta v g (T_w - T_\infty)}{U_0 v^2{}_0},\ Mh = \frac{\sigma B^2{}_0 v}{\rho_0 v^2{}_0}, Ek = \frac{U_0^2}{C_p (T_w - T_\infty)} \quad (11)$$

we get

$$u_t + v u_y = U' + Gr\theta + u_{yy} + M(U-u) - \frac{\lambda_1}{\kappa}(u - U) \quad (12)$$

$$\theta_t + v\theta_y = \frac{1}{\Pr}\theta_{yy} + Ek\left[\left(\frac{\lambda_1}{\kappa} - M\right)u^2 + (u_y)^2\right] \quad (13)$$

In the case of without dimensions, the boundary conditions are

$$u = 0\ at\ y = 0,\ u \to A\ e^{int}\ at\ y \to \infty$$
$$\theta = 1\ at\ y = 0, \theta \to 0\ at\ y \to \infty \quad (14)$$

3.0 Methodology

The non-linear linked partial differential equations (13) and (14) must be solved. We have applied one of the transformation methods as given below and also the solution

$$u(y,t) = e^{int} f(y) \quad (15)$$

$$\theta(y,t) = e^{int} g(y) \quad (16)$$

Using (15) and (16), we get the following system of equations from (12) and (13),

$$f''(y) - v_0\ B\ e^{int}\ f'(y) - \left(\frac{\lambda_1}{\kappa} + Mf\ in\right)(f(y) + U_0 A) + Grg(y) = 0 \quad (17)$$

$$\frac{1}{\Pr} g''(y) - v_0 B e^{int}\ g'(y) - in\, g(y) + Ee^{int}\left[\left(\frac{\lambda_1}{\kappa} - M\right)f^2 + f'^2\right] = 0 \quad (18)$$

The corresponding boundary constraints are

$f(y) = 0$ at $y = 0$, $f(y) \to A$ at $y \to \infty$

$$g(y) = 1 \text{ at } y = 0, g(y) \to 0 \text{ as } y \to \infty \quad (19)$$

4.0 Result and Discussion

Figure 1 shows how the non-linear analysis of a 2-Dl unstable Bossinesq viscid fluid using absorbent media confined by an infinite vertical permeable wall is affected by a transverse magnetic field. The governing equations for such a physical configuration result in a non-linear partial differential equation that is subjected to fluctuating perturbed free stream velocity far away from the permeable wall while maintaining a fluctuating suction velocity with constant temperature at the vertical permeable wall. The non-dimensional values that give birth to the non-dimensional parameters Pr, Gr, Mh, Ek, and are used to

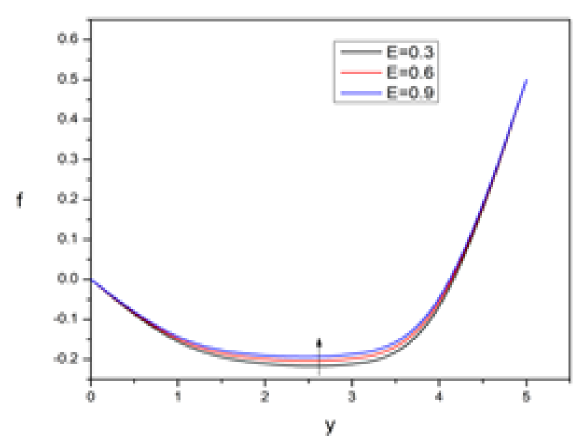

Figure 2. Velocity against y considering different amounts of *Eh* and Pr = 0.71, Mh = 5, K = 0.5, *Ek* = 0.01, ε = 0.1

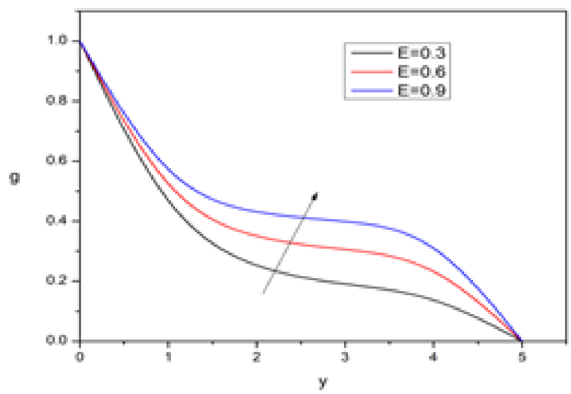

Figure 3. Velocity versus vconsidering different amounts of Eckert number Gr and the Pr = 0.71, Mh = 5, K = 0.5, *Ek* = 0.01, ε = 0.1

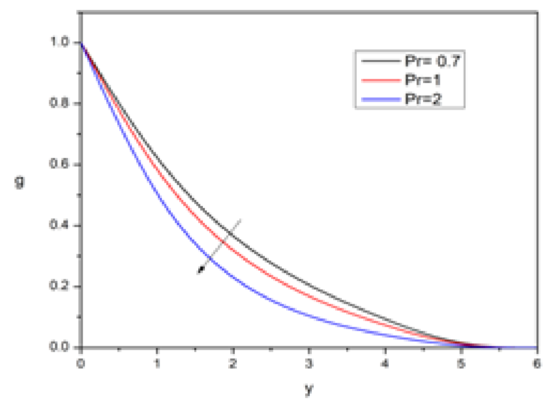

Figure 5. Velocity versus v considering different amounts of Prandtl number Pr and the other parameters Pr = 0.71, Mh = 5, K = 0.5, *Ek* = 0.01, ε = 0.1

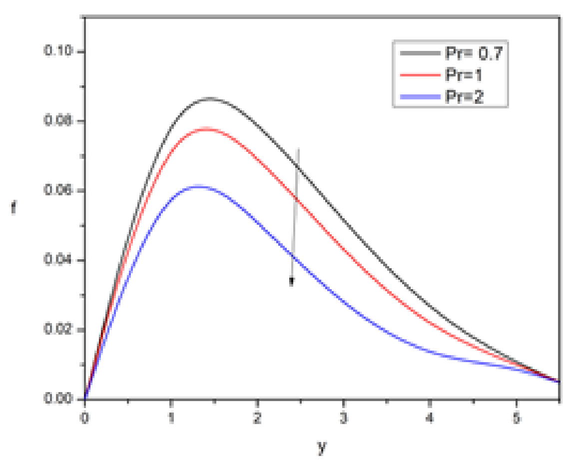

Figure 4. Temperature versus y considering different amounts of Prandtl number Pr and the other parameter. Pr = 0.71, Mh = 5, K = 0.5, *Ek* = 0.01, ε = 0.1

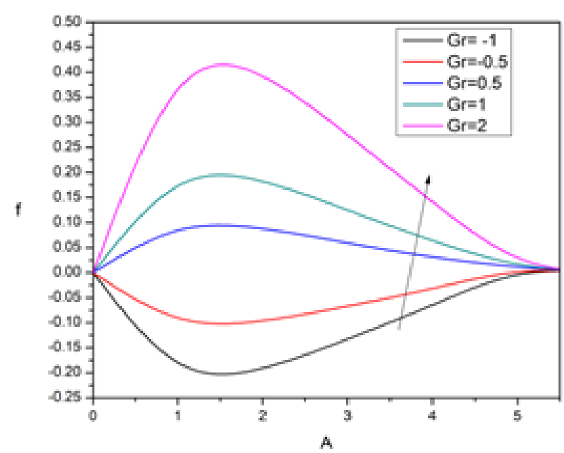

Figure 6. Velocity versus v considering different amounts of Grashof number Gr and the other parameters Pr = 0.71, Mh = 5, K = 0.5, *Ek* = 0.01, ε = 0.1

non-dimensionalise these partial differential equations. We use the twofold perturbation approach to convert non-linear PDE to linear ODE. Analytical solutions for velocity and temperature are discovered. Heat and momentum are computed numerically for a range of values of the non-dimensionalized factors that are part of the solution. Temperature and velocity variations are shown in Figure 2 for Gr and Eckert numbers Ek = 0.01 and Ek = 0.03. The vertical permeable wall cools more quickly when Ek is constant because of stronger circulation flows, which increase the average air speed.

Physically, this can be explained as follows. These results are useful for experimental verification. The same behavior may be seen when the value of Ek is set to 0.03 as seen in Figure 4. Natural forces of convection are moving the plate away from itself and into the free stream as the plate cools and because it is moving upward, the natural convection currents cause increase the mean velocity.

The velocity fluctuation for different negative values of the Grashof number Gr is shown in Figures 6 and 8. This suggests that as the plate heats up more than viscous heat dissipation increases, the mean air velocity

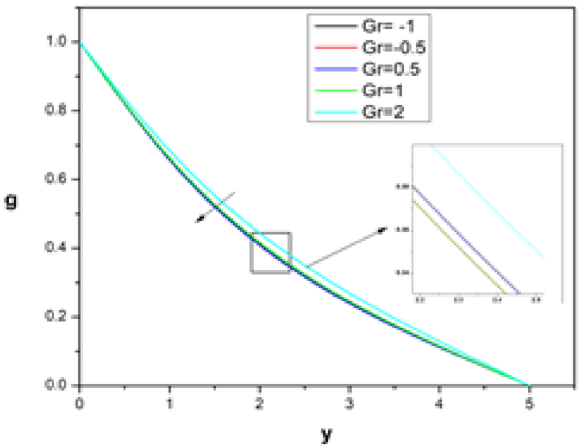

Figure 7. Velocity versus y Velocity versus considering different amounts of Grash of number Gr and the other parameters considering different amounts of Grash of number and the other parameters Pr = 0.71, Mh = 5, K = 0.5, *Ek* = 0.01, ε = 0.1

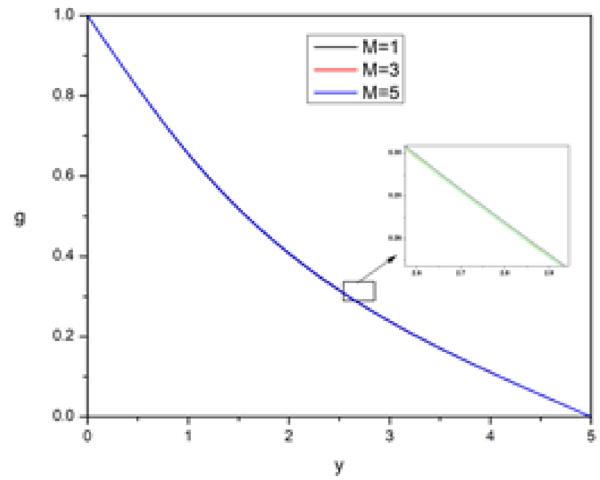

Figure 9. Temperature against v considering different amounts of Hartmann number Mh and the other parameters Pr = 0.71, Mh = 5, K = 0.5, *Ek* = 0.01, ε = 0.1

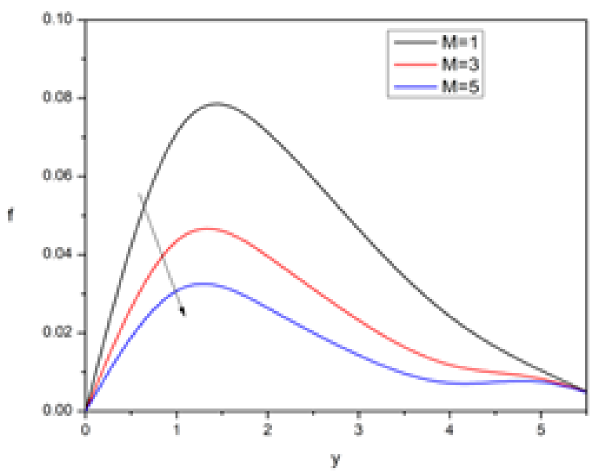

Figure 8. Temperature versus v considering different amounts of Hartman number Mh and the other parameters Pr = 0.71, Mh = 5, K = 0.5, *Ek* = 0.01, ε = 0.1

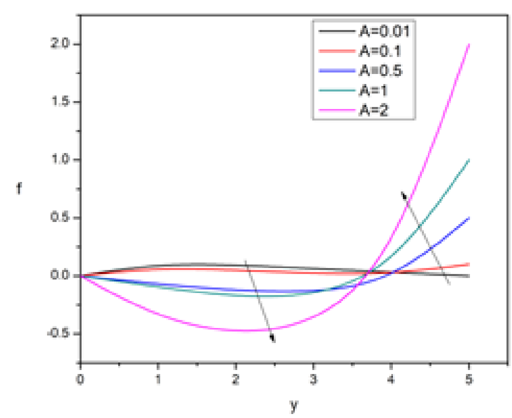

Figure 10. Temperature against v considering different amounts of A and Pr = 0.71, Mh = 5, K = 0.5, *Ek* = 0.01, ε = 0.1

is negative and decreases. This is physically possible since free convection currents running in the direction of the vertical permeable wall are now posing a challenge to the upward movement of air, both near and far from it, causing a decrease in mean velocity. The mean air flow is reversed when free convection currents heat the vertical permeable wall. Plots of temperature against y for a range of positive Grashof number Gr and Eckert number values are shown in Figures 3 and 5. Similarly, temperature vs. y shows the opposite tendency for various negative values of the Grashof number. Gr, in addition to the Eckert numbers expressed in Figures 7 and 9 as Ek = 0.01 and Ek = 0.03. This practically translates to higher heating of the vertical permeable wall. To understand how the changes

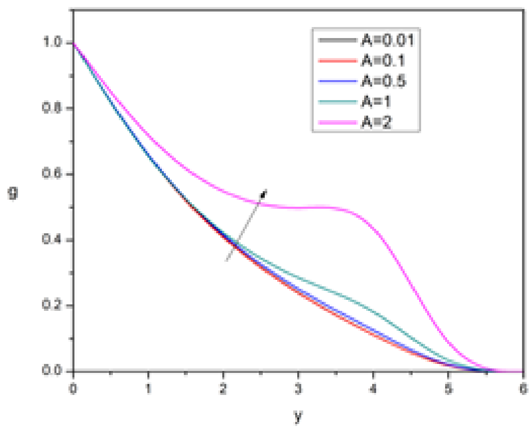

Figure 11. Velocity against considering different amounts of and Gr = 5, Mh = 5, K = 0.5, *Ek* = 0.01, ε = 0.1

in mean temperature and velocity for different fluids as seen in Figures 10 to 11 indicate that Pr has increased.

The velocity variance for a variety of fluids, from air to mercury, is displayed in Figures 10. For modest Prandtl values, 0.71 to 3, there is a significant shift in velocity that contributes to the cooling of the vertical permeable wall. Due to viscous dissipation, there is comparatively little variation in velocity when the Prandtl number is greater than 3. The behavior of temperature for different fluids is shown in Figure 11. Due to the vertical permeable wall's improved cooling, Pr = 0.71 to Pr = 7. For distinct Eckert number values, E = 0.01 and Ek = 0.03, Gr is greater than zero. For increasing Prandtl numbers for the constant value Ek = 0.01 in Figure 11.

5.0 References

1. Shojaefard MH, Noorpoor AR, Avanesians A, Ghaffarpour M. Numerical investigation of flow control by suction and injection on a subsonic airfoil. American Journal of Applied Sciences. 2005; 2(10):1474-80. https://doi.org/10.3844/ajassp.2005.1474.1480
2. Uwanta IJ, Hamza MM. Effect of suction/injection on unsteady hydromagnetic convective flow of reactive viscous fluid between vertical porous plates with thermal diffusion. Hindawi Publishing Corporation. International Scholarly Research Notices. 2014; 980270:14. https://doi.org/10.1155/2014/980270
3. Jhaa BK, Isahb BY, Uwantab I J. Combined effect of suction/injection on MHD free-convection flow in a vertical channel with thermal radiation. Ain Shams Engineering Journal. 2016. https://doi.org/10.1016/j.asej.2016.06.001
4. Mazzuoli M, Vittori G. Transition to turbulence in an oscillatory flow over a rough wall. J Fluid Mech. 2016; 792:6797. https://doi.org/10.1017/jfm.2016.61
5. Faladea JA, Ukaegbub JC, Egereb AC, Adesanyab SO. MHD oscillatory flow through a porous channel saturated with porous medium. Alexandria Engineering Journal. 2016. https://doi.org/10.1016/j.aej.2016.09.016
6. Oni MO. Combined effect of heat source, porosity and thermal radiation on mixed convection flow in a vertical annulus: An exact solution. Engineering Science and Technology, An International Journal. 2017; 20(2):518-27. https://doi.org/10.1016/j.jestch.2016.12.009
7. Upreti H, Pandey AK, Kumar M. MHD flow of Ag-water nanofluid over a flat porous plate with viscous Ohmic dissipation, suction/injection and heat generation/absorption. Alexandria Engineering Journal. 2018; 57(3):1839-47. https://doi.org/10.1016/j.aej.2017.03.018
8. Jha BK, Aina B. Role of suction/injection on steady fully developed mixed convection flow in a vertical parallel plate microchannel. Ain Shams Engineering Journal. 2018; 9(4):747-55. https://doi.org/10.1016/j.asej.2016.05.001
9. Jha BK, Oni MO, Aina B. Steady fully developed mixed convection flow in a vertical micro-concentric annulus with heat generating/absorbing fluid: An exact solution. Ain Shams Engineering Journal. 2018; 9(4):1289-301. https://doi.org/10.1016/j.asej.2016.08.005
10. Suresh Babu R, Rushi Kumar B, Dinesh PA. Effects of mixed convection on the oscillatory flow of a couple stress fluid through a vertical plate with variable fluid properties. Defect and Diffusion Forum. 2018; 388:328-43. https://doi.org/10.4028/www.scientific.net/DDF.388.328
11. Amaratunga M, Rabenjafimanantsoa HA, Time RW. Comparison of oscillatory flow conditions in Newtonian and non-Newtonian fluids using PIV and high-speed image analysis. Flow Measurement and Instrumentation. 2019; 70:101628. https://doi.org/10.1016/j.flowmeasinst.2019.101628
12. Ullah A, Alzahrani E, Shah Z, Ayaz, M, Islam, S. Nanofluids thin film flow of Reiner-Philippoff fluid over an unstable stretching surface with Brownian motion and thermophoresis effect. Coatings. 2019; 9(1):21. https://doi.org/10.3390/coatings9010021

13. Upreti H, Kumar M. Influence of non-linear radiation, Joule heating and viscous dissipation on the boundary layer flow of MHD nano fluid flow over a thin moving needle. Multidiscipline Modeling in Materials and Structures. 2019; 16(1):208–24. https://doi.org/10.1108/MMMS-05-2019-0097
14. Vo DD, Shah Z, Sheikholeslami M, Shafee A, Nguyen TK. Numerical investigation of MHD nanomaterial convective migration and heat transfer within a sinusoidal porous cavity. Physica Scripta. 2019; 94(11):115225. https://doi.org/10.1088/1402-4896/ab2ced
15. Javaid M, Imran M, Imran MA, Khan I, Nisar KS. Natural convection flow of a second-grade fluid in an infinite vertical cylinder. Second-Gradeeports. 2020; 8327. https://doi.org/10.1038/s41598-020-64533-z
16. Upreti H, Pandey AK, Kumar M. Thermophoresis and suction/injection roles on free convective MHD flow of Ag–kerosene oil nanofluid. Journal of Computational Design and Engineering. 2020; 7(3):386–96. https://doi.org/10.1093/jcde/qwaa031
17. Ajibade AO, Umar AM, Kabir TM. Analytical study on effects of viscous dissipation and suction/injection on a steady MHD natural convection couette flow of heat generating/absorbing fluid. Advances in Mechanical Engineering. 2021; 13(5):1–12. https://doi.org/10.1177/16878140211015862
18. Basavaraj MS, Shobha T, Aruna AS, Kumar V. MHD instability of the pressure-driven plane laminar flow in the presence of the uniform coplanar magnetic field: Linear stability analysis. Heat Transfer. 2021. https://doi.org/10.1002/htj.22148
19. AlmukhtarAllafia W, Al Zahrah Mohd Saata F, Maoc X. Fluid dynamics of oscillatory flow across parallel-plates in standing-wave thermoacoustic system with two different operation frequencies. Engineering Science and Technology, An International Journal. 2021; 24:41-9. https://doi.org/10.1016/j.jestch.2020.12.008
20. Sincomb S, Coenen W, Sanchez AL, Lasheras JC. A model for the oscillatory ow in the cerebral aqueduct. arXiv:2201.07568v1, Physics.flu-dyn. 2022. https://arxiv.org/abs/2201.07568
21. Sowbhagya. Outlook of density maximum on the onset of Forchheimer-Bénard convection with through flow. Journal of Mines, Metals and Fuels. 2022; 70(8A):32–40. https://doi.org/10.18311/jmmf/2022/32007
22. Vijaya Kumara VM, Aswatha, Banu Prakash Reddy V, Amit Datta D, Balaji V, Ashik AV. A numerical investigation of natural convection in a porous enclosure with lower wall heating. Journal of Mines, Metals and Fuels. 2023; 70(10A):195–201. https://doi.org/10.18311/jmmf/2022/31225

Nomenclature

Gr = Grashof Number

g = Acceleration due to gravity

Pr = Prandtl number

Mh = Hartmann number

Ek – Eckert number

n = Frequency

κ = Porous medium Permeability

P = Pressure

(u, v) = components of velocity along x and y directions

t = time

U – Dimensional free stream velocity

T_w = Temperature at the wall

T = Dimensional temperature

T_∞ = Temperature of the fluid far away from the wall

u_o = Mean velocity

v_o = Suction velocity

U_o = a constant velocity

u_1 = Fluctuating part of the velocity

$|B|$ = Amplitude of surface drag

q = Heat flux rate

$|Q|$ = Strength of the heat flux

θ = Dimensionless temperature

θ_0 = Mean temperature

β = Phase of the heat flux

θ_1 = Fluctuating part of the temperature

α = Phase of the skin friction

ρ = Density of the fluid

μ = Coefficient of viscosity

λ = Viscosity ratio

τ = Skin friction

β_0 = Coefficient of thermal expansion

(x,y) = Space coordinators

ρ_0 = Reference density

ν = Kinematic viscosity e