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On Edge Irregularity Strength of Mycielskian of Paths and Cycles

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Abstract

For a graph G having no loops and parallel edges, a labeling on the vertex set of $G,\Psi:V(G)\rightarrow\{1,2,\ldots,\alpha\}$ is refers to α-labeling. Let ab∈G be an edge. Then the weight the edge ab is $z_\Psi(ab)=\Psi(a)+\Psi(b)$. An α-labeling on the vertex set of G is refers to be an edge irregular α-labeling of G if $z_\Psi(a)\neq z_\Psi(b)$, where $a\neq b$ in G. The least number α for which the graph G has an edge irregular α-labeling is referred to the edge irregularity strength of G, written es(G). The edge irregularity strength of Mycielskian of paths and cycles is computed.

Keywords: *Edge Irregularity Strength, Mycielskian of Paths and Cycles*

1.0 Introduction

In a simple connected graph *G*, where *V(G)* and *E(G)* represent the vertex set and edge set, allocating positive integers to a set of vertices (points), edges (lines), or both, while meeting specific constraints, is referred to as *graph labeling*. Graph labeling has various applications in mines and mining operations to model and solve different problems in the mining industry, such as resource allocation and scheduling, safety and risk management, supply chain management, geological mapping, and exploration, etc.

For a given graph G, the edges are assigned the natural numbers so that the addition of the assignment of labels to the edges of every vertex is different. Such labeling is termed the irregular labeling of the graph G[1]. The least value among the maximum label which is assigned to an edge of a graph G is termed as the *irregularity strength* of of G, written s(G). Calculating *s(G)* seems to pose challenges, even when dealing with a graph *G* of a basic structure[1]. In their work, Ahmad *et al.*[2] proposed the concept of edge irregular α-labelings for graphs, drawing inspiration from *Chartrand et al.*[1]. A labeling Ψ on the vertices *V(G)* with values from the set {1, 2, ..., α} is refers to an α-*labeling*.

If, for any pair of distinct edges labeled as *a* and *b*, $z_\Psi(a)\neq z_\Psi(b)$, then an α-labeling on V(G) is refers to an edge irregular α-labeling of G. The minimum value of α for which the graph G possesses an edge irregular α-labeling defines the edge irregularity strength, denoted as es(G).

The construction of the Mycielskian graph for a given graph *G* is outlined in [3] as follows.

The *Mycielskian* of a graph G, represented by μ(G), where G has a vertex set $V(G)=\{a_1, a_2,\ldots,a_n\}$, is the graph

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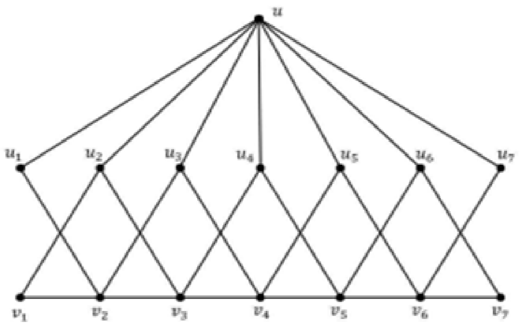

Figure 1. The Mycielskian of a path P_7.

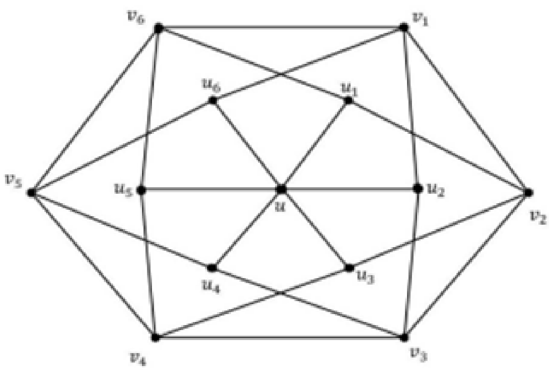

Figure 2. The Mycielskian of a cycle C_6.

with $V(\mu(G))=\{a_1, a_2,\ldots,a_n, b_1,b_2,\ldots,b_n,c\}$ such that $a_i\, a_j \in (\mu(G)) \Leftarrow a_i\, a_j \in E(G); a_i\, b_j \in E(\mu(G)) \Leftarrow a_i\, a_j\, E(G)$; and $b_i\, c \in E(\mu(G)), \forall\, i=1,2,\ldots,n$.

The Mycielskian of a path P_7 and a cycle C_6 is shown in Figures 1 and 2, respectively.

2.0 Initial Findings

Chartrand *et al.*[1] established both the upper and lower bounds for es(G) in a given graph G. They also determined the precise values of es(G) for paths P_n with order at least two, cycles C_n with order at least 3, star graph $K_{1,n}$ with order n+1 where n≥1, and double star $S_{m,n}$ with 3≤m≤n. Additionally, the authors[4,5] determined the edge irregularity strength for Toeplitz graphs and the corona product of graphs with paths, respectively. The exact value of es(μ(G)), where G is disjoint union of graphs is determined[6]. The es(μ(G)), where G is the sunlet graph is computed[7]; es(μ(L(G))) and es(μ(L_c (G)), where L(G) is the line graph and L_c (G) is the line cut-vertex graph of G, respectively[8]. For more information on finding the irregularity strength of graphs, see [9-20].

Inspired by the aforementioned research, we calculate es(μ(G)), with G representing both a path and a cycle.

The following theorem presented in[2] facilitates the exploration of either the precise value or the limits of es(G), as it gives the minimum value of es(G) for a given graph G.

Theorem 2.1 Consider Δ(G) to be the maximum degree within a simple graph *G*. Then

$$es(\mathrm{G}) \geq \max\left\{\left\lceil\frac{|E(G)|+1}{2}\right\rceil, \Delta(\mathrm{G})\right\}$$

3.0 Edge Irregularity Strength in the Mycielski Transformation of Paths

This section focuses on identifying the edge irregularity strength of Mycielski transformation of paths. The following result in[2] is well known.

Theorem 3.1 Consider C_n as a cycle of order on n≥3. Then

$$es(C_n) = \begin{cases} \left\lceil\frac{n}{2}\right\rceil & \text{for } n \equiv 1(\mathrm{mod}4) \\ \left\lceil\frac{n}{2}\right\rceil + 1 & \text{otherwise} \end{cases}$$

With the aid of Theorem 3.1, we establish es(μ(P_n)),n≥2 in the subsequent theorem.

Using Theorem 3.1, we find es(μ(P_n)),n≥2, in the next theorem.

Theorem 3.2 Let G=P_n be a path of order n≥2. Then es(μ(G))=[2n-1] for n=2,3.

Furthermore,

$$\lceil 2n-1\rceil \leq es(\mu(G)) \leq \left\lceil\frac{5n-4}{2}\right\rceil \text{ for } n \equiv 2(\mathrm{mod}2);$$

$$\lceil 2n-1\rceil \leq es(\mu(G)) \leq \left\lceil\frac{5n-3}{2}\right\rceil \text{ for } n \equiv 1(\mathrm{mod}2), n \neq 3.$$

Proof. Consider *G* to be a path P_n be a path of order n≥2. We have the cases as mentioned below.

Case 1: For G=P_2, G≅C_5. By Theorem 3.1, es(μ(G))=3=⌈2n-1⌉.

Case 2: For G=P_3, let V(μ(G)) and E(μ(G)) of μ(G) be as follows:

$$V(\mu(G)) \; = \{x, x_i: 1 \le i \le 3\} \cup \{y_j: 1 \le j \le 3\};$$

$$E(\mu(G)) \; = \{xx_i: 1 \le i \le 3\} \cup \{y_j y_{j+1}: 1 \le j \le 2\} \cup \{x_1y_2, x_3y_2, x_2y_1, x_2y_3\}$$

$$\cup \{x_1y_2, x_3y_2, x_2y_1, x_2y_3\}$$

Hence, |V(μ(G))|=7, |E(μ(G))|=9, and Δ(μ(G))=4.

According to Theorem 2.1, es(μ(G)) is guaranteed to be at least max{5, 4}. Thus es(μ(G))=5=⌈2n-1⌉.

Finally, it is enough to show the presence of an edge irregular ⌈2n-1⌉=5-labeling to prove the equality.

Let Ψ:V(μ(G))→{1,2,..,5} be a labeling on V(μ(G)) such that Ψ(x)=5; $\Psi(x_i)=i+2$ for

$1 \le i \le 3$; $\Psi(y_j)=1$ for $1 \le j \le 2$; $\Psi(y_3)=2$.

The edges carry the following weights:

$$z_\Psi(xx_i) = i + 7 \quad \text{for } 1 \le i \le 3;$$

$$z_\Psi(x_iy_{i+1}) = 2(i + 1) \quad \text{for } 1 \le i \le 2;$$

$$z_\Psi(x_3y_1) = 6; w_\phi(x_3y_3) = 7;$$

$$z_\Psi\left(y_jy_{j+1}\right) = j + 1 \quad \text{for } 1 \le j \le 2.$$

Clearly, every pair of distinct edges has a different edge weight. Therefore, es(μ(G))=⌈2n-1⌉=5.

Figure 3 shows the edge irregularity strength of es(μ(P_3)), where the maximum degree Δ=4 and |E(μ(G))|=9.

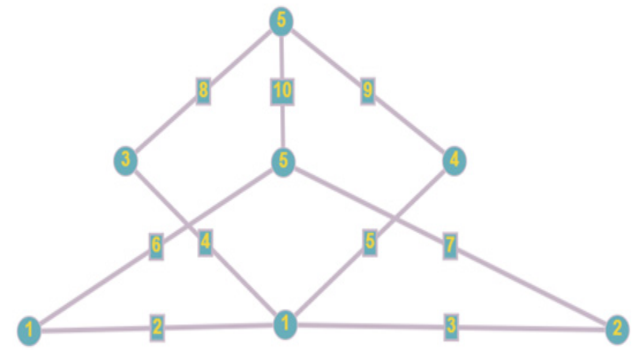

Figure 3. es(μ(P_3))

Case 3: : Let G=P_n be a path, where n≡1,2(mod2),n≠3.

Let V(μ(G)) and E(μ(G)) of μ(G) be as follows:

$$V(\mu(G)) = \{x, x_i: 1 \le i \le n\} \cup \{y_j: 1 \le j \le n\};$$
$$E(\mu(G)) = \{xx_i: 1 \le i \le n\} \cup \{y_jy_{j+1}: 1 \le j \le n-1\} \cup \{x_iy_{i+1}: 1 \le i \le n-1\} \cup \{x_iy_{i-1}: 2 \le i \le n\}.$$

Clearly, |V(μ(G))|=2n+1, |E(μ(G))|=4n-3, and Δ(μ(G))=n.

By Theorem 2.1, es(μ(G))≥max{⌈2n-1⌉,n}. As ⌈2n-1⌉ is greater than n for n≥4, it can be concluded that es(μ(G))>⌈2n-1⌉.

Let Ψ be a vertex labeling on V(μ(G)) in order to get the upper bound.

Let $\Psi: V(\mu(G)) \to \left\{1,2,\dots,\left\lceil\frac{5n-4}{2}\right\rceil\right\}$ such that

$$\Psi(x_{2i-1}) = n - 3(1 - i) \text{ for } 1 \le i \le \tfrac{n}{2};$$

$$\Psi(x_{2i}) = n - 2 + 3i \text{ for } 1 \le i \le \tfrac{n}{2};$$

$$\Psi(y_{2j-1}) = j \text{ for } 1 \le j \le \tfrac{n}{2};$$

$$\Psi(y_{2j}) = j \text{ for } 1 \le j \le \tfrac{n}{2}$$

The edges carry the following weights:

$$z_\Psi(xx_{2i-1}) = 3(n + i) - 4 \quad \text{for } 1 \le i \le \tfrac{n}{2};$$

$$z_\Psi(xx_{2i}) = 3(n - 1 + i) \quad \text{for } 1 \le i \le \tfrac{n}{2};$$

$$z_\Psi\left(y_jy_{j+1}\right) = j + 1 \quad \text{for } 1 \le j \le n - 1;$$

$$z_\Psi(x_iy_{i+1}) = 2i - 1 + n \quad \text{for } 1 \le i \le n - 1;$$

$$z_\Psi\left(y_jy_{j+1}\right) = 2j + n \quad \text{for } 1 \le j \le n - 1.$$

Clearly, every pair of distinct edges has a different edge weight. Hence Ψ is an edge irregular labeling with [(5n-4)/2] labels, that is, es(G)≤[(5n-4)/2] for n≡2(mod2).

Case 4: For n≡1(mod2),n≠3, in accordance with Theorem 2.1, es(μ(G))>⌈2n-1⌉.

For the upper bound, let

$\Psi: V(\mu(G)) \to \left\{1,2,\dots,\left\lceil\frac{5n-3}{2}\right\rceil\right\}$ be the vertex labeling on V(μ(G)) such that

$\Psi(x)=2n-1;\ \Psi(x_{2i-1})=n-3(1-i)$ for $1\le i\le\left\lceil\frac{n}{2}\right\rceil$;

$\Psi(x_{2i})=n-2+3i$ for $1\le i\le\left\lfloor\frac{n}{2}\right\rfloor$;

$\Psi(y_{2j-1})=j$ for $1\le j\le\left\lceil\frac{n}{2}\right\rceil$;

$\Psi(y_{2j})=j$ for $1\le j\le\left\lfloor\frac{n}{2}\right\rfloor$.

The edges carry the following weights:

$$z_\Psi(xx_{2i-1})=3(n+i)-4\ \text{ for } 1\le i\le\left\lceil\frac{n}{2}\right\rceil;$$

$$z_\Psi(xx_{2i})=3(n-1+i)\ \text{ for } 1\le i\le\left\lfloor\frac{n}{2}\right\rfloor;$$

$$z_\Psi\left(y_jy_{j+1}\right)=j+1\ \text{ for } 1\le j\le n-1;$$

$$z_\Psi(x_iy_{i+1})=2i-1+n\ \text{ for } 1\le i\le n-1;$$

$$z_\Psi\left(y_jy_{j+1}\right)=2j+n\ \text{ for } 1\le j\le n-1.$$

Clearly, every pair of distinct edges has a different edge weight. Hence Ψ is an edge irregular labeling with ⌈(5n-3)/2⌉ labels, that is, es(G)≤⌈(5n-3)/2⌉ for n≡1(mod2),n≠3. Hence, the proof.

4.0 Edge Irregularity Strength of Mycielskian of Cycles

This section focuses on identifying the edge irregularity strength of Mycielski transformation of cycles.

In the next theorem, we find es(μ(C_n)),n≥3.

Theorem 4.1 Consider G to be a cycle C_n, n≥3. Then,

$$\left\lceil\frac{4n+1}{2}\right\rceil\le es(\mu(G))\le 12i \text{ for } n=2(1+i), i\ge 1.$$

$$\left\lceil\frac{4n+1}{2}\right\rceil\le es(\mu(G))\le 12i-3 \text{ for } n=2i+1, i\ge 1.$$

Proof. Consider G to be a cycle C_n, n≥3. Initially, we establish the result for n=4.

For G=C_4, let V(μ(G)) and E(μ(G)) of μ(G be as follows:

$$V(\mu(G))\ =\{x,x_i\colon 1\le i\le 8\};$$

$$\begin{aligned}E(\mu(G))\ &=\{x_1x_4,x_ix_{i+1}\colon 1\le i\le 3\}\\ &\cup\{xx_i\colon 5\le i\le 8\}\\ &\cup\{x_1x_6,x_1x_8,x_2x_5,x_2x_7,x_3x_6,x_3x_8,x_4x_5,x_4x_7\}\end{aligned}$$

Hence |V(μ(G))|=9, |E(μ(G))|=16, and Δ(μ(G))=4.

According to Theorem 2.1, es(μ(G)) is guaranteed to be at least max{9,4}.

Hence es(μ(G))≥9=⌈(4n+1)/2⌉.

In conclusion, demonstrating the existence of an edge irregular 12- labeling is sufficient to establish equality.

Let ϕ:V(μ(G))→{1,2,…,5} be labeling on V(μ(G)) such that

$\Psi(x)=12;\ \Psi(x_i)=i$ for $1\le i\le 2$;

$\Psi(x_i)=(2+i)-2j$ for $3\le i\le 4, 0\le j\le 1$;

$\Psi(x_5)=2, \Psi(x_6)=11, \Psi(x_7)=2, \Psi(x_8)=10.$

The edges carry the following weights:

$$z_\Psi(xx_5)=14, z_\Psi(xx_6)=23,$$

$$z_\Psi(xx_7)=18, z_\Psi(xx_{10})=22;$$

$$z_\Psi(x_1x_2)=3, z_\Psi(x_2x_3)=7,$$

$$z_\Psi(x_3x_4)=9, z_\Psi(x_1x_4)=5;$$

$$z_\Psi(x_1x_6)=11, z_\Psi(x_2x_5)=4,$$

$$z_\Psi(x_3x_6)=16, z_\Psi(x_4x_5)=6;$$

$$z_\Psi(x_1x_8)=11, z_\Psi(x_2x_7)=8,$$

$$z_\Psi(x_3x_8)=15, z_\Psi(x_4x_7)=10.$$

Clearly, every pair of distinct edges has a different edge weight. Therefore, es(μ(G))=12.

Let G=C_n be a cycle of order n≥5.

$$V(\mu(G))\ =\{x,x_i\colon 1\le i\le 2\mathrm{n}\};$$

$$\begin{aligned}E(\mu(G))\ &=\{x_1x_n,x_ix_{i+1}\colon 1\le i\le \mathrm{n}-1\}\\ &\cup\{xx_i\colon \mathrm{n}+1\le i\le 2\mathrm{n}\}\\ &\cup\left\{\begin{matrix}x_1x_{n+2},x_2x_{n+1},x_ix_{2(n-1)-j}\colon\\ 3\le i\le n, 0\le j\le n-3\end{matrix}\right\}\\ &\cup\left\{\begin{matrix}x_1x_{2n},x_2x_{2n-1},x_ix_{2n-j}\colon\\ 3\le i\le n, 0\le j\le n-3\end{matrix}\right\}\end{aligned}$$

Case 1: : For n=2(i+1),i≥2,

|V(μ(G))|=2n+1, |E(μ(G))|=4n, and Δ(μ(G))=n.

By Theorem 2.1, es(μ(G))≥max{[(4n+1)/2],n}. Since [(4n+1)/2]>n for any n=2(i+1),i≥2,

es(μ(G))≥[(4n+1)/2].

Let Ψ:V(μ(G))→{1,2,…,12i},i≥2, such that

$\Psi(x)=n; \Psi(x_n)=-1+n;$

$$\Psi(x_i) = i \text{ for } 1 \le i \le n-1;$$

$$\Psi(x_{n+1}) = n;$$

$$\Psi(x_{n+2}) = 2n-1;$$

$$\Psi(x_{n+3}) = 2n+1;$$

$$\Psi(x_{n+4}) = 2(n+2);$$

$$\Psi(x_{n+i+4}) = 2n+4(1+i) \text{ for } 1 \le i \le n-4.$$

The edges carry the following weights:

$$z_\Psi(x_1x_n) = n;$$

$$w_\phi(x_ix_{i+1}) = 2i+1 \text{ for } 1 \le i \le n-2;$$

$$z_\Psi(x_ix_{i+1}) = 2i \text{ for } i = n-1;$$

$$z_\Psi(xx_{n+3}) = 7(n-1);$$

$$z_\Psi(xx_{n+i+1}) = 7n+2i-11 \text{ for } 1 \le i \le 2;$$

$$z_\Psi(xx_{n+i+3}) = 7n+4i-8 \text{ for } 1 \le i \le n-3;$$

$$z_\Psi(x_1x_{n+2}) = 2n; w_\phi(x_2x_{n+1}) = n+2;$$

$$z_\Psi\left(x_ix_{(2n-2)-j}\right) = 6n-3j-17 \text{ for } 3 \le i \le n-3, 0 \le j \le n-6;$$

$$z_\Psi\left(x_ix_{n-j+3}\right) = i+1-2(j-n) \text{ for } -2 \le i \le n-1; 0 \le j \le 1;$$

$$z_\Psi(x_nx_{n+1}) = 2n-1;$$

$$z_\Psi(x_1x_{2n}) = 6n-11;$$

$$z_\Psi(x_2x_{2n-1}) = 2(3n-7);$$

$$z_\Psi\left(x_ix_{2n-j}\right) = 6(n-1)-i-2j \text{ for } 3 \le i \le n; 0 \le j \le n-3.$$

Clearly, every pair of distinct edges has a different edge weight. Therefore, es(μ(G))=12i,i≥2.

Case 2: For n=2i+1,i≥1, by Theorem 2.1, es(μ(G))≥[(4n+1)/2].

Let Ψ:V(μ(G))→{1,2,…,12i-3},i≥1, such that

$$\Psi(x) = 5n-6; \Psi(x_i) = i \text{ for } 1 \le i \le n;$$

$$\Psi(x_{n+1}) = n+1; \Psi(x_{n+2}) = 2n+1;$$

$$\Psi(x_{n+2+i}) = 2n+i+3j+2$$

$$\text{for } 1 \le i \le n-2; 0 \le j \le n-3.$$

The edges carry the following weights:

$$z_\Psi(x_1x_n) = n+1; w_\phi(x_ix_{i+1}) = 2i+1 \text{ for } 1 \le i \le n-1;$$

$$z_\Psi(xx_{n+1}) = 6n-5; w_\phi(xx_{n+2}) = 7n-5;$$

$$z_\Psi(xx_i) = (7n-3)+4j \text{ for } n+3 \le i \le 2n, 0 \le j \le n-3;$$

$$z_\Psi(x_1x_{n+2}) = 2(n+1);$$

$$z_\Psi(x_2x_{n+1}) = n+3.$$

For $n = 3$, $w_\phi(x_nx_{n+1}) = 2n+1$.

For $n = 2i+1, i \ge 2$,

$$z_\Psi(x_ix_{2(n-1)-j}) = 6n-4-3j \text{ for } 3 \le i \le n-2, 0 \le j \le n-5;$$

$$z_\Psi(x_{n-1}x_{n+3}) = 3n, z_\Psi(x_nx_{n+1}) = 2n+1;$$

$$z_\Psi(x_1x_{2n}) = 6n-8; z_\Psi(x_2x_{2n-1}) = 6n-11;$$

$$z_\Psi(x_ix_{2n-j}) = 6(n-1)-3j \text{ for } 3 \le i \le n; 0 \le j \le n-3.$$

Clearly, every pair of distinct edges has a different edge weight. Therefore, es(μ(G))=12i+3,i≥1.

5.0 Conclusions

This paper presents findings on the limits of edge irregularity strength for Mycielskian transformations applied to paths and cycles. Nevertheless, discovering the precise value or limits for the edge irregularity strength of many other important classes of graphs, such as the line graph, total graph, middle graph, etc., of Mycielskian paths and cycles remain open. Graph labeling can be applied to represent the mining operations as a graph, where vertices represent tasks or activities, and edges represent dependencies or constraints. Labels can then be assigned to vertices or edges to indicate resource requirements, processing times, or other relevant parameters. This facilitates the optimization of resource allocation and scheduling for efficient mine operations.

6.0 References

1. Chartrand G, Jacobson MS, Lehel J, Oellermann OR, Saba F. Irregular networks. Congressus Numerantium. 1988; 64:187-192.
2. Ahmad A, Al-Mushayt O, Baca M. On edge irregularity strength of graphs. Appl Math Comput. 2014; 243:607-10. https://doi.org/10.1016/j.amc.2014.06.028
3. Lin W, Wu J, Lam PCB, Gu G. Several parameters of generalized Mycielskians. Discrete Appl Math. 2006; 154:1173-82. https://doi.org/10.1016/j.dam.2005.11.001
4. Ahmad A, Baca M, Nadeem MF. On the edge irregularity strength of Toeplitz graphs. Sci Bull Politeh Univ Buchar. 2018; 78:155-62.
5. Tarawneh I, Hasni R, Ahmad A. On the edge irregularity strength of corona product of graphs with paths. Appl Math E-Notes. 2016; 16:80-7.
6. Tarawneh I, Hasni R, Asim MA, Siddiqui MA. On the edge irregularity strength of disjoint union of graphs. Ars Combinatoria. 2019; 142:239-49.
7. Salma U, Nagesh HM. On the edge irregularity strength of sunlet graph. Bull Int Math Virtual Inst. 2022; 12(2):213-17.
8. Nagesh HM, Girish VR. On edge irregularity strength of line graph and line cut-vertex graph of comb graph. Notes on Number Theory and Discrete Mathematics. 2022; 28(3):517-524. https://doi.org/10.7546/nntdm.2022.28.3.517-524
9. Ahmad A, Bokhary U, Ahtsham S, Imaran M, BaigAQ. Vertex irregular labelings of cubic graphs. Util Math. 2013; 91:287-99.
10. Ahmad A, Baca M. On vertex irregular total labeling. Ars Combin. (in press).
11. Aigner M, Triesch E. Irregular assignemnts of trees and forests. SIAM J Discrete Math. 1991; 3(4). https://doi.org/10.1137/0403038
12. Al-Mushayt O. Irregular assignemnts of trees of certain families of graph with path P2. Ars Combin. (in press).
13. Amar D, Togni O. Irregualr strength of trees. Discrete Math. 1998; 190:15-38. https://doi.org/10.1016/S0012-365X(98)00112-5
14. Bohman T, Kravitz D. On the irregularity strength of trees. J Graph Theory. 2004; 45:241-54.
https://doi.org/10.1002/jgt.10158
15. Baca M, Jendrol S, Miller M, Ryan J. On irregular total labellings. Discrete Math. 2007; 307:1378-88. https://doi.org/10.1016/j.disc.2005.11.075
16. Cammack LA, Schelp RH, Schrag GC. Irregularity strength of full d-ary trees. Congr Number. 1991; 81:113-19.
17. Dimitz JH, Garnick DK, Gyarfas A. On the irregularity strength of the mxn grid. J Graph Theory. 1992; 16:355-74. https://doi.org/10.1002/jgt.3190160409
18. Frieze A, Gould RJ, Karonski M, Pfender F. On graph irregularity strength. J Graph Theory; 2002: 41:120-37. https://doi.org/10.1002/jgt.10056
19. Nierhoff T. A tight bound on the irregularity strength of graphs. SIAM J Discrete Math. 2000; 13:313-23. https://doi.org/10.1137/S0895480196314291
20. Przybylo J. Linear bound on the irregularity strength and total vertex irregularity strength of graphs. SIAM J Discrete Math. 2008; 23:511-16. https://doi.org/10.1137/070707385