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Continuous Wavelet Transform based Crack Detection in a Metallic Shaft

Bhupal Kumar[1], Ratnesh Kumar[2] and Ramnivas Kumar[1]*

[1]*Government Engineering College, Jamui – 811313, Bihar, India; ramnivas51152@gmail.com*
[2]*Purnea College of Engineering, Purnea – 854301, Bihar, India*

Abstract

In metals, the probability of fatigue crack initiation is significant in high load and high-speed machinery. Once the crack is initiated, it will grow with time and cause critical failure to the system. The condition based monitoring or predictive maintenance of the metallic shaft is essential before the crack reaches its critical size. The presence of excessive measurement noise increases the complexity to detect the crack; hence, there is a need of crack detection algorithm to detect the crack in the presence of excessive measurement noise. In the present work a Continuous Wavelet Transform (CWT) based algorithm is used to locate the crack in metallic shaft. Finite element analysis is used to generate the Operating Deflection Shape (ODS) of the shaft. The presence of crack in the shaft creates additional slope discontinuity in the shaft's ODS. This slope discontinuity is more dominant near the first natural frequency. However, exciting the shaft near the first natural frequency is hazardous. In order to make the algorithm more robust the excitation nearby the natural frequency is avoided. The CWT-based algorithm is developed to locate the crack by using the ODS at very low excitation frequency.

Keywords: *Continuous Wavelet Transform, Metal Crack, Operating Deflection Shape, Shaft*

1.0 Introduction

Rotating machineries play an important role in many industries. Metallic shaft is the critical part of rotating machinery. In rotating machineries such as gas and steam turbines, pumps, airplanes, automobiles, generators etc. shafts are the most arduous components because the power is transmitted through the rotating shaft. Although shaft is well designed, during operation it is very sensitive to severe damage due to fatigue loading. The occurrence of damage mostly comes due to the initiation and propagation of cracks without ample warning. Cracks are any unintentional discontinuities presents in the shaft material. Cracks initiates because of fluctuating bending stresses, stress concentration due to sudden changes of shaft cross section (for providing the gear and pulley slots, key holes etc.), manufacturing faults, slag inclusions, porosity, voids etc. Under the cyclic loading these cracks grow with time and causes total shaft failure, which leads to shut down the machinery system, labour hazards, increase the total cost.

The presence of crack in the shaft ODS generates supplementary flexibility nearby the crack. This ultimately alters the dynamic behaviour of the shaft. These changes in the dynamic behaviour of the shaft are widely used for finding the crack location. The dynamic behaviour of the shaft is captured by using the vibrational sensors. Vibration-based monitoring of shaft is widely used condition monitoring technique; as it can be employed in the running conditions of machine. Several researchers have presented a detailed review of vibration-based monitoring of shaft[1-3].

Gounaris *et al.*[4] simulated a transverse surface cracked beam and detected the crack location by measuring the

*Author for correspondence

coupled response. They excited the shaft in one direction and measured the response in another direction the displacement in another direction ensures the presence of crack. Sekhar *et al.*[5] used the mechanical impedance concept to detect the crack. They compared the differences of cracked and intact beam and found that there is a major difference in the mobility of cracked and intact beam, and on the basis of that they found the damage position along the shaft length. Kumar and Singh[6] used the variance-based algorithm to locate the crack in a beam. Hadjileontiadis *et al.*[7] used kurtosis to detect the crack location and size in a beam. Adams *et al.*[8] used the change in natural frequency to locate the cracks in fibre-reinforced plastics bar. However, the change in natural frequency is sensitive to environmental noise and wear. Pandey *et al.*[9] used curvature mode shape to locate the crack in a simply supported and cantilever beam. They found that the magnitude of curvature increased at damaged position due to the reduction in flexural stiffness. Recently wavelet-based crack detection algorithms are widely used as it possesses the excellent properties to extract the slope discontinuity in the presence of noise. Kumar *et al.*[10] used discrete wavelet transform with rbio2.2 wavelet of order to locate the crack in a beam. Sekhar[11] used CWT to detect the crack in a rotor system which was not possible to detect by Fast Fourier Transform (FFT). Han *et al.*[12] used the index of wavelet packet energy rate for the crack detection in beams. Rucka and Wilde and Rucka[13] used CWT to find the damage location in plate structures and beams by using the mode shape. Kumar and Singh[14] used CWT based crack detection algorithm to locate the crack in a beam. Doucka *et al.*[15] used CWT with sym4 mother wavelet to detect the crack position by using the mode shape of the beam. They also defined an intensity factor to relate the size of the crack to the coefficients of the wavelet transform. Rucka and Wilde[16] used CWT with gaus4 mother wavelet to detect the crack position in cantilever beams by using the beam mode shape. Rucka[17] used the higher order modes of the cantilever beam to detect the damage. They used CWT with gaus4 mother wavelet. Richardson[18] presented the difference between Operating Deflection Shape (ODS) and mode shape. ODS can be defined for nonlinear and non-stationary motion of the structure and depends on external applied load, while mode shape is the inherent properties of the structure and do not depend on the forces or load applied on the structure. Babu and Sekhar[19] developed amplitude deviation curve from ODS to detect the crack position in a simple rotor system. Exciting the shaft near the natural frequency is hazardous.

In the present work, the ODS at low frequency is used as an input signal for the CWT. The CWT based algorithm is able to locate the crack by using the ODS. Presence of noise forms difficulties to detect the crack position. Because noise creates fluctuation in the ODS. Timoshenko beam theory is used to model the shaft by using the finite element method. Further the algorithm is also tested by exciting the shaft in between the first and second natural frequency. The algorithm is also able to locate the crack by using the ODS, obtained by exciting the shaft in between the first and second natural frequency.

2.0 Mathematical Model of a Cracked Shaft Element

A Timoshenko shaft is formulated using finite element method. Open transverse surface crack in the middle of the element is considered. The shaft is assumed to be non-rotating and a harmonic exciter is used to excite the shaft. The excitation is assumed in the vertical direction only. Finite element discretisation of the shaft with single crack has been shown in Figure 1.

Cracked element with 4 degrees of freedom at each node is of elemental length is shown in Figure 2.

The general equation of vibration for intact shaft is[20]

$$[M]^e\{\ddot{X}\}^e + [D_{wc}]^e\{\dot{X}\}^e + [K_{wc}]^e\{X\}^e = \{F\}^e \quad (1)$$

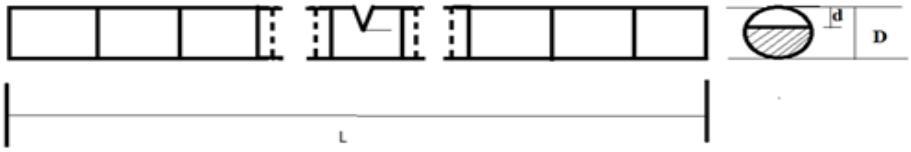


Figure 1. Finite element discretisation of the shaft with single crack.

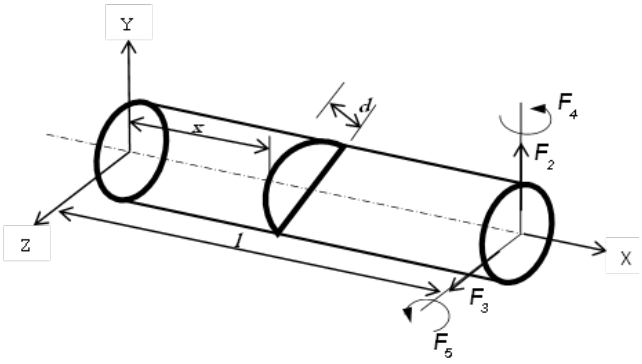


Figure 2. Shaft element with crack.

and

$$[M]^e\{\ddot{X}\}^e + [D_c]^e\{\dot{X}\}^e + [K_c]^e\{X\}^e = \{F\}^e \quad (2)$$

Where, $[M]^e$ is the elemental mass matrix; $[D]^e$ is elemental damping matrix; $[K]^e$ is elemental stiffness matrix; $\{F\}^e$ is excitation force of the shaft. $\{X\}^e$ is nodal displacement matrix of the shaft. The nodal displacement contains 4 motions, $\{X\}^e = [v\ w\ \theta\ \phi]^T$. Where, v ,w are the translational motions along y and z-axis, and θ ,ϕ are the rotational motions about the y and z-axis. The subscripts 'c' and 'wc' are used for cracked and intact shaft elements respectively. Proportional damping or Rayleigh damping is considered to mitigate the damping effect. The Damping matrix of shaft element without crack can be written as[21].

$$[D]_{wc} = a_0[M]^e + a_1[K_{wc}]^e \quad (3)$$

and with crack

$$[D]_c = a_0[M]^e + a_1[K_c]^e \quad (4)$$

Here a_0 and a_1 are the constants which can be calculated by using the natural frequencies.

The additional flexibility matrix is taken from article[22].

$$[K]^e = [T]\,[C]^{-1}\,[T]^T \quad (5)$$

where,

$$[C] = [C_0] + [C_c] \quad (6)$$

$[C_0]$ is the flexibility matrix of the element without crack, and $[C_c]$ is the flexibility matrix of the cracked element. The flexibility matrix of the cracked element can be obtained as[16].

$$[C_C]^e = \begin{bmatrix} C_{22} & 0 & 0 & 0 \\ 0 & C_{33} & 0 & 0 \\ 0 & 0 & C_{44} & C_{45} \\ 0 & 0 & C_{54} & C_{55} \end{bmatrix} \quad (7)$$

Here, the non-zero components C_{22}, C_{33}, C_{44}, C_{55} are the uncoupled coefficients and C_{45} and C_{54} are the coupled coefficients. The coupling of flexibility coefficients occurs due to the presence of crack[7]. The transformation matrix [T] is expressed as[22]

$$[T]^T = \begin{bmatrix} -1 & 0 & 0 & -l & 1 & 0 & 0 & 0 \\ 0 & -1 & l & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

The elemental matrix is combined to get the corresponding global matrix. For generating the cracked shaft response, the following parameters are used.

Table 1. Parameters for cracked shaft simulation

Parameters	Values for simulation
Dia of shaft (D)	0.01 m
Crack depth (d)	0.0035 m
Length of the shaft(l)	1 m
Modulus of elasticity (E)	2.06×
Number of finite elements	500
Location of the crack	162
Excitation frequency	10 rad/s.

The cracked and intact shaft response or Operating Deflection Shape (ODS) is shown in the Figure 3. The cracked shaft response is shown by solid line and the intact shaft response is shown by dotted line. The 500 finite element is taken over a one-meter shaft length. As the crack induced in the system than the stiffness of the system decreases, this allows shaft to deflect more. The presence of crack affects the shaft system globally but the effects dominant at the vicinity of crack.

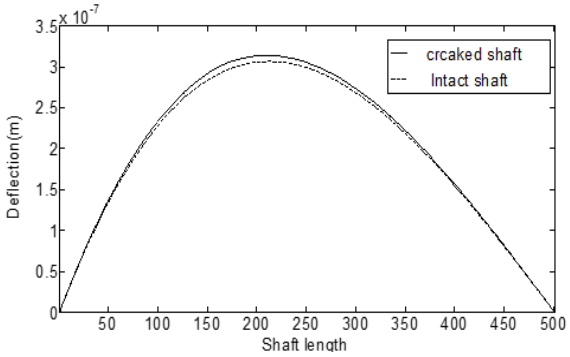


Figure 3. Shaft response at 10 rad/s.

As the crack depth increases than the amplitude, difference between the cracked and intact shaft also increases and this amplitude difference greatly depends on the excitation frequency of the exciter. The Figure 4 shows the sensitivity curve for the amplitude difference of the cracked and intact shaft. The maximum amplitude difference is taken along the y-axis and crack depth is taken along the x-axis. Therefore, the variation of the curve in the figure shows the sensitivity of the amplitude difference w.r.t crack propagation. The sensitivity of the amplitude difference at some excitation frequency is plotted in Figure 4.

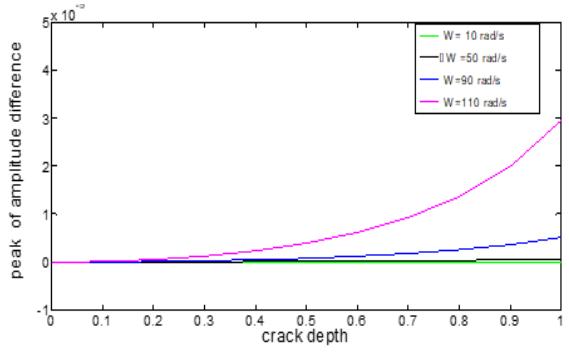


Figure 4. Variation of ODS with crack depth.

From the above Figure 4, it is seen that when the excitation frequency is less than the slope or rate of increase of the sensitivity curve is less. The curve becomes more sensitive when the excitation is above 50 rad/s. so for getting more realistic results the shaft is excited at very law excitation frequency here in this work it is taken 10 rad/s.

The variation of shaft response at different excitation frequency is shown in Figure 5. First two natural

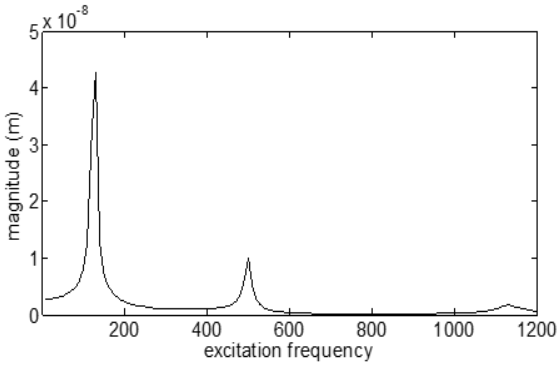


Figure 5. Variation of shaft displacement amplitude with excitation frequency.

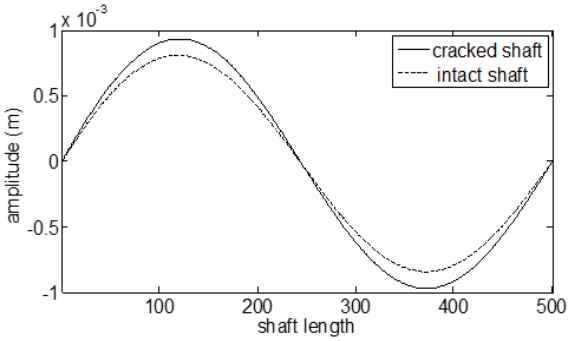


Figure 6. ODS at 480 rad/s.

frequencies of the intact shaft are found from the standard eigen value analysis. The first three natural frequencies are 126.78 rad/s, 506.93 rad/s, and 1140.02 rad/s. Sudden increase in amplitude variation is seen near the all-natural frequencies. The cracked and intact shaft response or Operating deflection shape at 480 rad/s (near the second natural frequency) is shown in Figure 6.

The crack position cannot be seen directly from the shaft response thus for extracting the information about the crack position some algorithm is needed to be applied to locate the crack position. Here, in this work, the CWT is applied to locate the crack. The details of CWT are presented in the next section.

3.0 Results and Discussions

3.1 Continuous Wavelet Transform

Wavelet has multi-resolution properties, due to this wavelet acts as a signal microscope and clearly shows the very subtle changes present in the signal.

A function Ψ(x) is said to be a wavelet if and only if its Fourier transform Ψ *(ω)*, fulfils the wavelet acceptability condition

$$\int_{-\infty}^{+\infty} \frac{|\Psi(\omega)|^2}{\omega} d\omega < \infty \quad (9)$$

This condition suggests that the average value of wavelet must be zero i.e. area underneath the curve must be zero or

$$\int_{-\infty}^{+\infty} \Psi(x)\, dx = 0 \quad (10)$$

And the Fourier transform of wavelet function at ω=0 must be zero

$$\Psi(0) = 0 \quad (11)$$

The CWT of a signal f(*x*) can be expressed as:

$$W_{u,s}(x) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(x)\, \Psi\left(\frac{x-u}{s}\right) dx \quad (12)$$

where, *u* is the translation parameter, s is the scale parameter and the variable *x* is the location along the shaft length. Translation means shifting the wavelet along the span, scaling means stretching or compressing the wavelet. Law scale means more compressed wavelet, it is used for higher frequency and better localization. Stretched wavelets helps in capturing the slowly varying changes in signal while compressed wavelets help in capturing the abrupt changes. This scaled wavelet is translated to the entire length of the signal and detects the subtle changes due to crack.

3.2 Noise Addition

For practical implementation the noise is added to the ODS at 10 rad/s of excitation frequency. Gaussian random number generator is used to generate the noise. For expressing the effect of noise in the shaft response signal, the noise level present in the signal is expressed into signal to noise (SNR) ratio. The SNR is expressed in dB as:

$$SNR = 20\log_{10}\left(\frac{\sigma_s}{\sigma_n}\right) \quad (13)$$

where, σ_s and σ_n are the standard deviation of the signal and noise respectively. Here, σ_s can be calculated by using the formula

$$\sigma_s = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

where, $\bar{x}$ is the mean value of x_i (data point of signal). For a given SNR the σ_n can be obtained and the noise with that particular variance is generated.

$$r = \sqrt{(-2.0 \times (\sigma_n{}^2) \times \log(\text{rand}(N, 1)))}$$

$$\emptyset = 2.0 \times \pi \times \text{rand}(N,1)$$

$$\text{noise} = x_0 + r \times \cos(\emptyset)$$

The generated White Gaussian Noise is added in the Figure 7.

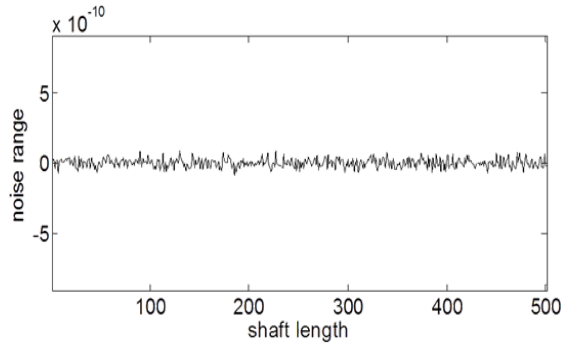


Figure 7. Noise sample.

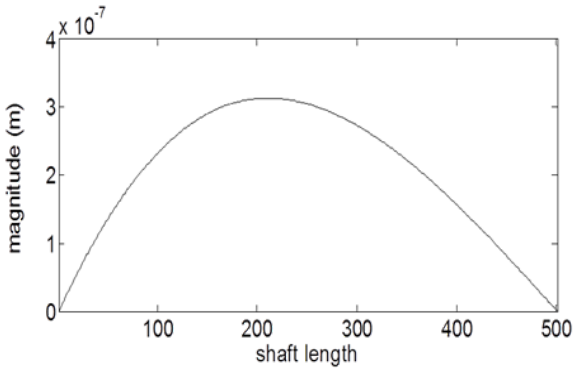


Figure 8. ODS of cracked shaft after addition of noise at the excitation frequency of 10 rad/s.

Figure 8, shows the noisy ODS of the shaft. The addition of noise suppresses the crack detection and make the crack detection more intricate. The CWT based crack detection is presented in the subsequent section.

3.3 CWT based Crack Detection

In order to test the robustness of the algorithm the cracked shaft is excited at 10 rad/s. However, the first natural frequency of the shaft is 125 rad/s. The white Gaussian noise is added to the haft ODS. The CWT is applied on the noisy ODS. The rbio2.2 wavelet is taken for the analysis. A cone of influence representing the position of the crack is shown in the CWT coefficients, as plotted in Figure 9. In order to test the noise robustness, the algorithm is tested for different noise conditions. It is also found that CWT is able to locate the crack position up to 60 dB SNR. The proposed CWT-based algorithm is unable to locate the crack below the 60 dB SNR, as crack information is suppressed due to the higher noise.

3.4 Crack Detection Algorithm near Second Mode Shape

The second natural frequency of the shaft is 506.93 rad/s. The shaft is excited at 480 rad/s to obtain the ODS. The ODS at 480 rad/s is shown in Figure 10. The white Gaussian noise is added to the simulated ODS. The noisy signal is taken as the input of the CWT-based algorithm. The CWT coefficients are plotted in the Figure 11. A ridge corresponding to the crack location is seen.

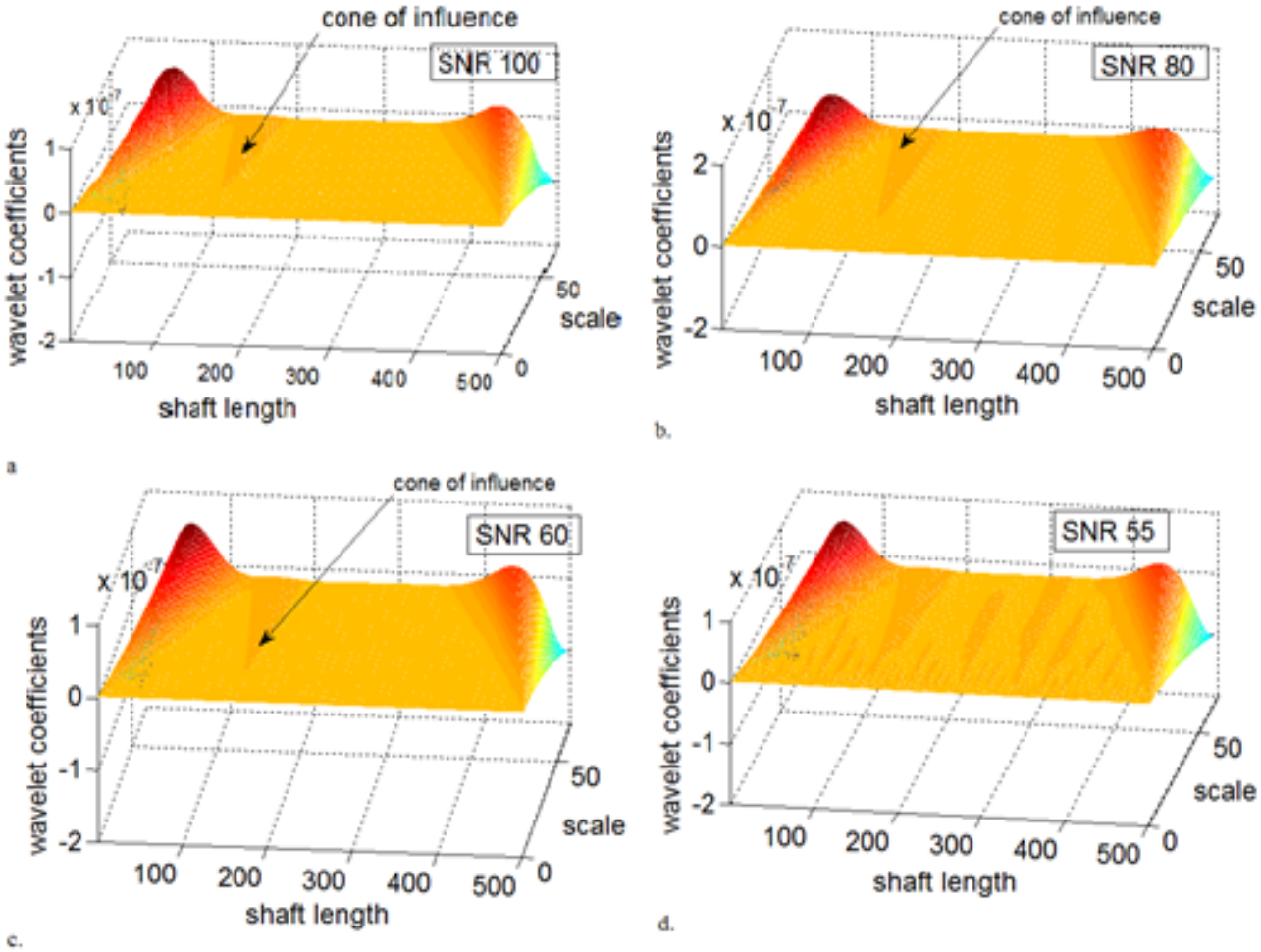


Figure 9. CWT of ODS at 10 rad/s.

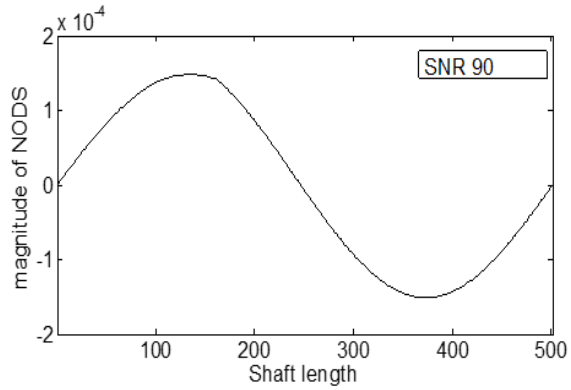


Figure 10. NODS at 480 rad/s.

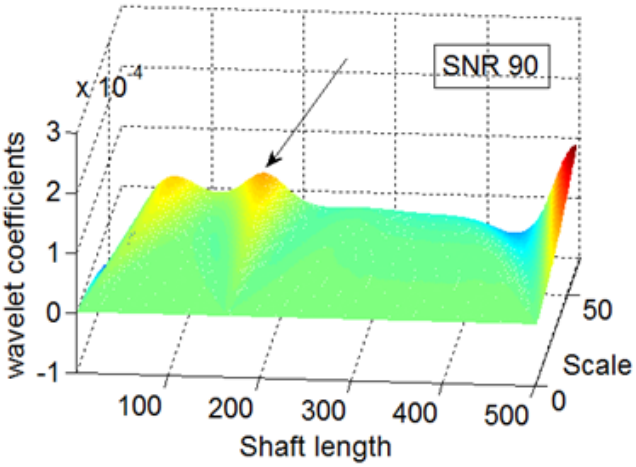


Figure 11. CWT coefficients of ODS excited at 480 rad/s.

4.0 Conclusions

Dynamic analysis of a rotor system with transverse surface crack has been presented. The Timoshenko beam theory is used to generated the ODS of the shaft. A CWT-based crack detection algorithm is used to locate the crack along the shaft length. The proposed algorithm is able to locate the crack by using the ODS. The ODS is obtained by exciting the shaft at very low excitation frequency. The proposed algorithm is able to detect the crack by using the ODS at low excitation frequency. The noise robustness of the algorithm is also tested by adding the white Gaussian noise at different SNR. The proposed algorithm is able to locate the crack at 60 SNR.

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