Journal of Mines, Metals and Fuels, *71(12C) : 154-158; 2023. DOI: 10.18311/jmmf/2023/47286*

Print ISSN : 0022-2755

Journal of Mines, Metals and Fuels

Contents available at: *www.informaticsjournals.com/index.php/jmmf*

Unique Studies of Butterfly Graph

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Abstract

Butterfly distribution plays a very important role while mining metals and fuels in various ways. Availability of metals, minerals, fuels can be found using the techniques/studies of Butterfly graph. We investigate the unique studies of Butterfly graph like its Characteristic polynomial, Complement, Graph Indices, Discrete Fourier transform of Diameter, Girth, Vertex connectivity and Edge connectivity using Fast Fourier Transform algorithm (Butterfly method), QR-Decomposition, its regular stochastic matrix and optimal prefix code of a Butterfly graph.

Keywords: Butterfly Graph, Discrete Fourier Transform, Optimal Prefix Code, QR-Decomposition, Regular Stochastic Matrix

1.0 Introduction

The industries like mining metals, minerals, fuels are required for the global economy, supplying essential raw materials to various sectors. Essential environmental study of distribution is required for the mining.

The Butterfly graph is denoted by $BF(V, E)$ we mean the graph obtained by joining two copies of the cycle graph C_3 to a common vertex.

Example. $BF(5, 6)$ a typical Butterfly graph and is

1.1 QR-Decomposition

Every $m \times n$ matrix, let's say A, can be factored into the sum of an upper (right) triangular matrix R and a matrix Q with orthonormal vectors for its columns. The QR-Decomposition of A is the product A= QR [1]. Q is a unitary matrix if A is square.

Note that one or more of the r_{kk} values computed in $r_{kk} = \|X_k\|_2$ will be zero if the columns are not linearly independent. It is necessary to change this step to calculate $r_{kk} = \|X_k\|_2$.

$$Q_k = \frac{X_k}{r_{kk}} \text{ if } r_{kk} \neq \|X_k\|_2$$

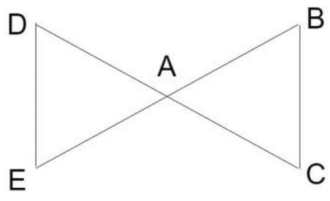
Select Q_k as any normalised vector that is orthogonal to $Q_1, Q_2, \ldots, Q_{k-1}$ if $r_{kk} = 0$.

1.2 Probability Vector

If each of the elements of a vector $V = \{v_1, v_2, \ldots, v_n\}$ is non-negative and the sum of those elements is equal to one, then vector is referred to as a probability vector.

1.3 Stochastic Matrix

A stochastic matrix is a square matrix $P = (p_{ij})$ in which each row represents a probability vector.



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1.4 Stochastic Matrix Regular

If all of the entries of a stochastic matrix P where $n \in Z^+$ [2], P^n are positive, then that matrix is said to be a regular stochastic matrix.

1.5 One-of-a-Kind Fixed Probability Vector

A single fixed probability vector $V = \{v_1, v_2, \ldots, v_n\}$ such that $VP = V$ exists for the stochastic matrix P.

1.6 Best Prefix Code

You can use the Huffman tree (also known as the optimum tree) to get a prefix code for the symbols that represent its leaves. On do this, we first apply the previously mentioned labelling process to the edges of the symbols 0 and 1. Binary sequences can then be used to identify the tree's vertices and leaves. The binary sequences that are used to identify the leaves produce a prefix code for the symbols that represent these leaves. This prefix code has the reputation of being the best prefix code[3,4,5].

2.0 The Findings and Discussion

2.1 The Characteristic Polynomial of the Butterfly

graph $BF(x) = -x^5 + 6x^3 + 4x^2 - 5x - 4$

The Complement graph of a Butterfly graph is Square and Singleton.

2.2 Graph Indices

i) Harary index 8 Stability index

ii) Cyclomatic number × Wiener index = 2×14= 28 =Detour index

iii) Hosoya index × Harary index = 12×8= 96 Molecular Topological index.

iv) Hosoya index × Stability index 12 × 8 = 96 Molecular Topological index.

v) Hosoya index = 12 = 6 × 2 = 6 × Cyclomatic number.

vi) Harary index = 8 = 4 × 2 = 4 × Cyclomatic number.

vii) Stability index 8 = 4 × 2 = 4 × Cyclomatic number.

viii) Wiener index = 14 = 7 × 2 = 7 × Cyclomatic number.

ix) Detour index = 28 = 14 × 2 = 14 × Cyclomatic number.

2.3 Topological Properties of a Butterfly Graph

Diameter = 2, Girth = 3, Vertex connectivity = 1, Edge connectivity = 2.

The Discrete Fourier transform (DFT) of Diameter, Girth, Vertex connectivity and Edge connectivity $x(n) = \{2, 3, 1, 2\}$ here $N = 4$ only then by using Fast Fourier transform (FFT) algorithm (Butterfly method), we have $X(k) = x(0)\left(w_4^k\right)^0 + x(1)\left(w_4^k\right)^1$

$$+ \; x(2)\left(w_4^k\right)^2 + x(3)\left(w_4^k\right)^3$$

when $k = 0, X(0) = 2(w_4^0)^0 + 3(w_4^0)^1 + 1(w_4^0)^2$

$$+2(w_4^0)^3 = 8$$

when $k = 1, X(1) = 2(w_4^1)^0 + 3(w_4^1)^1 + 1(w_4^1)^2$

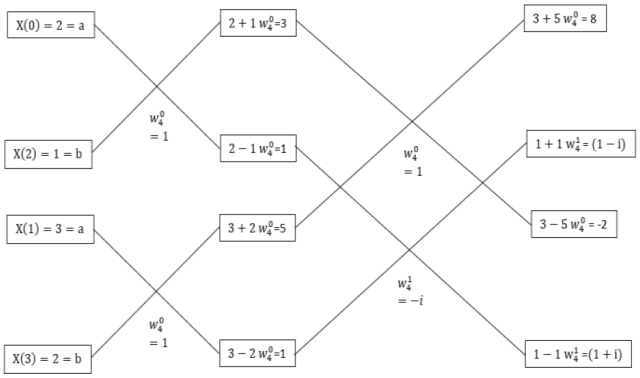
$$+ 2(w_4^1)^3 = (1 - i)$$

when $k = 2, X(2) = 2(w_4^2)^0 + 3(w_4^2)^1 + 1(w_4^2)^2$

$$+2(w_4^2)^3 = -2$$

when $k = 3, X(3) = 2(w_4^3)^0 + 3(w_4^3)^1 + 1(w_4^3)^2$

$$+2(w_4^3)^3 = (1 + i)$$



Therefore, DFT $[x(n)] = X(k) = \{8, (1 - i), -2,$

$$(1 + i)\} \text{ where, } w_4^k = \cos\left(\frac{k\pi}{2}\right) - i \sin\left(\frac{k\pi}{2}\right)$$

2.4 A Butterfly Graph's QR-Decomposition Results

Using the example from above, we have,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$X_1 = \begin{bmatrix}0\\1\\1\\1\\1\end{bmatrix}, X_2 = \begin{bmatrix}1\\0\\1\\0\\0\end{bmatrix}, X_3 = \begin{bmatrix}1\\1\\0\\0\\0\end{bmatrix}, X_4 = \begin{bmatrix}1\\0\\0\\0\\1\end{bmatrix}, X_5 = \begin{bmatrix}1\\0\\0\\1\\0\end{bmatrix}$$

$$A = QR$$

$$= [Q_1 \quad Q_2 \quad Q_3 \quad Q_4 \quad Q_5]\begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} & r_{15} \\ 0 & r_{22} & r_{23} & r_{24} & r_{25} \\ 0 & 0 & r_{33} & r_{34} & r_{35} \\ 0 & 0 & 0 & r_{44} & r_{45} \\ 0 & 0 & 0 & 0 & r_{55} \end{bmatrix}$$

First Iteration.

$$\therefore r_{11} = \|X_1\|_2 = \sqrt{4} = 2$$

$$Q_1 = \frac{X_1}{r_{11}} = \begin{bmatrix}0\\0.5\\0.5\\0.5\\0.5\end{bmatrix}$$

$$r_{12} =< X_2, Q_1 > = (1 \times 0) + (0 \times 0.5) + (1 \times 0.5) + (0 \times 0.5) + (0 \times 0.5) = 0.5$$

$$r_{13} =< X_3, Q_1 > = (1 \times 0) + (1 \times 0.5) + (0 \times 0.5) + (0 \times 0.5) + (0 \times 0.5) = 0.5$$

$$r_{14} =< X_4, Q_1 > = (1 \times 0) + (0 \times 0.5) + (0 \times 0.5) + (0 \times 0.5) + (1 \times 0.5) = 0.5$$

$$r_{15} =< X_5, Q_1 > = (1 \times 0) + (0 \times 0.5) + (0 \times 0.5) + (1 \times 0.5) + (0 \times 0.5) = 0.5$$

$$X_2 \leftarrow X_2 - r_{12}Q_1 = \begin{bmatrix}1\\-0.25\\0.75\\-0.25\\-0.25\end{bmatrix}$$

$$X_3 \leftarrow X_3 - r_{13}Q_1 = \begin{bmatrix}1\\0.75\\-0.25\\-0.25\\-0.25\end{bmatrix}$$

$$X_4 \leftarrow X_4 - r_{14}Q_1 = \begin{bmatrix}1\\-0.25\\-0.25\\-0.25\\0.75\end{bmatrix}$$

$$X_5 \leftarrow X_5 - r_{15}Q_1 = \begin{bmatrix}1\\-0.25\\-0.25\\0.75\\-0.25\end{bmatrix}$$

Similar like this, after the second, third, etc.,

$$Q = \begin{bmatrix} 0 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.7559 & -0.1890 & 0.5669 & -0.1890 & -0.1890 \\ 0.4781 & 0.7171 & -0.4781 & -0.1195 & -0.1195 \\ 0.3508 & -0.3508 & -0.3508 & -0.0877 & 0.7894 \\ 0.2774 & -0.2774 & -0.2774 & 0.8321 & -0.2774 \end{bmatrix}$$

and

$$R = \begin{bmatrix} 2 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0 & 1.3229 & 0.5669 & 0.5669 & 0.5669 \\ 0 & 0 & 1.1952 & 0.3586 & 0.3586 \\ 0 & 0 & 0 & 1.1402 & 0.2631 \\ 0 & 0 & 0 & 0 & 1.1094 \end{bmatrix}$$

$$\therefore QR = \begin{bmatrix} 0 & 0.6615 & 0.8811 & 1.0329 & 1.149 \\ 1.5118 & 0.1279 & 0.9484 & 0.2586 & 0.2147 \\ 0.9562 & 1.1877 & 0.0742 & 0.3379 & 0.3101 \\ 0.7016 & -0.2887 & -0.4428 & -0.2493 & 0.7034 \\ 0.5548 & -0.2283 & -0.3501 & 0.8307 & -0.2069 \end{bmatrix}$$

2.5 Findings from a Graph with a Unique Fixed Probability Vector for a Butterfly

In the example above, P and $P^2 = P \cdot P$ are known. Which is

$$P^2 = \begin{bmatrix} 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & 1/8 & 1/8 & 1/8 & 1/8 \\ 1/4 & 3/8 & 1/8 & 1/8 & 1/8 \\ 1/4 & 1/8 & 3/8 & 1/8 & 1/8 \\ 1/4 & 1/8 & 1/8 & 3/8 & 1/8 \\ 1/4 & 1/8 & 1/8 & 1/8 & 3/8 \end{bmatrix}$$

$$VP = V$$

$$[a \quad b \quad c \quad d \quad e]\begin{bmatrix} 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 & 0 \end{bmatrix} = [a \quad b \quad c \quad d \quad e]$$

$$\therefore V = [1/3 \quad 1/6 \quad 1/6 \quad 1/6 \quad 1/6]$$

2.6 Results on a Butterfly Graph's Optimal Prefix Code

The best prefix code to use for the message DECOMPOSITION OF A BUTTERFLY

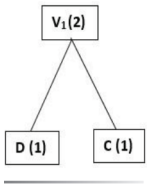
The following ideal prefix codes for the symbols we started with are obtained from the above-mentioned tree.

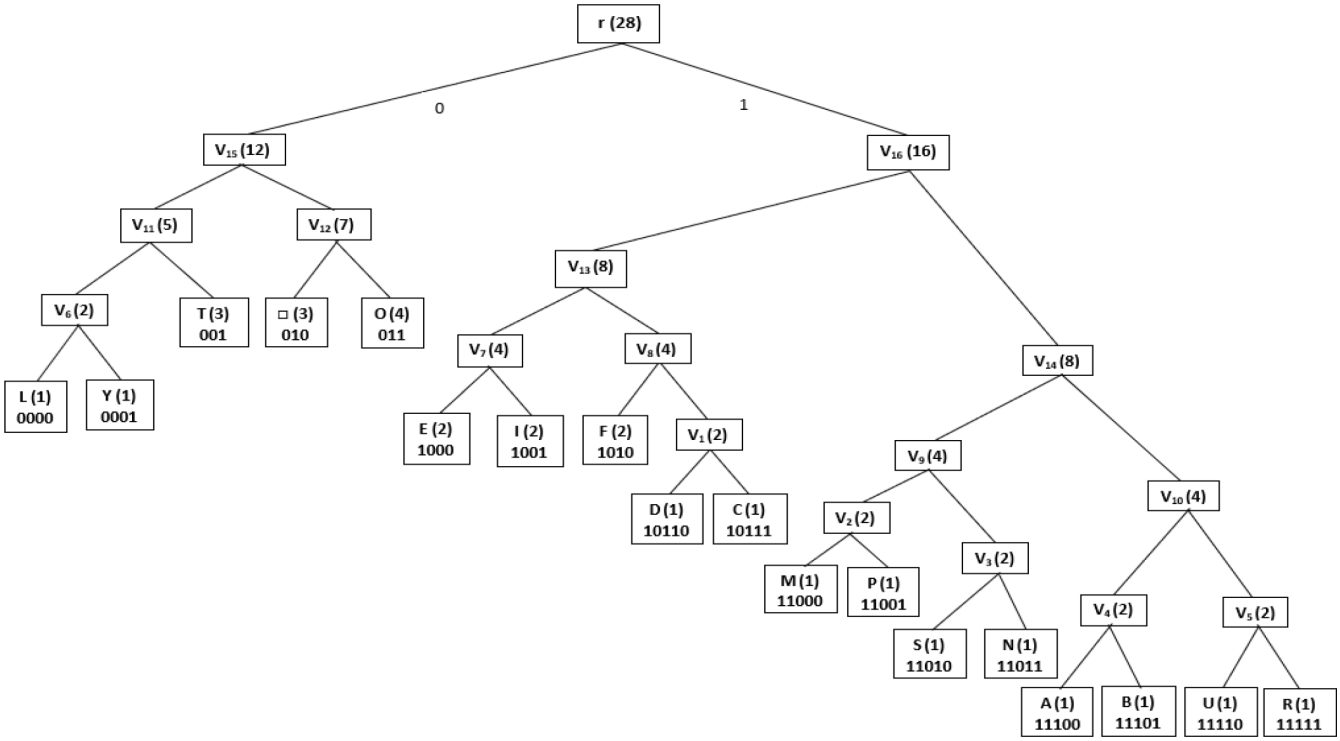
L: 0000, Y: 0001, T: 001, □: 010, O: 011 E: 1000, I: 1001, F: 1010, D: 10110,

D	E	C	O	M	P	S	I	T	N	□	F	A	B	U	R	L	Y
(1)	(2)	(1)	(4)	(1)	(1)	(1)	(2)	(3)	(1)	(3)	(2)	(1)	(1)	(1)	(1)	(1)	(1)

D	C	M	P	S	N	A	B	U	R	L	Y	E	I	F	T	□	O
(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(2)	(2)	(2)	(3)	(3)	(4)

M	P	S	N	A	B	U	R	L	Y	E	I	F	T	□	O
(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(1)	(2)	(2)	(2)	(3)	(3)	(4)





C: 10111, M: 11000, P: 11001, S: 11010, N: 11011, A: 11100, B: 11101,

U: 11110, R: 11111.

The code for the message DECOMPOSITION OF A BUTTERFLY is therefore

10110100010111011110001100101111010100100110010111101100111010010111000101110111100010011000111111101000000001.

The optimal tree's weight is

$$W(T) = (1\times 4) + (1\times 4) + (3\times 3) + (3\times 3) + (4\times 3) + (2\times 4) + (2\times 4) + (2\times 4) + (1\times 5) + (1\times 5) + (1\times 5) + (1\times 5) + (1\times 5) + (1\times 5) + (1\times 5) + (1\times 5) + (1\times 5) = 112$$

3.0 Conclusions

The above-mentioned results demonstrate the analytical characteristics polynomial, complement, graph indices, discrete Fourier transform of diameter, girth, vertex connectivity, and edge connectivity using the butterfly method of the Fast Fourier Transform algorithm, QR-Decomposition, and its regular stochastic matrix. Through this unique studies of Butterfly graph distribution, increment and decrement of mining metals and fuels can be identified.

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