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# Effect of Electrothermal Convection in a Rotating Layer of Viscoelastic (Walter's B) Fluid Saturated Porous Medium Heated from Below

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## Abstract

*We have looked at how a rotating layer of viscoelastic (Walter's B) fluid saturated porous medium heated from below is affected by a vertical DC electric field when it comes to the commencement of thermal convection. The technique called Galerkin is used to solve the eigen value problem in a numerical manner. The viscoelastic fluid "Walter's B" exhibits behavior similar to a standard Newtonian nanofluid when there is stationary convection. The stationary convection does not begin at the viscoelasticity criteria. The stationary convection occurrence for rigid rigid boundaries is obtained analytically. More stabilization is present in the system with increase in Ta and $Da^{-1}$. Heat is more effectively transferred and convection is hastened when a DC electric field is present. It's important to note that the specific application of electro-thermal convection in mining will depend on the geological and environmental conditions of the site, as well as the type of minerals or resources being extracted. The effectiveness and feasibility of such methods will also need to be carefully evaluated in each case.*

**Keywords:** *DC Electric Field, Electrothermal Convection, Rotation*

## 1.0 Introduction

Because of its various applications in geophysics, food processing, and nuclear reactors considerable interest has been evinced in the study of thermal instability in a porous medium. a detailed account of the thermal instability of a Newtonian fluid, under varying assumptions of hydrodynamics and hydro magnetics, has been given by Chandrasekhar[1]. Lapwood[2] has explored convective flow in porous medium using linearized stability theory. Wooding[3] has been considered the Rayleigh instability of a thermal boundary layer in flow through a porous medium. Many viscoelastic fluids cannot be characterized by Maxwell's constitutive relations Or Oldroyd's constitutive relations. One such class of fluids is Walters's (Model B) viscoelastic fluid having relevance in chemical technology and industry. Walters[4] reported that the mixture of polymethyl methacrylate and pyridine at 25°C containing 30.5 g of polymer per liter with density 0.98 g per liter behaves very nearly as the Walter's (Model B) viscoelastic fluid. Walters's (Model B) viscoelastic fluid forms the basis for the manufacture of many important polymers and useful products.

The investigation in porous media began with a simple darcy model and progressed to a Darcy-Brinkman model. An excellent account of convection problems in a porous medium is provided by Hadim and Vafai[5], Bejan and Nield[6] and Ingham and Pop[7]. Nield and Kuznetsov[8] investigated thermal instability in a saturated porous medium using the Brinkman model.

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Rana *et al.*[9] have studied the electrohydrodynamic thermal instability in a Walter's (Model B) rotating nanofluid layer. Lee *et al.*[10] have studied the effect of thermal modulation on the onset of thermal convection in a Walters B viscoelastic fluid in a porous medium. Rana and Sharma[11] have studied thermal instability of a Walters (Model B) viscoelastic model in the presence of a variable gravity field and rotation in a porous medium. Kango and Rana[12] have been studied the effect of rotation on thermal instability of compressible Walter's (Model B) elastic viscous fluid in a porous medium. Chand and Rana[13] the onset of electrohydrodynamic instability of an elastic-viscous Walter's (Model B) dielectric fluid layer. Given the importance of electrothermal convection in a variety of applications, the purpose of this paper is to investigate the effect of rotation on the onset of electrothermal convection in a Walter's (Model B) fluid heated from below in a saturated porous medium.

Fluid flow is integral to the efficient and safe operation of mining, metals, and fuel industries, influencing everything from material transport and processing to equipment operation, environmental management, and safety compliance.

## 2.0 Mathematical Formulation

As shown in Figure 1, we consider a horizontal layer of Walters B viscoelastic fluid-saturated porous medium of thickness d in the presence of gravity. The time-dependent temperature of the porous layer's lower and upper surfaces is imposed externally and is given by

$$T = V_0 + \frac{\Delta V}{2} \qquad at \qquad z = 0 \tag{1}$$

$$T = V_0 - \frac{\Delta V}{2} \qquad at \qquad z = d \tag{2}$$

With the reference temperature is $T_0$. In addition, a vertical DC electric field across layer is also imposed; the potential between the top and lower surfaces is maintained alternately $V_0 + \Delta T/2$ and $V_0 - \Delta T/2$, respectively. With the origin at the porous layer base and the z-axis pointing upwards in the vertical direction, a Cartesian coordinate system that is X, Y, Z is employed.

Under Boussinesq approximations, the following equations are relevant:

$$\Delta.\vec{q} = 0 \tag{3}$$

$$\rho_0\left[\frac{1}{\phi_p}\frac{\partial\vec{q}}{\partial t} - \frac{2}{\phi_p}\left(\vec{q}\times\vec{\Omega}\right)\right] = -\nabla p + \rho\vec{g} - \frac{1}{k}\left(\eta - \eta_v\frac{\partial}{\partial t}\right)\vec{q} + f_e \tag{4}$$

$$\frac{\partial T}{\partial t} + \left(\vec{q}.\nabla\right)T = K\nabla^2 T \tag{5}$$

$$f_e = \rho_e\vec{E} - \frac{1}{2}\vec{E}.\vec{E}\nabla_\varepsilon + \frac{1}{2}\nabla\left(\rho\frac{\partial\varepsilon}{\partial\rho}\vec{E}.\vec{E}\right) \tag{6}$$

$$\rho = \rho_0\left\{1 - \alpha\left(T - T_0\right)\right\} \tag{7}$$

$k$ is the effective thermal diffusivity, $\vec{q}$ as velocity, The porous medium's permeability is '$K$', '$p$' as pressure, $\vec{g}$ acceleration owing to gravity, temperature is $T$, the porosity of medium $\varepsilon$, the coefficient of volumetric expansion is $\alpha$, '$\mu$' as viscosity, $\mu_v$ is Walters B liquid viscoelastic constant, and $\rho_0$ is density. Electric field as '$\vec{E}$', $\Omega$ = (0,0, $\Omega$) which is the angular velocity, $\rho_e$ as free charge density, The density ($\rho$) is equivalent to $\rho_0$ at reference temperature. Where the quantities have their pre-defined meaning.

The Coulomb force, which includes the free charge, is the initial term on the right side of Eq. (6). The forces resulting from the non-uniformity of the dielectric constant are the last two terms.

The equations by Maxwell that are pertinent are,

$$\nabla\times\vec{E} = 0 \tag{8}$$

From (7), $\vec{E}$ written as

$$\vec{E} = -\nabla V \tag{9}$$

Wherein the dieelectric constant as '$\varepsilon$', as 'V', $p_e = \nabla.(\varepsilon,\vec{E})$, is the charge density. Based on the assumption that dielectric constant is linearly related to temperature




**Figure 1.**

$$\sigma = \sigma_0(1-\beta(T-T_0)) \tag{10}$$

The electrical conductivity's coefficient of thermal expansion is

$$(\beta = \sigma_0^{-1}(\partial\sigma/\partial T)$$
$$\partial p_e/\partial t + \nabla.J = 0 \tag{11}$$

$$J = \sigma\vec{E} + \rho_e\vec{q} \tag{12}$$

## 2.1 Basic State

Quiescence is the basic state and is characterized by

$$\vec{q} = \vec{q}_b = 0, T = T_b(z),\ p = p_b(z),\ \vec{E} = E_b(z)$$
$$\vec{\rho} = \rho_b(z)\ \ \varepsilon = \varepsilon_b(z)\ \ v = v_b(z) \tag{13}$$

Wherein the basic state is indicated by the subscript b.

$$\rho_b\vec{g} - \nabla p_b + f_e = 0 \tag{14}$$

$$\frac{dT_b}{dz} = -1 \tag{15}$$

$$\rho_b = \rho_0\{1+\alpha\beta z\} \tag{16}$$

$$\nabla\times\vec{E}_b = 0,\quad \nabla.(\varepsilon\vec{E}_b) = 0 \tag{17}$$

$$\nabla.\sigma_b\nabla v_b = 0 \tag{18}$$

$$\rho_{eb} = \in(\nabla.\overrightarrow{E_b}) \tag{19}$$

$$V_b = V_O - \Delta v\left[1/2 - z/d\right]$$

## 2.2 Linear Stability Analysis

To examine the stability of the fundamental state, we introduce an infinitesimally a small disruption in the following manner:

$$\vec{q} = \overrightarrow{q'}\ \ p = p_b + p',\quad \vec{E} = \vec{E}_b + \overrightarrow{E'}\ \ T$$
$$= T_b + T', \rho = \rho_B + \rho', \varepsilon = \varepsilon_b + \varepsilon' \tag{21}$$

where $q', p', E', T', \rho'$ and $\varepsilon'$ represents the perturbs. Substituting Eq. (21) into Eq. (3)-(12), dispensing with the pressure by working two times curl, and holding the upward part, we get (subsequent to disregarding the primes).

$$\left\{\frac{1}{\varnothing_p}\frac{\partial}{\partial t} - \frac{\eta}{k\rho_0}\left(1-\frac{\eta_v}{\eta}\frac{\partial}{\partial t}\right)\right\}\nabla^2 w$$
$$= -\alpha g\nabla^2{}_b T - \frac{2\Omega}{\varepsilon}\frac{\partial\xi}{\partial z} - \frac{\Delta v\Delta^2(\Delta^2 v_b)}{d} \tag{22}$$

$$\left\{\frac{1}{\varnothing_p}\frac{\partial}{\partial t} + \frac{\eta}{k\rho_0}\left(1-\frac{\eta_v}{\eta}\frac{\partial}{\partial t}\right)\right\}\xi = \frac{2\grave{U}}{\varnothing_p}\frac{\partial w}{\partial z} \tag{23}$$

$$\left(\frac{\partial}{\partial t} - k\nabla^2\right)T = w \tag{24}$$

$$\frac{\partial}{\partial t}\nabla^2 v' = -\sigma_0\Delta^2 v' - \sigma_0\beta T'\frac{\Delta v}{d}\frac{dT'}{dz} \tag{25}$$

Non-dimensional zing the equations by setting

$$(x^*, y^*, z^*) = \left(\frac{x}{d}, \frac{y}{d}\frac{z}{d}\right),\quad T^* = \frac{T}{\Delta T},$$
$$w^* = \frac{w}{k/d},\ \ t^* = \frac{t}{d^2\varepsilon/k},$$
$$\vec{q}^{'} = \frac{k}{d},\quad \xi' = \frac{k}{d^2},\ V' = \gamma E_0\Delta T \tag{26}$$

and substituting Eq. (26) into Eq. (22) – (25), we obtain, respectively,

$$\left\{\frac{1}{pr}\frac{\partial}{\partial t} + Da^{-1}\left(1-\frac{\Gamma p}{pr}\frac{\partial}{\partial t}\right)\right\}\nabla^2 w$$
$$+R_t\nabla^2{}_b T + Ta^{1/2}\frac{\partial\xi}{\partial z} + R_{ed}\nabla^2(\nabla^2 v_b) = 0 \tag{27}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)T = w \tag{28}$$

$$\left\{\frac{1}{pr}\frac{\partial}{\partial t} + Da^{-1}\left(1-\frac{\Gamma p}{pr}\frac{\partial}{\partial t}\right)\right\}$$
$$\xi - Ta^{1/2}\frac{\partial w}{\partial z} = 0 \tag{29}$$

$$\left[\rho_e\frac{\partial}{\partial t} + 1\right]\nabla^2 V = -\frac{\partial T}{\partial z} \tag{30}$$

Where $R_t = \alpha g\ddot{A}Td^3/vk$ is the thermal Rayleigh number, $R_{ed} = \gamma^2\varepsilon_0 E^2{}_0(\ddot{A}T)^2 d^2/kv$ is the DC electric Rayleigh number, $Ta = 4\Omega^2\frac{d^4}{v^2}$ is the Taylor number,

$pr = \nu / k\varepsilon^2$ is the modified Prandtl number, the elastic parameter is $\Gamma p = \mu_v \varepsilon / \rho_0 d^2, \rho_{e=\varepsilon k / d^2 \sigma_0}$ is the electric Prandtl number the Darcy number is $Da = k / d^2$ and $\xi = \partial v / \partial x - \partial u / \partial y$ vorticity of the z-component.

It's been assumed that the porous layer's margins are rigid-rigid and have a constant temperature and at boundaries the electric potential are

$$w = \frac{\partial w}{\partial z} = T = \xi = V = 0 \tag{31}$$

## 2.3 Normal Mode Study

We apply the normal mode procedure to perform the linear stability analysis, the result of the form

$$(w, T, V, \xi) = (W, \theta, \phi, Z)(Z) e^{(ilx + imy + \omega t)} \tag{32}$$

in which the numbers of the horizontal waves in the direction x and y are denoted by "l" and "m," correspondingly, and growth rate $\omega = \omega_r + i\omega_i$. Substituting Eq. (32) into Eq. (26) – (29), we obtain

$$\begin{aligned} &[\rho_e w + 1]\left\{\frac{w}{pr} + Da^{-1}\left(1 - \frac{\Gamma p}{pr} w\right)\right\} \\ &\left(D^2 - a^2\right) W + [\rho_e w + 1] \\ &R_t a^2 \theta + [\rho_e w + 1] Ta^{1/2} DZ + R_{ed} a^2 D\theta = 0 \end{aligned} \tag{33}$$

$$\left\{\frac{w}{pr} + Da^{-1}\left(1 - \frac{\Gamma p}{pr} w\right)\right\} Z - Ta^{1/2} DW = 0 \tag{34}$$

$$\left(w - \left(D^2 - a^2\right)\right)\theta - w = 0 \tag{35}$$

where $D = d/dz$ and $a = \sqrt{l^2 + m^2}$ is the dimensionless horizontal wave number,

A rigid-rigid boundary condition must be considered when solving the above equations.

$$W = DW = \theta = Z = \varnothing = 0 \quad at \ z = 0, 1 \tag{36}$$

# 3.0 Numerical Solutions

It has been seen that nonstationary convection is not happening as instability mode favored by viscoelastic fluids, it will occur iff the (Prandtl number) $Pr<1$ and Ta exceeds a threshold. But in this study $r>1$ . (For instance, with Prandtl numbers being 480 for corn oil, 100 for silicone oil, and 10,000 for castor oil), the Galerkin method is employed to solve the resulting eigenvalue problem. An exact solution is not possible. Factors are expressed as follows:

$$W = \sum A_i W_i \quad \theta = \Sigma B_i \theta_i \quad Z = \Sigma C_i Z_i \tag{37}$$

The respective boundary conditions satisfying the power series functions are $W_i$, $\theta_i$ and $Z_i$ the constant are $A_i$,$B_i$ and $C_i$. Using Eq. (37) into Eq. (33) – (35) (after noting $\omega = 0$).

multiplying the resulting Eq. (33) (momentum equation) by $W_j(z)$, (vorticity equation) Eq. (34) by $Z_j(z)$, (energy equation) Eq. (35) by $\theta_j(z)$ using the boundary conditions and acting the integration by parts with respect to between z=0 and z=1, this leads us to the subsequent set of algebraic equations of linear homogeneous:

$$A_i E_{ji} + B_i F_{ji} + C_i G_{ji} = 0 \tag{38}$$

$$A_i J_{ji} + B_i K_{ji} = 0 \tag{39}$$

$$\mathrm{A_i H_{ji}} + \mathrm{C_i I_{ji}} = 0 \tag{40}$$

Where,

$$E_{ji} = [\rho_e w + 1]\left\{\frac{w}{pr} + Da^{-1}\left(1 - \frac{\Gamma p}{pr} w\right)\right\} W_j \left(D^2 - a^2\right) W_i$$

$$F_{ji} = [\rho_e w + 1](R_t a^2 W_j \theta_i + R_{ea} a^2 W_j D\theta_i)$$

$$G_{ji} = [\rho_e w + 1] Ta^{1/2} W_j DZ_i$$

$$H_{ji} = -Ta^{1/2} Z_j DW_i$$

$$I_{ji} = \left\{\frac{w}{pr} + Da^{-1}\left(1 - \frac{\Gamma p}{pr} w\right)\right\} Z_j Z_i$$

$$J_{ji} = -\theta_j \ W_j$$

$$K_{ji} = \theta_j \left[ Aw - \left(\left(D^2 - a^2\right)\right)\right] \theta_i$$

A non-trivial solution to the aforementioned set of homogeneous algebraic equations is possible only in the following circumstances:

$$\begin{bmatrix} E_{ji} & F_{ji} & G_{ji} \\ H_{ji} & 0 & I_{ji} \\ J_{ji} & K_{ji} & 0 \end{bmatrix} = 0 \tag{41}$$

The suitable rigid-rigid boundary conditions as follows:

$$W_i = (2z^4 - 5z^3 + 3z^2)T^*_{i-1}, \quad \theta_i = \left(z - z^2\right)T^*_{i-1} = Z_i \quad (42)$$

Using the above trial functions is $T^*_{i-1}\ (i = 1,2,...n)$, Equation (41) yields the equation of the characteristic equation by expanding the determinant and which inreturn gives the R_t (thermal Rayleigh number) or the R_ed (DC electric Rayleigh number) as a wave number's function and other parameters.

## 4.0 Result Analysis

The study looks at the electrothermal convection effect in Walter's B fluid rotating layer within a porous medium that is saturated and is being heated from below. Since in the expression for $R_t$ does not contain the viscoelastic parameter f, found no difference in the viscous fluid and viscoelastic in stationary convection case. Rigid–rigid velocity boundary condition is considered for the study.

In Figure 2 $R_{tc}$ verses $R_{ed}$ is plotted for various $Ta$ values with $Da^{-1}$ (constant). It can be observed that as the $R_{ed}$ value increases, so does the critical Rayleigh number. It is also noted that the system become more stable as Ta's value rises. For stationary convection, this means that the rotation has stabilizing effects.

Figure 3 gives the plotting of $R_{tc}$ verses $R_{ed}$ for diverse values with (constant). The critical number for Rayleigh is observed to rise as the value of increases. It is also

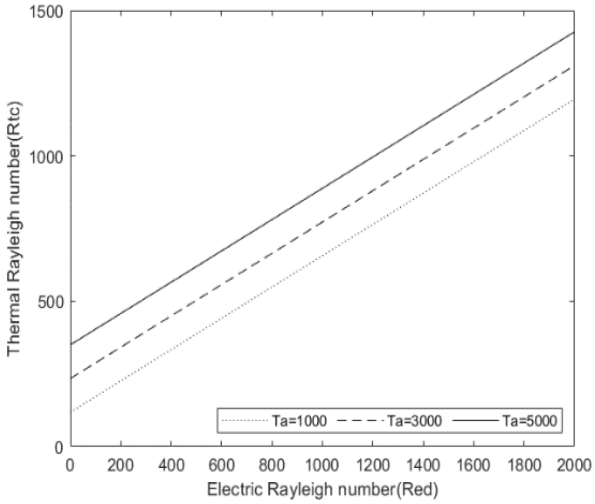



**Figure 2.** $R_t$ versus $R_{ed}$ at $Da^{-1}$ = 10, $T\alpha$=1000,3000,5000 for Rigid-Rigid boundaries.

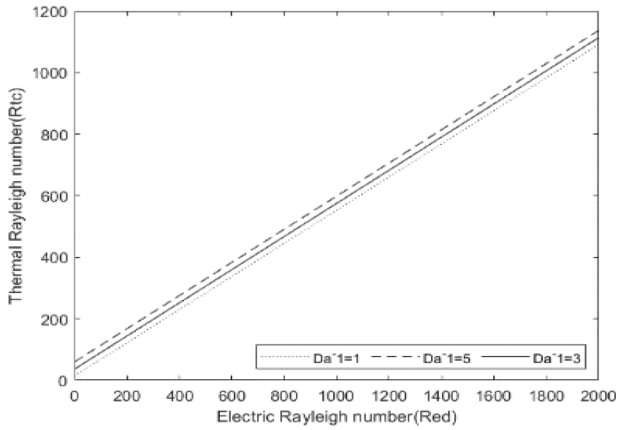



**Figure 3.** $R_t$ versus $R_{ed}$ at $T\alpha$=1000, $Da^{-1}$ = 1,3,5 for Rigid-Rigid boundaries.

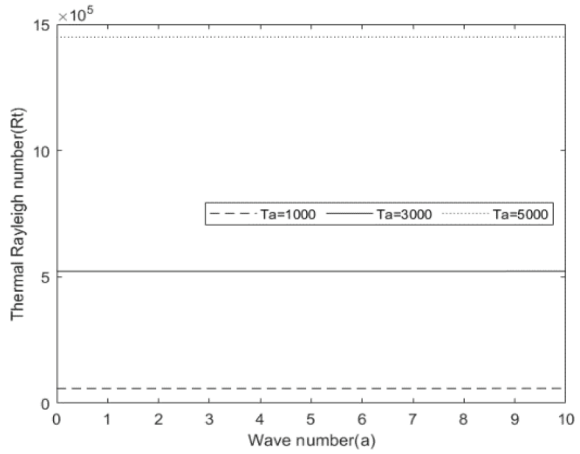



**Figure 4.** $R_t$ versus $a$ at $Da^{-1}$ = 5, $T\alpha$=1000,3000,5000 for Rigid-Rigid boundaries.

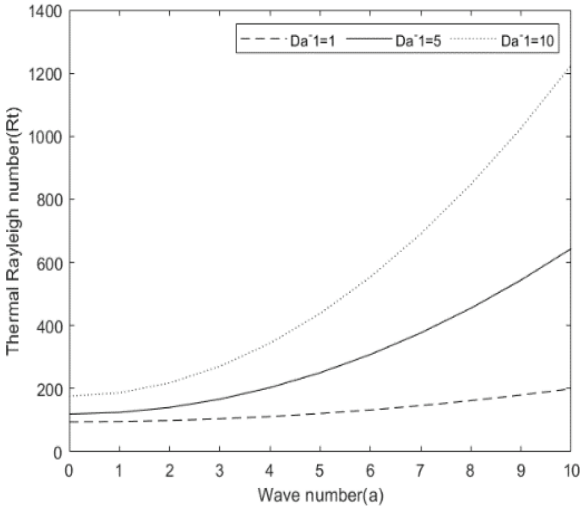



**Figure 5.** $R_t$ versus $a$ at $T\alpha$=1000, $Da^{-1}$ = 1,5,10 for Rigid-Rigid boundaries.

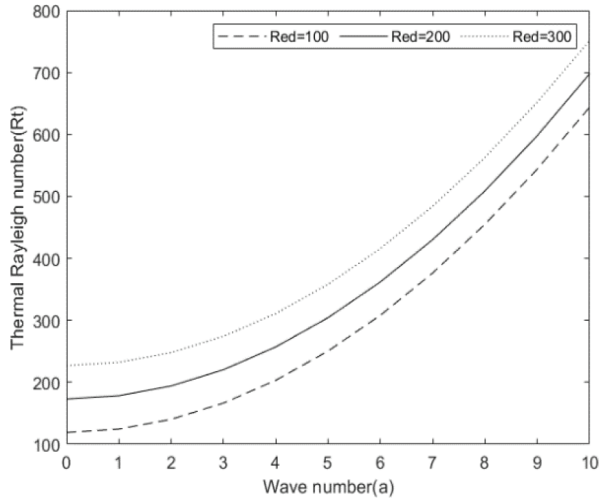



**Figure 6.** $R_t$ versus $a$ at $T\alpha$=1000, $Da^{-1} = 5$ $R_{ed}$ = 100,200,300 for Rigid-Rigid boundaries.

noted that the system become more stable with $Da^{-1}$ value enhancement. Therefore, for stationary convection, the Darcy number exerts a destabilizing impact.

In Figure 4, $R_t$ verses $a$ is plotted for diverse $T\alpha$ values with $Da^{-1}$ (constant). It is found that the system become more stable with $T\alpha$ value enhancement. For stationary convection, this means that the rotation has stabilizing effects.

In Figure 5, $R_t$ verses $a$ for diverse $Da^{-1}$ values with $T\alpha$ (kept constant) is shown. It is found that the system become stable with $Da^{-1}$ value enhancement. Therefore, for stationary convection, the Darcy number exerts a destabilizing impact.

In Figure 6, $R_t$ for diverse $R_{ed}$ values with constant $Da^{-1}$ & $T\alpha$ is given. It's been found that Rayleigh numbers enhances with the wave number a values enhancement. It is also noted that the system become more stable with increase in the value of $R_{ed}$. Therefore for stationary convection, this means that the DC electric field stabilizes it.

## 5.0 Conclusion

In the rigid-rigid boundaries' case, the impact of rotation on the start of electrothermal convection in a porous medium saturated with Walter's B fluid and heated from below has been studied. An equation for the stationary convection's occurrence in the case of rigid- rigid boundaries is found analytically. More stabilization is present in the system with increase in the Ta and Heat is more effectively transferred and convection is hastened when a DC electric field is present. Electrothermal convection refers to the process of heat transfer driven by electric currents in a conducting fluid. This phenomenon can be utilized in various applications, and in the context of mining, it may play a role in several ways those are heating of Subsurface Layers, Permafrost Melting, Enhanced Oil Recovery (EOR), Ore Processing, Thermal Fracturing, Deicing and Snow Melting, Ground Stabilization. Fluid flow plays a crucial role in various industries such as mining, metals, and fuels. Here are some ways fluid flow is managed and optimized in these industries: for example

**Fluid Control and Regulation**: Valves, regulators, and control systems are employed to control fluid flow rates, pressures, and directions within mining, metal processing, and fuel production facilities. Properly designed and maintained control systems help optimize process efficiency, ensure product quality, and enhance safety.

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