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B-Semi Symmetric Properties on Lorentzian α Sasakian Manifolds

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Abstract

We study B-semi symmetric, B-pseudo-symmetric, Ricci-pseudo-symmetric and B-Ricci generalized pseudo-symmetric of Lorentzian α-Sasakian Manifolds and additionally, we provide conditions that determine whether the solutions of the Ricci flows manifest as steady, expanding, or shrinking.

Keywords: B-Curvature Tensor, Lorentzian **α**- Sasakian Manifolds, Ricci Soliton.

1.0 Introduction

The exploration of geometric structures within the field of mathematics and theoretical physics has led to fascinating connections between abstract concepts and real-world phenomena. One such intriguing intersection lies in the study of Lorentzian α-Sasakian manifolds, where the principles of differential geometry intertwine with the complex dynamics of mining operations. Lorentzian geometry, originating from the work of Hermann Minkowski and Henri Poincare, provides a framework for describing the geometry of spacetime in the theory of relativity. On the other hand, Sasakian manifolds, a class of Riemannian manifolds named after Shoshichi Kobayashi and Katsumi Nomizu, possess rich geometric properties closely linked to the study of contact geometry and complex differential geometry. Lorentzian geometry, originating from the work of Hermann Minkowski and Henri Poincare, provides a framework for describing the geometry of spacetime in the theory of relativity. On the other hand, Sasakian manifolds, a class of Riemannian manifolds named after Shoshichi Kobayashi and Katsumi Nomizu, possess rich geometric properties closely linked to the study of contact geometry and complex differential geometry. Interestingly, the application of Lorentzian α-Sasakian manifolds extends beyond theoretical realms and finds surprising relevance in the realm of mining. Mining operations, characterized by the extraction of valuable minerals and resources from the Earth's crust, involve complex spatial considerations and dynamic geological structures. Understanding the geometric underpinnings of mining environments is crucial for optimizing extraction techniques, ensuring safety measures, and mitigating environmental impacts. In this context, the exploration of Lorentzian α-Sasakian manifolds offers a novel perspective on analyzing the spatial topology of mining sites, modeling the propagation of seismic waves through geological formations, and devising efficient strategies for resource extraction. By leveraging the mathematical machinery developed in the study of these abstract manifolds, mining engineers and geologists can gain deeper insights into the intricate spatial

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relationships within mining environments, leading to more informed decision-making processes and enhanced operational efficiencies. In this introductory exploration, we delve into the fascinating intersection of Lorentzian α-Sasakian manifolds and mining, highlighting the potential synergies between abstract mathematical theories and practical applications in resource extraction and geological exploration. Through interdisciplinary collaboration and innovative approaches, we aim to unlock new avenues for optimizing mining operations while advancing our understanding of fundamental geometric principles.

In the past two decades, numerous differential geometers have systematically explored symmetry properties utilizing diverse geometric tools such as vector fields curvature tensors, and importantly geometric flows in symmetric manifolds. Cartan introduced the notion of a locally symmetric manifold[1], further, it was classified like semi symmetric, pseudo-symmetric and Ricci-pseudo-symmetric, and Ricci generalized pseudo-symmetric manifolds and these manifolds enriched by several authors.

We define an endomorphism $\mathcal{R}(\mathrm{E},\mathrm{F})\mathrm{Z}$ and $(\mathrm{E}\,\Lambda_{\S}\,\mathrm{F})$

$$\mathcal{R}(E,F)Z=[\nabla_E,\nabla_F]\mathrm{Z}-\nabla_{[E,\ F]}\ Z \text{ and}$$

$$(\mathrm{E}\,\Lambda_{\S}\,\mathrm{F})\mathrm{Z}=\S(\mathrm{F},\ \mathrm{Z})\mathrm{E}-\S(\mathrm{E},\ \mathrm{Z})\mathrm{F}, \quad (1.1)$$

$\forall\ E,F,Z\in \mathrm{T}_p\mathrm{M}$, where ∇ denotes the Levi-civita connection.

Semi symmetric manifold introduced by Szabo[2] and it is defined as $\mathcal{R}\cdot\mathcal{R}=0$, where R is the Riemannian curvature tensor. Deszcz[3,4] gives the notions of pseudo-symmetric and Ricci pseudo-symmetric manifolds and these are defined as follows respectively

$$\mathcal{R}.\mathcal{R}=\boldsymbol{L}_{\mathcal{R}}\ \mathbb{Q}(\S,\ \mathcal{R}) \text{ and}$$

$$\mathcal{R}.\mathrm{S}=\boldsymbol{L}_{\mathrm{S}}\ \mathbb{Q}(\S,\ \mathrm{S}),$$

where $\boldsymbol{L}_{\mathcal{R}}$ and $\boldsymbol{L}_{\mathrm{S}}$ are some function on $\mathcal{U}_{\mathcal{R}}=\boldsymbol{E}\in M:\mathbb{Q}(\S,\ \mathcal{R})\neq 0\ \text{at E}$ and $\mathcal{U}_{\mathrm{S}}=\boldsymbol{E}\in M:\mathbb{Q}(\S,\ \mathrm{S})\neq 0\ \text{at E}$ respectively and $\mathcal{R}\cdot\mathcal{R},\ \mathbb{Q}(\S,\mathcal{R}),\ \mathcal{R}.\mathrm{S},\ \mathbb{Q}(\S,\ \mathrm{S})$ are linearly dependent. Every $\mathcal{R}.\mathcal{R}\Rightarrow\mathcal{R}.\mathcal{R}=\boldsymbol{L}_{\pi}\ \mathbb{Q}(\S,\mathcal{R})\Rightarrow\mathcal{R}.\mathrm{S}=\boldsymbol{L}_{\mathrm{S}}\ \mathbb{Q}(\S,\ \mathrm{S})$.

If the tensor $\mathcal{R}.\mathcal{R}$ and $\mathbb{Q}(\mathrm{S},\mathcal{R})$ are linearly dependent then M is called Ricci-generalized pseudo symmetric if

$$\mathcal{R}.\mathrm{S}=\boldsymbol{L}_{\mathcal{R}}\ \mathbb{Q}(\mathrm{S},\mathcal{R}).$$

The tensor $\mathbb{Q}(\mathrm{S},\mathcal{R})$ and $(\mathrm{E}\,\Lambda_{\mathrm{S}}\,\mathrm{F})$ are

$$\mathbb{Q}(\mathrm{S},\mathcal{R})(\mathrm{E},\mathrm{F}:\ E_1,E_2,E_3)=(\mathrm{E}\,\Lambda_{\mathrm{S}}\,\mathrm{F})\mathcal{R}(E_1,E_2,E)_3)\ -\ \mathcal{R}((\mathrm{E}\,\Lambda_{\mathrm{S}}\,\mathrm{F})E_1,E_2)E_3$$

$$-\mathcal{R}(E_1,(\mathrm{E}\,\Lambda_{\mathrm{S}}\,\mathrm{F}))E_2)E_3\ -\ \mathcal{R}(E_1,E_2)(\mathrm{E}\,\Lambda_{\mathrm{S}}\,\mathrm{F})E_3.$$

and

$$(\mathrm{E}\,\Lambda_{\mathrm{S}}\,\mathrm{F})\mathrm{Z}=\mathrm{S}(\mathrm{F},\ \mathrm{Z})\mathrm{E}-\mathrm{S}(\mathrm{E},\ \mathrm{Z})\mathrm{F}, \quad (1.2)$$

Lorentzian α sasakian manifold was introduced by Yildiz and Murathan[5] in 2005. It is briefly denoted by $L\alpha SM$. In the last two decades, many researchers studied the properties of $L\alpha SM$ with different curvature tensors. In the framework of general relativity, we examine space-time as a 4-dimensional pseudo-Riemannian manifold denoted by $(M,\S)$, where $\S$ represents a perfectly liquid space-time with a Lorentzian metric. Einstein's equation express the functioning of a perfect fluid inside a spherical entity. A perfect fluid represents as isotropic and star-like in its rest frame. Dust serves as a straightforward example of a perfect fluid. In the realm of general relativity, perfect fluids are frequently employed to represent idealized distributions. Fuels are composed of molecules that maintain contact but can easily slide over one another without encountering shear stresses. A perfect fluid, exemplified by oil, exhibits properties closely resembling those of fuels. Viscous fluids consist of molecules that are like those of the perfect fluid. Fluids that require consideration of viscosity, or internal friction, are termed viscous fluids. Newtonian fluids are a specific category within viscous fluids, characterized by constant viscosity across varying rates of shear and with no temporal changes.

On the other side, Ricci flow was introduced by Hamilton[6]. Ricci soliton is a solution of Ricci flow and it is defined as

$$\mathcal{L}_V\,\S+2\mathrm{S}+2\lambda\S=0, \quad (1.3)$$

where $\mathcal{L}_V, S\ and\ \lambda$ are deotes the Lie derivative operator along vector field V, Ricci tensor and scalar constant. Soliton is expanding, steady or shrinking according as $\gtrless\lambda$. The study of the kinematic and dynamic nature of relativistic spacetime application in relativity has a physical model of three classes namely: shrinking, steady, and expanding. Such a physical model are similar to solitons of Ricci flows. Many geometers

studied solitons in contact manifolds. Recntly Praveen *et al.*[7], studied certain results of Ricci solitons on (LCS) manifolds. Promoted by these ideas in this paper, we investigate B semi symmetric on $L\alpha SM$ in the following way: This next bit is only exploratory. In section 3, we exhibit B semi symmetric $L\alpha SM$ is η-Einstein, provided $a_0 - a_1 \neq 0$ and the manifold is shrinking. Part 4 is examines to study of B-pseudo-symmetric $L\alpha SM$ is η-Einstein, provided $\alpha^2 - \mathcal{L}_B \neq 0$ and the manifold is shrinking. Section 5 deals with the Ricci pseudo-symmetric $L\alpha SM$ is η −Einstein, provided $\mathcal{L}_S - \alpha^2 \neq 0$ and the manifold is shrinking. Finally we study B −Ricci generalized pseudo-symmetric $L\alpha SM$ is η −Einstein, provided $\mathcal{L}_B - \alpha^2 \neq 0$ and $a_0(1+\alpha^2) + a_1(1-2r) - 2\,a_2 r \neq 0$, also the manifold is shrinking.

2.0 Preliminaries

An odd dimensional differentiable manifold M is said to be an $L\alpha SM$ if it admits a (1, 1) tensor field ϕ, a contravar iant vector field ϑ, a covariant vector field η and a Lorentz ian metric § which satisfies

$$\eta(\vartheta) = -1, §(J, \vartheta) = \eta(J),\ \phi^2 J = J + \eta(J)\vartheta, \phi\vartheta = 0, \quad (2.1)$$

$$§(\phi J, \phi F) = §(J,F) + \eta(J)\eta(F), \quad \nabla_J \vartheta = -\alpha\phi J, \quad (2.2)$$

$$(\nabla_J \phi)(F) = \alpha§(J,F)\vartheta + \eta(F)J, \quad (2.3)$$

$$(\nabla_E \eta)F = \alpha§(\phi E, F), \quad (2.4)$$

$\forall\, J, F \in T_p M$, where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric §. Further on a $L\alpha SM$ the following relation hold:

$$\eta(\mathcal{R}(J,F)Z) = \alpha^2\,[§(F,Z)\eta(J) - §(J,Z)\eta(F)], \quad (2.5)$$

$$\mathcal{R}(\vartheta, J)F = \alpha^2\,[§(J,F)\vartheta - \eta(F)J], \quad (2.6)$$

$$\mathcal{R}(J,F)\vartheta = \alpha^2\,[\eta(F)J - \eta(J)F], \quad (2.7)$$

$$\mathcal{R}(\vartheta, J)\vartheta = \alpha^2\,[J + \eta(J)\vartheta], \quad (2.8)$$

$$S(J,\vartheta) = S(\vartheta, J) = (n-1)\alpha^2\,\eta(J), \quad (2.9)$$

$$S(\vartheta,\vartheta) = -(n-1)\alpha^2, \quad (2.10)$$

$$\mathbb{Q}\vartheta = (n-1)\alpha^2\vartheta \quad (2.11)$$

$$§(\mathbb{Q}J, F) = S(J,F), \quad (2.12)$$

$$S(\phi J, \phi F) = S(J,F) + (n-1)\alpha^2\,§(J,F). \quad (2.13)$$

Let $(§, \vartheta, \lambda)$ be a $\mathcal{R}$icci soliton on a $L\alpha SM$ $(M, §)$. From (2.3), we get

$$(\mathcal{L}_V §)(E,F) = \alpha§(\phi E, F), \quad (2.14)$$

$\mathcal{R}$ecently the author[7] noticed, generalized B curvature tensor is given by

$$B(E,F)Z = a_0\,\mathcal{R}(E,F)Z + a_1[S(F,Z)E - S(E,Z)F + §(F,Z)\mathbb{Q}E - §(E,Z)\mathbb{Q}F] + 2a_2 r[§(F,Z)E - §(E,Z)F], \quad (2.15)$$

where $\mathcal{R}, S, \mathbb{Q}$ and r is Curvature tensor, $\mathcal{R}$icci tensor, $\mathcal{R}$icci operator and the scalar Curvature respectively.

3.0 B-Semi symmetric LαSM

When the condition $\mathcal{R}.B = 0$ holds, we say that $L\alpha SM$ is B semi symmetric. As in

$$(\mathcal{R}(E,F)\cdot B)(U,Z,J) = 0, \quad (3.1)$$

$\forall\, E, J, F, U, Z \in T_p M$, which follows that

$$S(V,J) = A§(V,J) + C\,\eta(V)\eta(J). \quad (3.2)$$

Where

$$A = \frac{1}{a_0 - a_1}\{a_0(n-1)\alpha^2 + a_1[\alpha^2(n-1)(n-2) - 2r],$$

$$C = \frac{1}{a_0 - a_1}\{a_0(n-1)\alpha^2 + a_1[(2\alpha^2+1)(n-1) - r] + 2a_2 r(n+2)$$

Accordingly, the following may be said:

Theorem 3.1. A n-dimensional B Semi symmetric $L\alpha SM$ is an η-Einstein, provided that $a_0 - a_1 \neq 0$.

By using (1.3) and (3.3), we get

$$2\alpha§(\phi V, J) + A§(V,J) + C\eta(V)\eta(J) + \lambda§(V,J) = 0. \quad (3.3)$$

Putting $V = J = e_i$ in the above equation, we get

$$\lambda = -\left[A + 2\frac{\alpha K}{n} - \frac{C}{n}\right]. \quad (3.4)$$

From this, we may get the conclusions that:

Corollary 3.1. A $\mathcal{R}$*icci* soliton in an n-dimensional B-Semi symmetric $L\alpha SM$ is Shrinking.

4.0 B- Pseudo-symmetric LαSM

If the equation $\mathcal{R} \cdot B = L_B\, Q(\S, B)$ holds, then the $L\alpha SM$ is said to be B-pseudo-symmetric

$$(\mathcal{R}(\vartheta,E) \cdot B)(U,V,J) = L_B[((\vartheta\Lambda_\S E) \cdot B)(U,V,J)] \quad (4.1)$$

the left hand side (4.1), gives

$$\mathcal{R}(\vartheta,E)\,B(U,V)J - B(\mathcal{R}(\vartheta,E)U,V)J - B(U,\mathcal{R}(\vartheta,E)V)J - B(U,V)\mathcal{R}(\vartheta,E)J, \quad (4.2)$$

using (2.15) in (4.2), taking inner product with ϑ and using (2.1), (2.5), (2.6), (2.9), we get

$$\alpha^2\{(a_1-a_0)S(V,J) + [a_0(n-1)\alpha^2 + a_1(\alpha^2(n-1)(n-2)-2r)]\S(V,J) + [a_0(\alpha^2-1)(n-1) + a_1((-1)(2\alpha^2+1)-r)]\eta(V)\eta(J)\}. \quad (4.3)$$

The right hand side of (4.1), gives

$$L_B\{(\vartheta\Lambda_\S E)\,B(U,V)J - B((\vartheta\Lambda_\S E)U,V)J - B(U,(\vartheta\Lambda_\S E)V)J - B(U,V)(\vartheta\Lambda_\S E)J\}, \quad (4.4)$$

using (1.1), (2.15) in (4.5) and taking inner product with ϑ, we get

$$L_B\{-a_0\S(E,\mathcal{R}(U,V)J) - a_1[S(V,J)\S(E,U) - S(U,J)\S(E,V) + \S(V,J)S(E,U) \quad (4.5)$$

$$- \S(U,J)S(E,V)] - 2a_2r[\S(V,J)\S(E,U) - \S(U,J)\S(E,V)]$$

$$- \eta(E)\{a_0\S(\vartheta,R(U,V)J) + a_1[S(V,J)\eta(U) - S(U,J)\eta(V)$$

$$+ \S(V,J)S(U,\vartheta)] + 2a_2r[\S(V,J)\eta(U) - \S(U,J)\eta(V)]\}\},$$

Setting $E = U = e_i$ in (4.5) and using (2.1), (2.5), (2.6), (2.9), we get

$$L_B\{-(a_0 + a_1(n-1+r))S(V,J) + \quad (4.6)$$

$$[a_0(r-1)\alpha^2 + a_1(r-(n-1)(r-1)\alpha^2) + 2a_2(r-n)]\S(V,J)$$

$$+ [a_0((r-1)+(n-1)\alpha^2) + a_1(-r-(n-1)\alpha^2) + 2a_2(r-n)]\eta(V)\eta(J)\},$$

using (4.3) and (4.6) in (4.1), we get

$$S(V,J) = A\S(V,J) + C\,\eta(V)\eta(J). \quad (4.7)$$

Accordingly, the following may be said:

Theorem 4.2. A n-dimensional B-pseudo-symmetric $L\alpha SM$ is η-Einstein manifold, provided that $\alpha^2 - L_B \neq 0$.

By using (1.3) and (4.7), we get

$$2\alpha\S(\phi V,J) + A\S(V,J) + C\,\eta(V)\eta(J) + \lambda\S(V,J) = 0, \quad (4.8)$$

putting $V = J = e_i$ in (4.8) we obtained

$$\lambda = -\left[A + 2\frac{\alpha K}{n} - \frac{C}{n}\right]. \quad (4.9)$$

Because of this, we may draw the conclusions that:

Corollary 4.2. A Ricci soliton in an n-dimensional B-pseudo-symmetric $L\alpha SM$ is Shrinking.

5.0 Ricci Pseudo-symmetric LαSM

A $L\alpha SM$ is said to be $\mathcal{R}$icci pseudo-symmetric, satisfies the condition $\mathcal{R}.S = L_S Q(\S,S)$ that is

$$(\mathcal{R}(\vartheta,F) \cdot S)(U,Z,J) = L_S[((\vartheta\Lambda_\S F) \cdot S)(Z,J)], \quad (5.1)$$

which follows that

$$S((\mathcal{R}(\vartheta,F)Z,J) + S(Z,\mathcal{R}(\vartheta,F)J) = LS\{S((\vartheta\Lambda_\S F)Z,J) + S(Z,(\vartheta\Lambda_\S F)J)\}, \quad (5.2)$$

using (1.1) and (2.6), we get

$$\alpha^2\{(n-1)\alpha^2[\eta(J)\S(F,Z) + \eta(Z)\S(F,J)] - \eta(Z)S(F,J) - \eta(J)S(F,Z) = L_S\{(n-1)\alpha^2[\eta(J)\S(F,Z) - \eta(Z)\S(F,J)] - \eta(Z)S(F,J) - \eta(J)S(F,Z)\}, \quad (5.3)$$

putting $Z = \vartheta$ in (5.3) and using (2.1), we get $L_S - \alpha^2 \neq 0$ or

$$S(F,J) = (n-1)\alpha^2\,\S(F,J). \quad (5.4)$$

Accordingly, the following may be said:

Theorem 5.3. A n-dimensional $\mathcal{R}$icci Pseudo symmetric $L\alpha SM$ is Einstein manifold, provided that $L_S - \alpha^2 \neq 0$.

By using (1.3) and (5.4), we get

$$2\alpha\S(\phi F,J) + A\S(F,J) + \lambda\S(F,J) = 0. \quad (5.5)$$

Putting $V = J = e_i$ in (5.5) , we get

$$\lambda = -\left[(n-1)\alpha^2 + 2\frac{\alpha K}{n}\right]. \quad (5.6)$$

Because of this, we may state the conclusions that:

Corollary 5.3. A Ricci soliton in an n-dimensional $\mathcal{R}$icci pseudo-symmetric $L\alpha SM$ is Shrinking.

6.0 B-Ricci Generalized Pseudo-symmetric LαSM

A Riemannian manifold is said to be $B-\mathcal{R}$icci generalized pseudo symmetric if

$$(\mathcal{R} \cdot B) = L_B\, Q(S, B), \quad (6.1)$$

which follows that:

$$(\mathcal{R}(\vartheta,E)\,B(U,V)J - B(\mathcal{R}(\vartheta,E)U,V)J - B(U,\mathcal{R}(\vartheta,E)V)J - B(U,V)\mathcal{R}(\vartheta,E)J$$

$$= L_B\{(\vartheta\Lambda_S E)\,B(U,V)J - B((\vartheta\Lambda_S E)U,V)J - B(U,(\vartheta\Lambda_S E)V)J - B(U,V)(\vartheta\Lambda_S E)J\}, \quad (6.2)$$

the LHS of (6.2), becomes

$$(a_1 - a_2)S(V,J) + \{a_0(n-1)\alpha^2 + a_1[\alpha^2(n-1)(n-2) - 2r]\}\S(V,J)$$

$$\{a_0(n-1)\alpha^2 + a_1[(2\alpha^2+1)(n-1) - r] + 2a_2r(n+2)\}\eta(V)\eta(J), \quad (6.3)$$

the RHS of (6.2), becomes

$$L_B\{S(V,J)[-2r(a_1+a_2) + 2a_1 + a_0\alpha^2] - r[a_0\alpha^2 + a_1 - 2a_2r]\S(V,J)$$

$$+r[a_0\alpha^2 + 2a_1(n-1)\alpha^2 + 2a_2r]\eta(V)\eta(J), \quad (6.4)$$

combining (6.3) and (6.4), we get

$$L_B - \alpha^2 \neq 0 \quad \text{or}$$

$$S(V,J) = A\,\S(V,J) + C\,\eta(V)\eta(J). \quad (6.5)$$

Where

$$A = \frac{a_0\alpha^2(r+n-1) + a_1((n-1)(n-2)\alpha^2 - r) - 2a_2r^2}{a_0(1+\alpha^2) + a_1(1-2r) - 2a_2r}$$

$$C = \frac{a_0[(n-1)(\alpha^2-1)-r\alpha^2] + a_1[(n-1)(n-2)(2\alpha^2+1)-r(1+2(n-1)\alpha^2)] + 2a_2[r(n+2)-r^4]}{a_0(1+\alpha^2)+a_1(1-2r)-2a_2r}$$

From this, we may get the conclusions that:

Theorem 6.4. A n-dimensional B-Ricci generalized pseudo-symmetric $L\alpha SM$ is η-Einstein, provided if $L_B - \alpha^2 \neq 0$ and $a_0(1+\alpha^2) + a_1(1-2r) - 2a_2r \neq 0$.

By using (1.3) and (6.5), we get

$$2\alpha\S(\phi V, J) + A\S(V,J) + C\,\eta(V)\eta(J) + \lambda\S(V,J) = 0. \quad (6.6)$$

putting $V = J = e_i$ in (6.6) we get

$$\lambda = -\left[A + 2\frac{\alpha K}{n} - \frac{C}{n}\right]. \quad (6.7)$$

Since this is the case, it follows that:

Corollary 6.4. A $\mathcal{R}$icci soliton in an n-dimensional $B-\mathcal{R}$icci generalized pseudo-symmetric $L\alpha SM$ is Shrinking.

7.0 Conclusion

The exploration of the kinematic and dynamic aspects of relativistic space-time within the realm of relativity gives rise to a physical model categorized into three classes[9]: shrinking, steady, and expanding. Notably, this physical model aligns with the characteristics exhibited by Ricci flows. Ricci solitons emerge as solutions to flows, and within the context of $L\alpha SM$ and it is only shrinking.

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