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# Unsteady MHD Natural Convection Cu-Water Homogeneous Nanofluid Flow above the Surface with Source/Sink

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## Abstract

*Unsteady magnetohydrodynamics (MHD) free convective flow of single-phase Cu-water nanofluid to study thermal radiation and sink or source of heat over the vertical plate have been considered. It has various range of technological and manufacturing applications. Crank-Nicolson method is applied to obtain dimensionless equations numerically. The effect of several non-dimensional factors such as radiation, dimensionless time, magnetic parameter, solid volume fraction and heat source or sink on temperature and velocity profiles have been investigated. The present numerical work is verified with experimental correlation results of regular fluid for average Nusselt number and are coinciding accurately.*

**Keywords:** *Convection, Crank–Nicolson Method, Magnetohydrodynamics, Nanofluid*

## 1.0 Introduction

Magnetohydrodynamic convective nanofluids heat transfer gained great consideration of investigators due to wide range of uses in many industries. Cooling of solar water heating, electronics equipment, nuclear reactors cooling system, microwave tubes, biomedical applications, high-power lasers and refrigerators are few applications. Oils, water, glycol and ethylene are conservatively used to transfer heat fluids. Due to very low thermal conductivity, these fluids have limitations to transfer heat. To overcome from this, nanoparticles are suspended in heat transfer fluids to enhance their performance massively. Hence it is necessary to know the individualities of nanofluids to transfer heat with different conditions. Several investigators explored various effects of thermophysical bounds on steady free convection nanofluids flow[1-5].

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However, in several real-world applications the flow of fluids and heat related problems are not steady in nature because of impulsive fluctuations in external temperature. In such circumstances, the flow adjacent to the surface progresses gently then normal steady flow is obtained gradually. Hamad and Pop[6] used small perturbation approximations to resolve MHD unsteady flow of convective nanofluids above permeable oscillating plate. It was stated that characteristics of transfer of heat have been varied with base fluid when nanoparticles were added. Bachok *et al.*,[7] investigated unsteady flow nano fluid above the porous shrinking or expanding sheet for transmission of heat.

Nandkeolyar *et al.*,[8] used Laplace transform approach to observe the effect of radiation on hydrodynamic, unsteady, water based natural convection nanofluid flow over a perpendicular plate and temperature was

maintained constantly and found that Nusselt number declines with surge in radiation and solid volume fraction. Turkyilmazoglu[9] presented the transient transfer of heat and nanofluids flow above an infinite affecting plate mounted vertically in existence of radiation. Further, stated that highest heat transfer exhibited by Cu-water and lowest heat transfer exhibited by $TiO_2$-water. Das and Jana[10] examined unsteady, hydromagnetic water based nanofluid free convective flow over plate in existence of radiation. Haroun *et al.,*[11] applied spectral relaxation method to observe the impact of Soret-Dufour number and generation of heat on MHD, unsteady, free and forced convection fluid flow above the shrinking or expanding surface with injection or suction. Further, stated that surge in rate of expansion and volume fraction leads in growth of skin friction.

Influence of volume concentration of nanofluid and generation of heat studied by Loganathan *et al.,*[12] by considering nanofluid free convection flow above a plate mounted vertically. MHD, unsteady, boundary layer convective nanofluid flow over the porous expanding surface have been carried out by Freidoonimehr *et al.,*[13] and stated that Cu-water and $Al_2O_3$-water give most and least co-efficient of skin friction whereas $TiO_2$-water and Cu-water gives out lowest and highest rates of heat transfer. Belhaj and Ben-Beya[14] numerically investigated the influence of sinusoidal heating on MHD, unsteady, convection flow of CNT-water nanofluid in a four-sided cavity which is heated at the bottom. The influence of sloping magnetic field on unsteady natural convective flow of various types of magnetic nanofluid enclosed in a square have investigated numerically by Al-Balushi and Rahman[15]. Daniel *et al.,*[16] examined the impact of suction, chemical reaction, thermal radiation and viscous dissipation on electrical MHD nanofluid flow over an enlarging slip by Keller-Box method.

Mjankwi *et al.,*[17] numerically carried out variable properties of unstable, MHD flow of nanofluid on an extending sheet. A MHD, unsteady, convective fluid flow with Cu- nano particle over a vertical plate with boundary condition depending on time has been examined by Moli and Naikoti[18]. Shuaib *et al.,*[19] examined the unsteady, MHD Couette nanofluids move in a parallel, horizontal pattern between two porous plates by neglecting the thermal effects, and the Crank-Nicolson finite-difference method has implemented to solve the Navier-Stokes equations. Recently, Neeraja *et al.,*[20] studied heat and mass transition in a curved channel of Casson nanofluid through peristalsis, in two dimensions using BVP.

With all the above literature survey, the furthermost of work carried on 2-D, unsteady free convective flow to transfer heat in occurrence of radiation, magnetic field, heat source or sink are not sufficient. Therefore, the aim of this paper to study MHD, unsteady, free convection, single phase nanofluid flow above the vertical surface maintained at persistent temperature with sink or source. The obtained equations have been non-dimensionalised and Crank-Nicolson method has been implemented to obtain equations of temperature and velocity and have been discussed graphically. The present model finds its uses in industrial thermal management, heat exchangers, and engineering devices designing. Nano fluids as a mixture of nano particles of metal oxides or metals and base fluid are used in different areas of medicine, mining engineering, electronics and others.

## 2.0 Mathematical Analysis

A 2-D, natural convective and unsteady flow of a single-phase nanofluid above an infinite plate mounted vertically. Lateral to the plate $x'$ axis and normal to the plate $y'$ axis is situated. The nanofluid and plate are at uniform temperature $T'_\infty$ at $t' \le 0$. The plate temperature is suddenly increased to $T'_w$ at $t' > 0$ and $B_0$ is the uniform strength of magnetic field is imposed in $y'$ axis. The induced magnetic field and magnetic Reynolds number are assumed to be neglected. The non-dimensional Boussinesq's approximation of mass equations, energy and momentum equation for natural convective boundary layer nanofluid flow under the influences of heat source/sink and thermal radiation are as follows:

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y}u\frac{\partial u}{\partial x} + = a_2\theta + a_1\frac{\partial^2 u}{\partial y^2} - a_3 Mu \tag{2}$$

$$\frac{\partial \theta}{\partial t} + \frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = Qa_4\theta + \frac{1}{Pr}a_4(a_5 + R)\frac{\partial^2 \theta}{\partial y^2} \tag{3}$$

The symbols $a_1, a_2, a_3, a_4,$ and $a_5$ are given as,

$$a_1 = \frac{1}{\left[\phi\frac{\rho_s}{\rho_f} + (1-\phi)\right]} * \frac{1}{(1-\phi)^{2.5}},$$

$$a_2 = \frac{1-\phi+\frac{\phi(\beta\ )_s}{(\beta\ )_f}}{\phi\frac{\rho_s}{\rho_f}+1-\phi}, \quad \sigma_r = \frac{\sigma_s}{\sigma_f},$$

$$a_3 = \left[\frac{(\sigma_r - 1)3\phi}{2+(\sigma_r)-\phi(\sigma_r - 1)} + 1\right] \times \frac{1}{\left[\phi\frac{\rho_s}{\rho_f} + (1-\phi)\right]},$$

$$a_4 = \frac{1}{\left[\phi\frac{(\rho C_P)_s}{(\rho C_P)_f} + (1-\phi)\right]},$$

$$a_5 = \frac{k_{nf}}{k_f} = \left[\frac{k_s - 2\phi(k_f - k_s) + 2k_f}{k_s + \phi(k_f - k_s) + 2k_f}\right],$$

where components of velocity in $x'$ and $y'$ directions are $u'$ and $v'$ respectively. Temperature of the fluid far away from the plate is $T'_\infty$, $q_r$ is radiative heat flux in $y'$ - direction, acceleration due to gravity is g, temperature of the fluid near the plate is $T'$, temperature of the plate is $T'_w$, dimensional heat source/sink parameter $Q_0$, time is $t'$, dimensionless coordinate axis are $x$ and $y$, $L$ is the characteristic length, dimensionless fluid velocity in $x$ and $y$ directions are $u$ and $v$ respectively, dimensionless temperature $\theta$, Grashof number $Gr$, dimensionless time $t$, base fluid kinematic viscosity $v_f$, radiation parameter $R$, Prandtl number Pr, magnetic field parameter $M$, heat source/sink parameter $Q$, solid volume fraction of nanoparticle $\phi$, density of base fluid $\rho_f$, density of nanoparticle $\rho_s$, thermal expansion coefficient of base fluid $\beta_f$, thermal expansion coefficient of nanoparticle $\beta_s$, electrical conductivity of base fluid $\sigma_f$, $\mu_f$ is base fluid viscosity, heat capacitance of base fluid $(C_p)_f$, thermal conductivity of base fluid $k_f$, heat capacitance of nanoparticle $(C_p)_s$, thermal conductivity of nanoparticle $k_s$ and $\sigma_s$ is nanoparticle electrical conductivity. The correlation equations for thermophysical properties of nanofluid are employed as defined in [14].

The appropriate boundary and initial conditions are as follows:

$$\begin{aligned} &t \le 0 : v = 0,\ u = 0,\ \theta = 0 \ for\ all\ x,\ y \ge 0 \\ &t > 0 : \begin{Bmatrix} v = 0,\ u = 0,\ \theta = 1 \ at\ y = 0 \\ v = 0,\ u = 0,\ \theta = 0 \ at\ x = 0 \\ v \to 0,\ u \to 0,\ \theta \to 1 \ as\ y \to \infty \end{Bmatrix} \end{aligned} \tag{4}$$

The following dimensionless parameters are introduced:

$$\left.\begin{aligned} &y = \frac{y'}{L}Gr^{1/4},\ x = \frac{x'}{L},\ u = \frac{u'L}{V_f}Gr^{-1/2},\ t = \frac{t'V_f}{L^2}Gr^{1/2}, \\ &\theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},\ v = \frac{v'L}{v_f}Gr^{-1/4},\ R = \frac{16\sigma^* T'^3_\infty}{3k_f k^*}, \\ &\mathrm{Pr} = \frac{V_f(pC_p)_f}{k_f},\ Gr = \frac{g\beta_f L^3(T'_w - T'_\infty)}{v_f^2} \\ &Q = \frac{QL^2}{(pC_p)_f v_f}Gr^{-1/2},\ M = \frac{\sigma_f B_0^2 L^2}{v_f p_f}Gr^{-1/2} \end{aligned}\right\} \tag{5}$$

## 3.0 Numerical Technique

Crank-Nicolson type of an implicit finite difference scheme have been implemented to resolve equations (1) to (3) under condition (4) numerically. Further, rectangular computational domain with sides $y_{\max}(=7)$ and $x_{\max}(=1)$ which bounds both energy and momentum boundary layer equations were solved for dissimilar values of chosen parameters.

Based on Carnahan *et al.*[21] computational procedures, the numerical results have been computed. Step sizes of $\Delta x = 0.01$, $\Delta y = 0.05$, with step time $\Delta t = 0.01$ of mesh independent results have been obtained after some repeated trials. $10^{-6}$ is considered for convergence criterion of each independent variable. Further, Thomas algorithm have been applied iteratively to the resulted tridiagonal system of equations. If the relative error percentage is less than $10^{-6}$ then iterations were terminated in all cases to ensure that the computed results are correct to at least four significant figures.

## 4.0 Result and Discussion

The focus of this paper is on transient MHD free convective nanofluid boundary layer flow over a plate mounted vertically with heat source/sink, constant temperature and thermal radiation. Numerical computations have been done for particular physical parameters for copper-water (Cu-water) nanofluid. Table 1 shows nanoparticles and thermo physical characters of water. The influence of different dimensionless constraints on heat and flow have been graphically depicted in Figures 1-7 with various values of $Q$, $t$, $\phi$, $R$, and $M$. In this case Prandtl number (Pr) of nanofluid is fixed at 6.2. To check the precision of present results, of average Nusselt number with Chen *et al.*,[22] when, $M=0$, $\phi=0$, $Q=0$ and $R=0$ for varying Pr has been done and mentioned in Table 2. It is been perceived that the results have been correlated by the present work.

The impact of time $t$ on heat and flow of the fluid is seen in Figures 1 and 2. It is observed that the profiles enhances with surge in time till steady state is reached. Nanofluid (Cu-water) achieves state of steady condition at $t$ = 5.4 whereas $M$ = 0.5, $P$ = 6.2, $Q$ = 0.5, $R$ = 5, ø = 0.05 are fixed is seen in Figures 1 and 2. Heat profiles diminishes when $\theta=0$ along y-axis when it reaches to free stream and heat profiles grows to peak near the plate and gradually diminishes when reaches to free stream as $t$ develops. Fluid flow rises from zero to some value of $y$ at the plate thereby drops slowly faraway from plate to reach free stream flow is seen in Figure 2.

The variation of heat and fluid flow against various values of sink or source $Q$ is represented in Figures 3 and 4, denoted heat source $Q>0$, absence of sink or source $Q=0$ and $Q<0$ heat sink. The profiles increase with surge in heat source, this is because of generation of more energy inside the system which in turn develops the buoyancy effect in nanofluid, therefore flow also surges and vice-versa is seen when heat sink increases and is seen in Figures 3 and 4. Also, temperature rises to peak as heat source increases and deliberately drops to free stream whereas same temperature drops in case of heat sink when far from the plate. As seen in Figure 4, the fluid velocity first rises to peak and then falls gradually along $y$ near the plate. The solid volume fraction $\phi$ and radiation effect R on temperature is demonstrated in Figure 5. The rise in R resulted in growth of heat profile, this shows that $q_r$ radiative heat flux grows because of drop in $k^*$ co-efficient of Rosseland mean absorption, also heat profile drops at greater values of R rather at lesser R values.

**Table 1.** Thermophysical properties of nanoparticle and base fluid

| Property | Water | Copper (Cu) |
|---|---|---|
| $\rho$ (Kg/m$^3$) | 997.1 | 8933 |
| $C_p$ (J/Kg · K) | 4179 | 385 |
| $k$ (W/m · K) | 0.613 | 401 |
| $\beta \times 10^5$(K$^{-1}$) | 21 | 1.67 |
| $\sigma$ ($\Omega^{-1}$ · m$^{-1}$) | 0.05 | 5.96×10$^7$ |

**Table 2.** Comparison of mean Nusselt number when $M=0, \phi=0$, $Q=0$ and $R=0$ with results of correlation for normal fluids

| Pr | Average Nusselt Number | |
|---|---|---|
| | Chen *et al.*[22] | Present |
| 1 | 0.5318 | 0.5390 |
| 7 | 0.9914 | 1.0016 |
| 10 | 1.1000 | 1.1107 |
| 15 | 1.2345 | 1.2459 |
| 20 | 1.3379 | 1.3498 |
| 50 | 1.7167 | 1.7304 |
| 100 | 2.0626 | 2.0774 |
| 500 | 3.1271 | 3.1381 |
| 1000 | 3.7310 | 3.7336 |

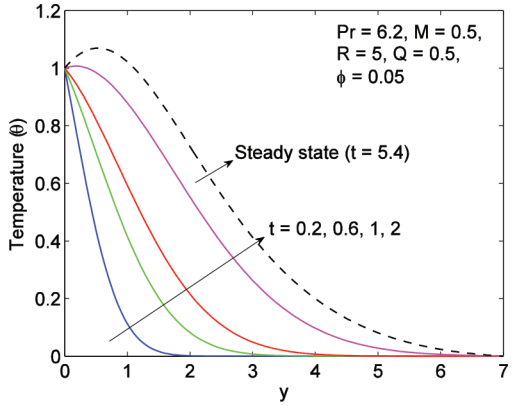



**Figure 1.** Temperature profiles for different $t$ values.

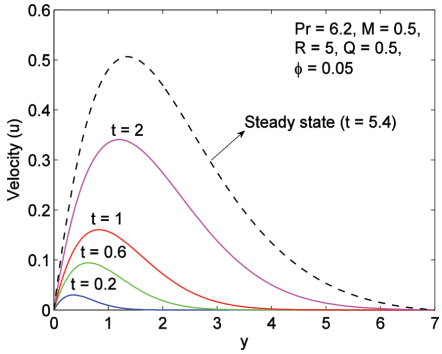



**Figure 2.** Velocity for *t* values.

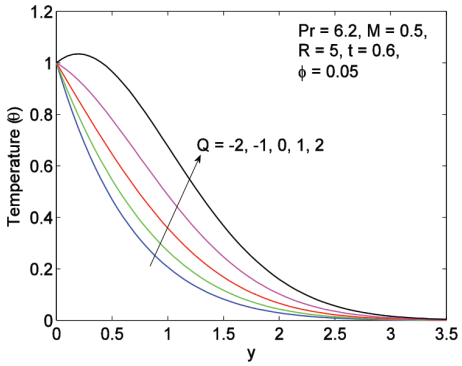



**Figure 3.** Temperature for *Q values*.

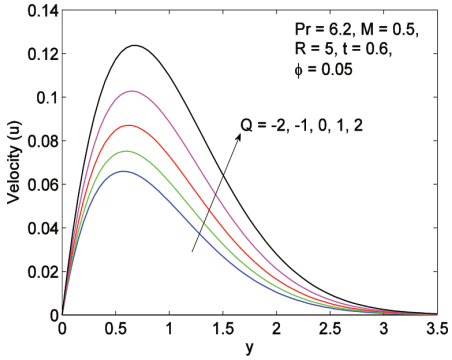



**Figure 4.** Velocity for *Q* values.

Further, similar effect is seen in Figure 5 when $\phi$ rises, this is because of addition of nanoparticles to the fluid which in turn rises the thermal conductivity and also the temperature.

Radiation R and Magnetic *M* parameter with velocity profiles are represented in Figure 6. The rise in R value surges the velocity distributions whereas converse tendency is resulted in surge of M, this is due to Lorentz force works contrary to the flow and shrinks the boundary layer thickness of momentum and velocity. Therefore, the

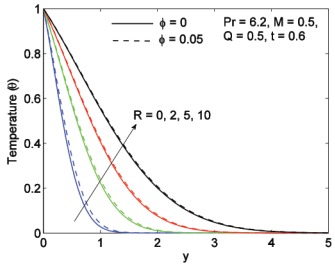



**Figure 5.** Temperature for $\phi$ and R values.

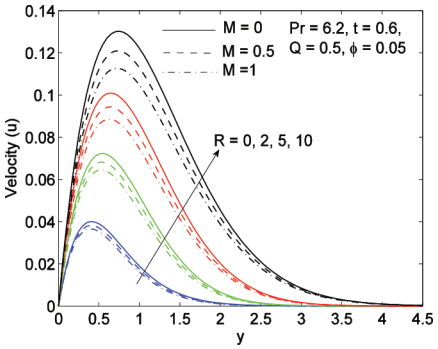



**Figure 6.** Velocity for M and R values.

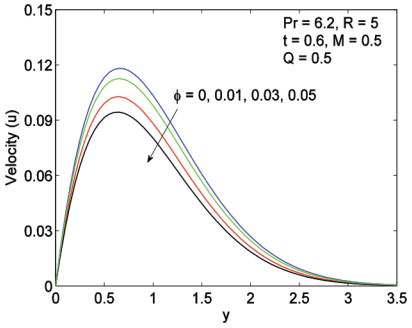



**Figure 7.** Velocity profiles for different $\phi$.

profile drastically increases at plate and gradually drops at free stream. The impact of $\phi$ solid volume fraction on velocity is represented in Figure 7. It clearly shows the major fall of velocity with increase in $\phi$ this is due to presence of nanoparticle withdraws the flow.

## 5.0 Conclusions

The effect of heat sink or source and radiation on MHD, unsteady, natural convective flow of single-phase nanofluid uniform temperature on a plate is examined. The accurate finite-difference method of second order have been implemented to find solution numerically.

The current numerical results of mean Nusselt number and previous experimental values are coinciding greatly. The influence of certain parameters, i.e., period, heat source/sink parameter, radiation parameter, magnetic parameter and solid volume fraction on heat and flow of fluid have been discussed with graphs. The main highlights are velocity and heat profiles surge by time till steady state is reached. Both the profiles enhance with rise in radiation parameter and heat source. The rise in solid volume fraction increases heat profile but drops the flow. No effect of magnetic field parameter is seen on temperature profiles, yet it reduces the fluid flow at the boundary layer.

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