Journal of Mines, Metals and Fuels, *71(12C) : 276-285; 2023. DOI: 10.18311/jmmf/2023/47297*

Print ISSN : 0022-2755

Journal of Mines, Metals and Fuels

Contents available at: *www.informaticsjournals.com/index.php/jmmf*

Numerical Study of Nanomaterial Flow with Joule Effect, Coefficient of Rate of Chemical Reaction along with Viscous Dissipation and Heat Source

P. A. Dinesh[1*], R. Sumithra[2], K. R. Roopa[1], Sweeti Yadav[1], H. R. Shrigowri[2], M. V. Gowthami[2], R. Jyothsna[2], N. S. Shivakumar[2] and R. Aparna[2]

[1]Department of Mathematics, M S Ramaiah Institute of Technology, Bangalore – 560054, Karnataka, India; dineshdpa@msrit.edu, krroopa870@gmail.com, sweetiyadav1989@gmail.com

[2]Department of Mathematics, Nrupathunga University, Bangalore – 560001, Karnataka, India; sumitradiya@yahoo.com, hrsgowri@gmail.com, gowthamimv1@gmail.com, jyothsnar123@gmail.com, shivakumarapril6@gmail.com, aparna20667@gmail.com

Abstract

This research, investigate the flow of nanomaterials with heat radiation, viscous dissipation, Joule effect. The paper also investigates the rate of chemical reactions, which reveals how the concentration of nanoparticles in the nanofluid varies over time and space. A physical representation guided us to a nonlinear paired partial differential equation, which was then reduced to an ordinary differential equation using the similarity approach for different presumptions on the real situation. The computational effort is carried out by compressing the Runge-Kutta-Felberg method with the Newton-Raphson iterative strategy to solve non-linear linked ordinary differential equations by changing the boundary value issue to an initial value problem by selecting a guess value. Later, computations for numerous non-dimensional factors relevant in the investigation of fluid flow, such as temperature, are anticipated. It has been noted that the reaction rate parameter had a major impact on the concentration profiles, and that when the reaction rate parameter rises, the boundary layer's concentration thickness increases.

Keywords: Chemical Reaction, Nanofluids, Radiative, Unsteady

1.0 Introduction

Nanoparticles (1–50 nm) are mixed with base liquids to create nanofluids, which have improved thermal properties. Due of their excellent thermo-physical and rheological (measuring deformation) qualities, they are primarily employed in the transport of heat. Usually, metals, metal oxides, carbide particles, or carbon-based nanotubes are employed to create the nanoparticles that are used in nanofluids. Because nanofluids have several uses in engineering, geophysics, industrial systems, the medical profession, the pharmaceutical business, and many other disciplines, understanding how they behave on a stretched sheet is crucial. It is crucial to comprehend the characteristics of nano fluids, such as thermal conductivity, viscosity, and specific heat, in order to use them in a variety of applications. The peculiar characteristics of nanomaterials are caused by the existence of Zinc (Zn), Copper (Cu), Manganese (Mn), Iron (Fe), Titanium Dioxide (Ti) and other metallic nanoparticles. It is thought that more research and experimentation on the flow of nanofluids or nanomaterials over stretched sheets,

**Author for correspondence*

as well as on their viscosity and heat transfer properties, would greatly expand the range of sectors in which they may be used.

Choi[1] analysed the nanofluids for enhancing the thermal conductivity of fluids. Before 1995 the base fluids are used which has less sufficiency of heat transfer, hence, the researcher introducing nanofluids which consists of copper nano phase materials used to exhibit higher thermal conductivity compared to base fluids. These nanofluids provide potential advantages in enhancement of heat transfer and it reduces heat exchanger pumping power. This study creates a route for developing industrial applications by several research studies.

Keblinski *et al.*[2] investigated the study of nano layers and their impact on thermo physical behaviour of nanofluids. Nano layers are solid like structure and it is the inter connection between solid surface and the first layer of the fluid. Influence of heat transfer diffusive property and thermophoresis are discussed. The nano layers behave like a thermal bridge for heat transfer process in solid liquid interfaces by enhancing the thermal conductivity of the fluid. They discuss that increase in vander Waals force of attraction among them at high nano particle loading and promoting the formation of nano clusters.

Yu[3] examined a literature review, assessment, comparison and developments of current status of the nanofluids for applications of heat transfer and thermal conductivity. The several experimental results are reviewed by multiple research groups. Later they recast the results for comparisons of data, next identify heat transfer and thermal property. There result of the study shows that enhancement of heat transfers to be in range of 15-40%. Mahabudul[4], explored the comprehensive review of viscosity of nanofluids with various factors such as temperature, preparation methods, shape, volume fraction, particle size was studied. Particle size influencing effects of viscosity of nanofluids and experimental result shows that in higher volume concentration, the viscosity increases and when temperature rise viscosity gradually decreases. Andersson *et al.*[5] contemplated the unsteady case of heat transfer in a liquid film over a stretching surface are investigated. The boundary layers time dependent equations can be reducing or transformed into ordinary differential equations using similarity transformation and parameters are solved using numerical methods. The result describes that temperature from the elastic sheet towards the free surface simultaneously increasing except at the surface temperature approaches of the sheet because, it has high diffusive limit. A low stretch ingrate has high values of 's' which tends to reduces the surface temperature. The graphical representation concludes that heat flux decreases with 's' with pr<0:1 from liquid to elastic sheet and increases with increased unsteadiness of parameter 's' with pr>1. Chen[6] explores the study of heat transfer of laminar boundary layer on a linearly stretching surface subjected to blowing or suction. The two cases such as heat flux and temperature are studied. They are found to be match and expressed in terms of using Kummer's functions. The effects of different parameters such as suction, temperature, Prandtl number and heat flux on the temperature distribution are discussed.

The foundation for the movement of comparable fluids through porous plate was laid by French civil engineer Henry Darcy during his 1856 research on the movement of liquid across sand ridges. Darcy law is ineffective whenever boundary and frictional effects occur at a greater flow rate. Philippes Forchheimer, a Dutch scientist, improved the Darcian frequency equation in 1901 from consist of the square velocity factor in the equation for momentum in order to predict the motion of viscosity and boundary layer flow. Many researchers[7-11] incorporate Darcy Forchheimer model.

Chen[12] contemplated viscous dissipation with the heat transfer of non-Newtonian fluids on a thin liquid film over an unsteady stretching surface. The results for parameters like free surface temperature gradient, power law index, velocity profile, free surface velocity and wall shear stress are studied. When fluid is heating then the fluid temperature decreases and when the fluid is cooled the fluid temperature decreases. The wall temperature takes higher value for negative Eckert number and lower minimum value for positive Eckert number. To check the obtained results from the study, compare those results without viscous dissipation. Mjankwi[13] revealed the unsteady MHD flow of nanofluid variable properties over a stretching sheet in the presence of thermal radiation and chemical reaction are investigated. The graphical representation shows when the increase in the rate of porosity parameter and magnetic fields results into a decrease in the rate of skin friction, mass and heat transfer. Hayat[14] performed the study of MHD flow of a third-grade fluid in presence of chemical reaction past an exponentially stretching sheet. The effects of parameters such as velocity, skin friction, local Sherwood number and concentration are discussed. Tarakaramu[15] emphasized study of the MHD

mass transfer flow electrical conducting and incompressible nanofluid flow past an unsteady porous stretching surface. Where unsteady parameter represents unsteadiness of the flow and found that skin friction and Sherwood number is opposite to each other. Waini[16] established the study of heat transfer properties of hybrid nanofluid over unsteady stretching surface. The results analyse the dual solutions and its stability are showing some impacts on nanofluids for heat transfer applications.

The atomic ions that make up the conductor's body and the flowing charged particles that make up the current interact to generate the Joule heating. It comes about as the outcome of the collision of moving particles. A portion of the energy that moves is converted into heat in this process, raising body temperature as a result. Numerous studies[17-20] investigated the effects of Joule heating on heat transfer and fluid flow under various circumstances and discovered that it has a significant effect on MHD flows.

Pattnaik[21] describes the chemical reaction effects of unsteady nanofluid boundary layer flow over a stretching surface with heat source or sink. Megahed[22] discussed variable viscosity and variable conductivity parameters of Newtonian MHD laminar flow over an unsteady stretching sheet with thermal radiation and variable heat flux. The study results in the conclusion that the rate of heat transfer is decreased when the magnetic parameter and viscosity parameter increases. The rate of cooling increases when thermal conductivity, radiation, unsteadiness increases.

Nuclear reactors, aviation, drying processes, cosmic fluid mechanics, exchangers for heat, geothermal energy, chemical technology, construction of buildings, solar power, solar energy systems, and oil recovery all rely heavily on the analysis of the movement of heat and mass on fluids over a substantially expanding surface about chemical reaction effect. Numerous researchers[23,24] had their chemical reactions evaluated across a stretched sheet. Jamshed[25] studied unsteady Casson non-Newtonian nano fluid flow when the fluid is allowed to flow over a stretching sheet with the effects of solar thermal radiation and entropy. In the solution part they used Roseland approximation for boundary layer flow. The analysis of heat transfer was made for two different types of nanofluids titanium water and copper water, resulting conclusion was that the heat transfer in copper water is much better than the titanium water.

The main objective of this paper is to study behaviour and flow characteristics of incompressible fluids. The study of boundary layer nanofluid flow on the stretching sheet with the influences of heat source and radiation of nanofluids. The research addresses the study behaviour of nanofluids flowing on a stretching energy after adding joule heating to energy equation and chemical rate co-efficient constant to concentration equation. Using nanofluids thermal performance is enhanced which can be implemented in Industries, medical fields and etc., for better application.

2.0 Mathematical Formulation

Consider a boundary layer nanofluid flow on a stretching medium under the influences of Joule heating, heat

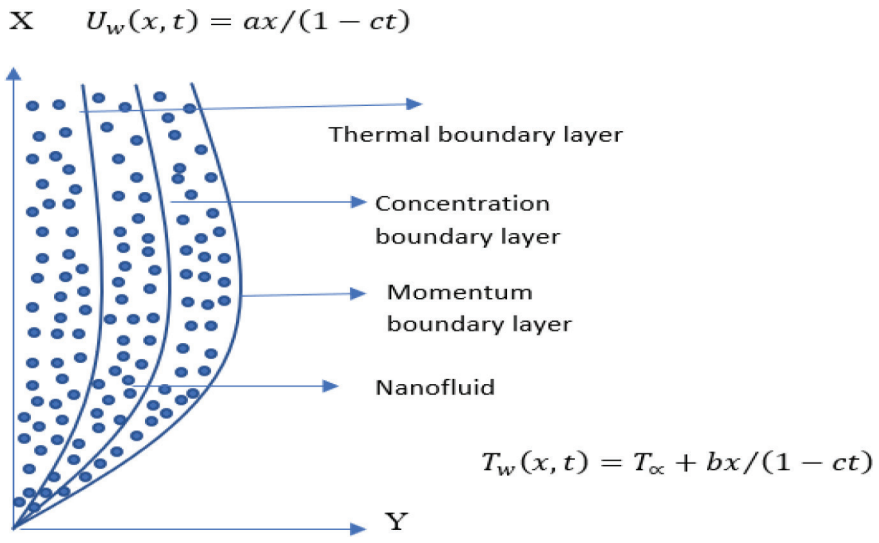


Figure 1. Geometry

source, radiation, viscous dissipation. Also consider the n^{th} order chemical reaction rate in concentration equation. The fluid taken here is electrically conducting and also assumes that the magnetic field is acting normal to the surface. Let the surface is stretched with velocity $U_w(x,t)=\frac{ax}{1-ct}$ *and* $T_w(x,t)=T_\infty+\frac{bx}{1-ct}$ along x-axis.

By using all these assumption equations obtained are,

Equation of Motion:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum Equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma}{\rho}B^2u \tag{2}$$

Energy Equation:

$$\begin{aligned}&(pc)_f\left(\frac{\partial \mathrm{T}}{\partial \mathrm{t}}+\mathrm{u}\frac{\partial \mathrm{T}}{\partial \mathrm{x}}+\mathrm{v}\frac{\partial \mathrm{T}}{\partial \mathrm{y}}\right)=\mathrm{k}\frac{\partial^2 \mathrm{T}}{\partial \mathrm{y}^2}\\&+(\mathrm{pc})_p\left\{\mathrm{D_B}\frac{\partial \mathrm{C}}{\partial \mathrm{y}}\frac{\partial \mathrm{T}}{\partial \mathrm{y}}+\frac{\mathrm{D_T}}{\mathrm{T}_\infty}\left(\frac{\partial \mathrm{T}}{\partial \mathrm{y}}\right)^2\right\}\\&-\frac{-}{pc_p}\frac{\partial q_r}{\partial y}+\frac{\mu}{pc_\mathrm{p}}\left(\frac{\partial u}{\partial y}\right)^2+\frac{Q_0}{\mathrm{pc_p}}(T-T_\infty)-\frac{\sigma B_0^2u^2}{pc_p}\end{aligned} \tag{3}$$

Diffusion Equation:

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2}+\chi\left(C-C_\infty\right)^n \tag{4}$$

Boundary Conditions:

$$u(x,0,t)=U_w, v(x,0,t)=0, T(x,0,t)=T_w, C(x,0,t)=C_w,$$

$$u(x,\infty,t)=0, T(x,\infty,t)=T_\infty, C(x,\infty,t)=C_\infty \tag{5}$$

Here, the parameters of the magnetic field and heat source are assumed to be

$B=\frac{B_0}{\sqrt{1-ct}}$ and $Q=\frac{Q_0}{\sqrt{1-ct}}$, respectively, with B_0 and Q_0 as constants in order to get a similarity solution for the relevant problem.

By utilising the Rosseland approximation

$\mathrm{q_r}=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial z}$ where, k* is the Rosseland mean captivative constant and σ^* denotes the content of the Stefan- Boltzman.

The classification of the similarity transformations is [26]

$$\eta=y\left(\frac{U}{\nu x}\right)^{1/2},\ \varphi=\sqrt{U_w\nu x}\,f(\eta),\ \Phi(\eta)=\frac{c-c_\infty}{c_w-c_\infty},$$

$$\Theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{6}$$

Equation (1)'s fulfilling stream function is defined as

$$u=\frac{\partial \psi}{\partial y},\ v=-\frac{\partial \psi}{\partial x} \tag{7}$$

Equation (1), which is automatically fulfilled, using (6), (2-4) then converted to

$$f'''+ff''-f'^2-Mf'-A\left(f'+\frac{1}{2}A\eta f''\right)=0 \tag{8}$$

$$\begin{aligned}&A\left(\theta+\frac{1}{2}\eta\theta'\right)+f'\theta-f\theta'=\left(\frac{1+R}{P_r}\right)\theta''\\&+N_b\phi'\theta'+N_t\left(\theta'\right)^2+\gamma\theta+E_cf''^2+Jf'^2\end{aligned} \tag{9}$$

$$\frac{1}{Sc}\varnothing''-A(\eta 2^{-1}\varnothing'+\varnothing)+f'\varnothing-f\varnothing'+\frac{Nt}{Nb}\theta''+C_r\varnothing^n=0 \tag{10}$$

Dimensionless boundary constraints are:

$$f=0, f'=1, \theta=\phi=1 as\, n\to 0$$

$$f'=\theta=\phi=0 as\, n\to\infty$$

where,

$$M=\frac{\sigma B_0^{\ 2}}{\rho a},\ A=\frac{c}{a},\ E_c=\frac{U_w^{\ 2}}{C_p\left(T_w-T_\infty\right)},$$

$$\gamma=\frac{Q_0}{\rho C_p a},\ P_r=\frac{\nu}{\kappa},\ S_c=\frac{\nu}{D_B},$$

$$N_b=\frac{(\rho c)_p D_B\left(C_w-C_\infty\right)}{\nu(\rho c)_f},\ N_t=\frac{(\rho c)_p D_T\left(T_w-T_\infty\right)}{\nu(\rho c)_f T_\infty},$$

$$C_r=\chi\frac{\left(C_w-C_\infty\right)^{n-1}}{a}\phi^n,\ J=\frac{\acute{o}\mathrm{H}^2\vartheta}{\rho C_p\left(T_W-T_\infty\right)}$$

3.0 Solution Procedure

Ordinary Differential Equations (ODEs) or boundary value problems can be solved numerically by converting them into initial value issues using the shooting approach. In order to achieve the required boundary conditions, repeated adjustments are made after "shooting" an initial guess for the unknown variables. Here is a brief explanation of the shooting method. By assuming one boundary condition and considering the other as an unknown parameter, you may convert a boundary value problem into an initial value problem. Use a numerical ODE solver (such as Runge-Kutta techniques) to solve the ensuing initial value issue after making an educated guess for the unknown parameter or parameters. Compare the initial value problem's answer to the required boundary condition you didn't use in your original guess. Adjust the estimated parameter and repeat steps 2 and 3 until the solution is obtained and it satisfies the specified boundary condition within a defined tolerance. After the solution converges to the necessary boundary condition, you have found a numerical solution to the initial boundary value issue. The original assumption and the solution's behaviour both affect how well the shooting approach works. It's a process of trial and error that calls for cautious tweaks and convergent monitoring.

4.0 Results and Discussion

Figure 2 shows the graph for the values of magnetic field parameter M, which is plotted to dimensionless velocity versus span wise coordinate. The graph shows that velocity drops as M values rise. This could be the case because a rising magnetic field results from an increasing M. More Lorentz force is produced by the interaction of a magnetic field with an electrically conducting fluid. These forces can push against the fluid's motion and lower its velocity. The graph in Figure 3 is set up to show the change in temperature as a function of the values of magnetic field parameter M. This graph demonstrates how a change in the magnetic field parameter causes the system's heat to decrease. It suggests that the system is being cooled by the magnetic field. The interaction between the magnetic field and the nanofluid moving across the stretching medium is to blame for this. The thermal behaviour of the nanofluid is probably affected by the magnetic field, presumably by changing the movement or arrangement of the nanoparticles inside the fluid, which in turn modifies the characteristics of heat transmission and results in a reduction in heat The concentration of the nanoparticles is represented as a function of the change in values of the magnetic field parameter M. Figure 4 demonstrates that when M values rise, the effective concentration of the nanoparticle in the nanofluid drops. As M increases, the magnetic field likewise grows, and this interaction between the magnetic field and the nanoparticles may change how the fluid flows. The concentration and dispersion of the nanoparticles within the fluid are affected by these modifications. Owing to nanoparticle aggregation, this lowers the effective concentration of the particles in the nanofluid. Dimensionless velocity is displayed against an unstable parameter A in Figures 5-7. The velocity will decrease when the parameter 'A', which denotes unsteadiness, is increased. Due to enhanced heat transfer effects, a parameter 'A' increase that represents unsteadiness might cause a drop in temperature. By disrupting the flow pattern and the distribution of energy within the pattern, this might cause velocity to drop.

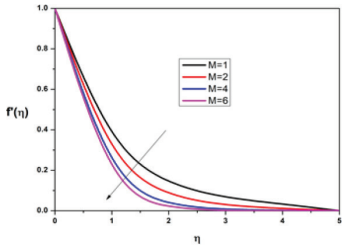


Figure 2. Plot on magnetic field parameter M versus dimensionless velocity '(η).

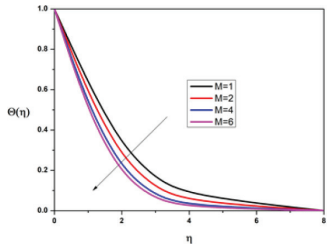


Figure 3. Plot on magnetic field parameter M versus temperature (η).

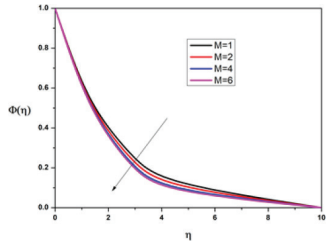


Figure 4. Plot on magnetic field parameter M versus Concentration Φ(η).

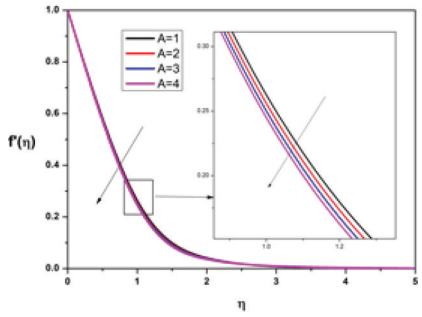


Figure 5. Velocity profile is displayed against an unstable parameter A.

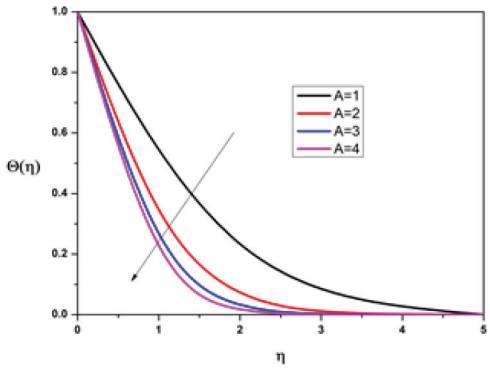


Figure 6. Temperature profile is displayed against an unstable parameter A.

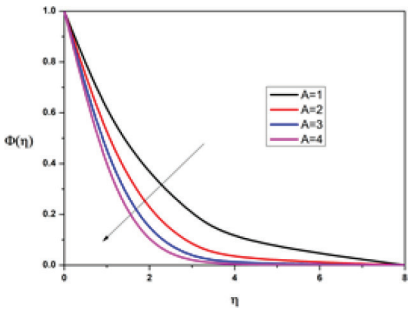


Figure 7. Concentration profile is displayed against an unstable parameter A.

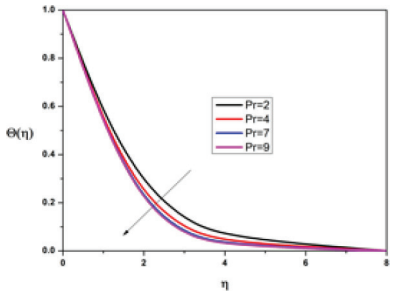


Figure 8. θ(η)is displayed against the Prandtl number Pr.

Figure 8 explains how a rise in the Prandtl number causes the system's heat to fall. It will demonstrate that, in comparison to the viscous effect, the influence of thermal diffusion is becoming more substantial. Figure 9 Plotting concentration against Prandtl number Pr results in the graph. Figure 9 demonstrates that when Pr values rise, the effective concentration of the nanoparticle in the nanofluid also rises. As Pr rises, momentum diffusivity becomes comparatively larger than thermal diffusivity. Nanoparticle concentration increases towards the stretching surface due to momentum diffusivity's uniform velocity and efficient mixing of the particles inside the fluid.

Figure 10 demonstrates how a rise in the Eckert number causes the system to get hotter. Eckert number greater denotes more or predominance of kinetic energy over internal energy changes. This might improve convective heat transfer, where heat is transferred layer by layer and leads in an increase in the system's overall heat content as heat accumulates inside it. Figure 11 demonstrates that when Ec values rise, the effective

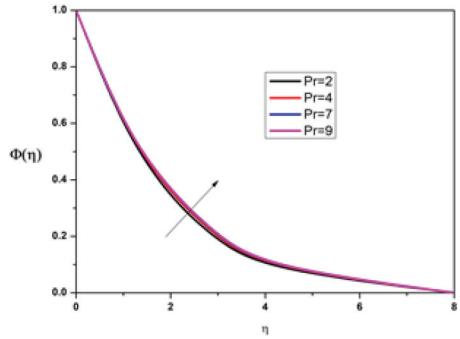


Figure 9. $\Phi(n)$ is displayed against the Prandtl number Pr.

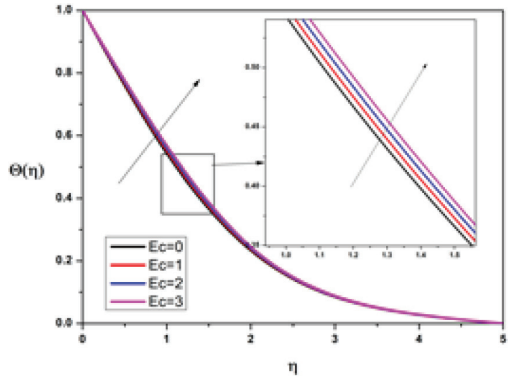


Figure 10. Heat Prandtl θ(η) is shown against Eckert number Ec.

concentration of the nanoparticle in the nanofluid falls. Ec rise refers to the stretching medium's velocity, which signifies that the fluid is moving more quickly. This means that the concentration of nanoparticles close to the stretching surface is decreasing as a consequence of the nanoparticles being carried away from the surface by the fluid. Since there is less of a temperature differential, the concentration of the nanoparticles must be higher yet greater fluid flow will dominate the thermophoretic motion of the nanoparticles by fault flowing fluid, causing the concentration to fall. Figure 12 demonstrates how a rise in R values causes the system's temperature to rise. R is greater, which indicates that ambient temperature is greater and has an impact on the system's overall thermal behaviour. Due to the fact that electrical resistance rises with temperature, the joule heating effect increases as ambient temperature rises. The amount of heat produced by joule heating increases the system's total temperature.

Figure 13 demonstrates that when R values rise, the effective concentration of the nanoparticle in the nanofluid drops. R rising indicates a high ambient temperature, which might enhance the thermal motion of nanoparticles and result in clustering, sedimentation, or nanoparticle disintegration. The effective concentration of the nanoparticle decreases in each of these situations. Figure 14 demonstrates how a rise in Cr levels causes the system's heat to rise. A rise in Cr suggests a faster rate of reaction, which signifies that the reactants are being transformed into products. As a result, the system's internal heat is reduced since more heat energy is emitted per unit of time. Figure 15 shows the curve showing the relationship between concentration and the chemical reaction parameter Cr. The graph demonstrates that when Cr values rise, the effective concentration of the nanoparticle in the nanofluid also rises. Figure 16 demonstrates how a rise in Ga (γ) values causes a rise in system heat.

Figure 17 depicts the impact of the Joule heating effect. The graphical data show that while the Joule heating increases, the internal temperature of the solution rises, yet the mass density decreases as shown in Figure 18. Because the Joule heat parameter creates warmth in the fluid, as the amount of the Joule parameter J grows, the internal temperature of the fluid rises and concentration profiles decline.

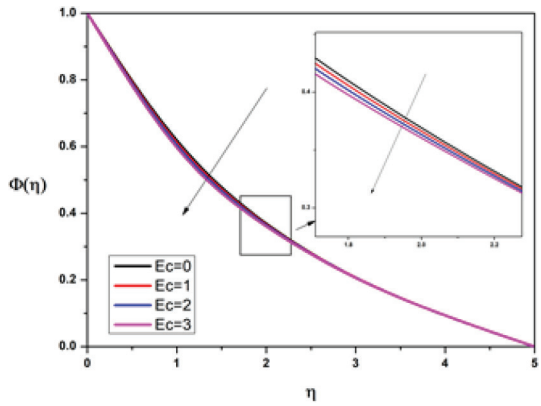


Figure 11. Concentration Φ(η) vs the Eckert number Ec.

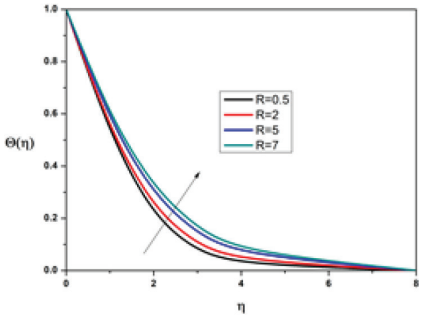


Figure 12. Heat θ(η) is displayed against Radiation parameter R.

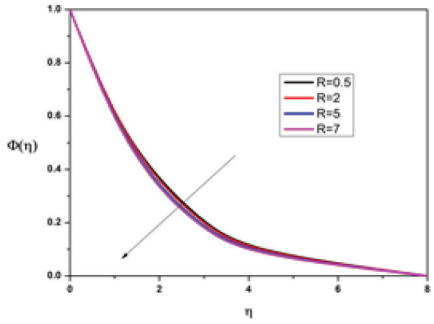


Figure 13. Concentration versus Radiation parameter R.

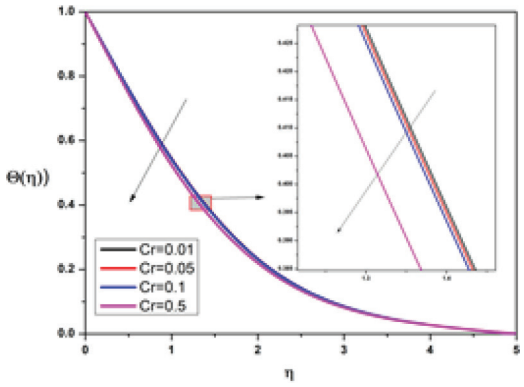


Figure 14. Heat θ(η) is shown against the chemical reaction parameter Cr.

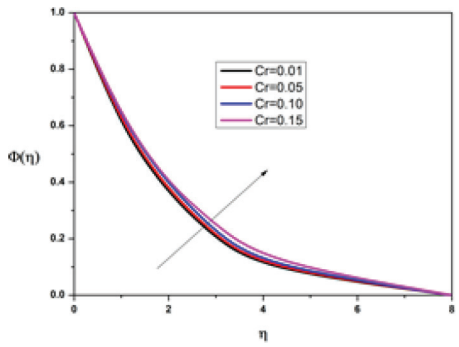


Figure 15. Concentration versus chemical reaction parameter Cr.

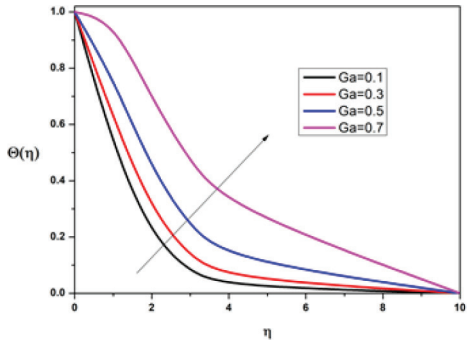


Figure 16. Heat θ(η) is displayed against Ga.

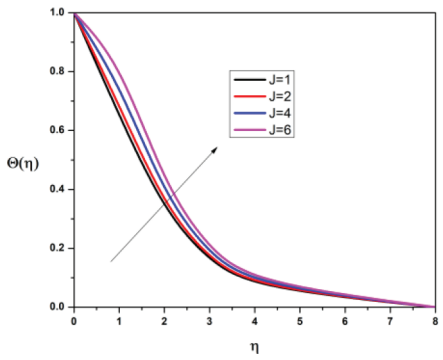


Figure 17. Plot of temperature verses Joule heating effect J.

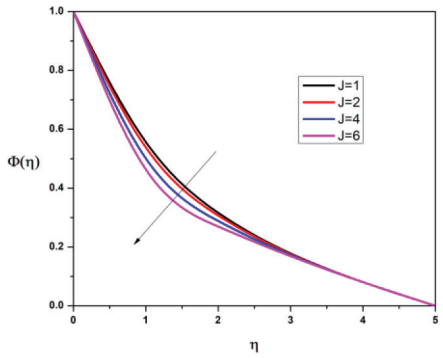


Figure 18. Plot of concentration verses Joule heating J.

Table 1. The skin friction coefficients for a number of parameters.

Pr	A	M	R	Ishak [27]	Saleem *et al.* [26]	Present Result
0.72	0	0	0	0.8086	0.80861	0.808612
1				1	1.00024	1.000241
3				1.9237	1.92361	1.923611
0.72	1	1	1	NA	0.70864	0.708642
1				NA	0.8678	0.86782
3				NA	1.6081	1.60814

Table 1 compares the skin coefficients of friction obtained for different values of Pr, A, M, and R in the absence of nanofluid to Saleem *et al.*[26] and Ishak[27]. The Nusselt number is compared in Table 2 for different amounts of Pr, Sc, Ec, and R. It also displays the current results for J and C_r. The result is relevant to the current work.

5.0 Conclusions

The unsteady nanofluid flow over a stretching surface in the presence of suction, subjected to viscous dissipation, heat source, joule heating and chemical reaction are studied. The main findings of this study are:

- Velocity reduces when the magnetic field parameter M is raised. Magnetic field therefore opposes velocity. Additionally, the temperature of the system and the concentration of nanoparticles are both inversely influenced by the magnetic field.
- Rise in unstable parameter A causes the temperature of the system and the fluid's viscosity to drop.
- A rising Pr value causes the temperature distribution to fall. The system's highly viscous fluid causes a reduction in the thickness of the boundary layer. Increasing Pr values also causes the volume fraction to rise. As a result, the volume fraction directly affects kinematic viscosity.
- Ec raises the temperature and kinetic energy. Therefore, when Ec rises, the system's temperature rises along with Ec, and concentration falls as Ec rises.
- As R value rises, system heat increases. Additionally, a higher R value causes the volume fraction to rise.
- The system's temperature rises along with the increase in Ga.

Table 2. The Heat transfer rate coefficient for a number of parameters

Sc	Pr	Ec	γ	R	C_r	J	Nusselt number Saleem *et al.* [26]	Nusselt number (Present)	Sherwod number Saleem *et al.*[26]	Sherwood number (Present)
0.5							0.14731	0.147311	0.20656	0.206566
1							0.14986	0.149863	-0.03305	-0.033051
1.5							0.1511	0.15112	-0.13568	-0.135685
	4						-0.68762	-0.687623	0.67507	0.675074
	8						-0.26558	-0.265581	1.04382	1.043822
	12						2.25205	2.252053	1.15988	1.15988
		0.5					-0.04382	-0.043822	0.22857	0.228573
		1					0.0859	0.08591	0.28849	0.288496
		1.5					0.21567	0.215674	0.34841	0.348414
			0.1				0.14757	0.147571	0.18064	0.180643
			0.2				0.10086	0.100863	0.19703	0.197032
			0.3				0.05304	0.053041	0.21353	0.213533
				0.5			0.11933	0.119335	0.22857	0.228571
				1			0.11035	0.110351	0.28849	0.288492
				1.5			0.09219	0.092196	0.34841	0.348413
					0.2		-	0.204575	-	0.354456
					0.4		-	0.200013	-	0.345568
					0.6		-	0.199355	-	0.333444
						0.1	-	0.251436	-	0.294578
						0.3	-	0.264587	-	0.306579
						0.5	-	0.273465	-	0.299945

- Increasing the parameter for chemical reactions Cr causes the system's temperature to drop and its concentration of nanoparticles to rise.
- The study sheds light on the migration of viscous nanomaterials like metals, metal oxides, carbide particles, or carbon-based nanotubes.

6.0 References

1. Choi SUS. Enhancing thermal conductivity of fluids with nanoparticles. Am Soc Mech Eng Fluids Eng Div FED. 1995; 231:99-105.
2. Keblinski P, Phillpot S, Choi SU, Eastman J. Mechanisms of heat flow in suspensions of nano-sized particles (nanofluids). Int J Heat Mass Transf. 2002; 45(4):855-63. https://doi.org/10.1016/S0017-9310(01)00175-2
3. Yu W, France DM, Routbort JL, Choi SUS. Review and comparison of nanofluid thermal conductivity and heat transfer enhancements. Heat Transf Eng. 2008; 29(5):432-60. https://doi.org/10.1080/01457630701850851
4. Mahbubul IM, Saidur R, Amalina MA. Latest developments on the viscosity of nanofluids. Int J Heat Mass Transf. 2012; 55(4):874-85. https://doi.org/10.1016/j.ijheatmasstransfer.2011.10.021
5. Andersson HI, Aarseth JB, Dandapat BS. Heat transfer in a liquid film on an unsteady stretching surface. Int J Heat Mass Transf. 2000; 43(1):69-74. https://doi.org/10.1016/S0017-9310(99)00123-4
6. Chen CK, Char MI. Heat transfer of a continuous, stretching surface with suction or blowing. J Math Anal Appl. 1988; 135(2):568-80. https://doi.org/10.1016/0022-247X(88)90172-2
7. Jawad M, Hameed MK, Nisar KS, Majeed AH. Darcy-Forchheimer flow of maxwell nanofluid flow over a porous stretching sheet with Arrhenius activation energy and nield boundary conditions. Case Stud Therm Eng. 2023; 44:102830. https://doi.org/10.1016/j.csite.2023.102830
8. Rasool G, Shafiq A, Khalique CM, Zhang T. Magnetohydrodynamic Darcy-Forchheimer nanofluid flow over a nonlinear stretching sheet. Phys Scr. 2019; 94(10):105221. https://doi.org/10.1088/1402-4896/ab18c8

9. Cui J, Jan A, Farooq U, Hussain M, Khan WA. Thermal analysis of radiative Darcy-Forchheimer nanofluid flow across an inclined stretching surface. Nanomaterials. 2022; 12(23):4291. https://doi.org/10.3390/nano12234291 PMid:36500914 PMCid:PMC9739262
10. Hayat T, Aziz A, Muhammad T, Alsaedi A. Darcy-Forchheimer three-dimensional flow of Williamson Nanofluid over a convectively heated nonlinear stretching surface. Commun Theor Phys. 2017; 68(3):387. https://doi.org/10.1088/0253-6102/68/3/387
11. Sadiq MA, Hayat T. Darcy-Forchheimer flow of magneto Maxwell liquid bounded by convectively heated sheet. Results Phys. 2016; 6:884-90. https://doi.org/10.1016/j.rinp.2016.10.019
12. Chen CH. Effect of viscous dissipation on heat transfer in a non-Newtonian liquid film over an unsteady stretching sheet. J Nonnewton Fluid Mech. 2006; 135(2-3):128-35. https://doi.org/10.1016/j.jnnfm.2006.01.009
13. Mjankwi MA, Masanja VG, Mureithi EW, James MN. Unsteady MHD Flow of Nanofluid with variable properties over a stretching sheet in the presence of thermal radiation and chemical reaction. Int J Math Math Sci. 2019; 2019:1-14. https://doi.org/10.1155/2019/7392459
14. Hayat T, Khan MI, Waqas M, Alsaedi A, Yasmeen T. Diffusion of chemically reactive species in third grade fluid flow over an exponentially stretching sheet considering magnetic field effects. Chinese J Chem Eng. 2017; 25(3):257-63. https://doi.org/10.1016/j.cjche.2016.06.008
15. Tarakaramu N, Satya Narayan PV. Unsteady MHD nanofluid flow over a stretching sheet with chemical reaction. IOP Conf Ser Mater Sci Eng. 2017; 263:062030. https://doi.org/10.1088/1757-899X/263/6/062030
16. Waini I, Ishak A, Pop I. Unsteady flow and heat transfer past a stretching/shrinking sheet in a hybrid nanofluid. Int J Heat Mass Transf. 2019; 136:288-97. https://doi.org/10.1016/j.ijheatmasstransfer.2019.02.101
17. Srinivasacharya D, Jagadeeshwar P. Effect of Joule heating on the flow over an exponentially stretching sheet with convective thermal condition. Math Sci. 2019; 13(3):201-11. https://doi.org/10.1007/s40096-019-0290-8
18. Ibrahim SM, Kumar P V., Lorenzini G, Lorenzini E. Influence of joule heating and heat source on radiative MHD flow over a stretching porous sheet with power-law heat flux. J Eng Thermophys. 2019; 28(3):332-44. https://doi.org/10.1134/S1810232819030044
19. El-dawy HA, El-Amin MF, Raizah ZA. Joule heating and viscous dissipation effects on a stretching/shrinking cannel filled by micropolar hybrid nanofluid in presence thermal/solar radiation. J Nanofluids. 2023; 12(3):738-44. https://doi.org/10.1166/jon.2023.1957
20. Abdel-Wahed M, Emam T. Effect of joule heating and hall current on MHD flow of a Nanofluid due to a rotating disk with viscous dissipation. Therm Sci. 2018; 22(2):857-70. https://doi.org/10.2298/TSCI160312218A
21. Pattnaik PK, Mishra N, Moduly MM, Mohapatra NB. Effect of chemical reaction on nanofluid flow over an unsteady stretching sheet in presence of heat source. 2018; 8(8):142-66.
22. Megahed AM, Ghoneim NI, Reddy MG, El-Khatib M. Magnetohydrodynamic fluid flow due to an unsteady stretching sheet with thermal radiation, porous medium, and variable heat flux. Adv Astron. 2021; 2021:1-9. https://doi.org/10.1155/2021/6686883
23. Reddy NN, Rao VS, Reddy BR. Chemical reaction impact on MHD natural convection flow through porous medium past an exponentially stretching sheet in presence of heat source/sink and viscous dissipation. Case Stud Therm Eng. 2021; 25:100879. https://doi.org/10.1016/j.csite.2021.100879
24. Khalili NNW, Samson AA, Aziz ASA, Ali ZM. Chemical reaction and radiation effects on MHD flow past an exponentially stretching sheet with heat sink. J Phys Conf Ser. 2017; 890:012025. https://doi.org/10.1088/1742-6596/890/1/012025
25. Jamshed W, Devi SSU, Goodarzi M, Prakash M, Nisar KS, Zakarya M, *et al*. Evaluating the unsteady Casson nanofluid over a stretching sheet with solar thermal radiation: An optimal case study. Case Stud Therm Eng. 2021; 26:101160. https://doi.org/10.1016/j.csite.2021.101160
26. Saleem S, Nadeem S, Rashidi MM, Raju CSK. An optimal analysis of radiated nanomaterial flow with viscous dissipation and heat source. Microsyst Technol. 2019; 25(2):683-9. https://doi.org/10.1007/s00542-018-3996-x
27. Ishak A. Unsteady MHD flow and heat transfer over a stretching plate. J Appl Sci. 2010; 10(18):2127-31. https://doi.org/10.3923/jas.2010.2127.2131