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Effect of Forchheimer and Joule Term in the Study of Radiated Nanomaterial Flow - a Numerical Approach

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Abstract

The current study presents the numerical evaluation of transmitted viscous nanomaterials by stretching the surface comprising Forchheimer drag force and joule effect. Heat source or sink and radioactive effects are also incorporated. For the various assumptions on the real problem, a physical model led us to a non-linear coupled partial differential equation which is then transformed to ordinary differential equation using similarity technique. The computational work is carried out using shooting technique compresses of Runge-Kutta-Felberg method with Newton-Raphson iterative scheme to solve the non-linear coupled ordinary differential equations with converting a boundary value problem to initial value problem by choosing a guess value. Later, numerical solutions are predicted for various non-dimensional parameters involved in the study of the fluid flow, temperature. In the absence of the Forchheimer and joule terms, the results are coinciding with the earlier works. Further, the outcomes of this study have a major role in many industrial application problems.

Keywords: *Joule Heating, Magnetic Field, Nanofluid, Viscous Dissipation*

1.0 Introduction

Fluids are substances that can flow and do not have a definite shape. They include liquids and gases. The basic properties of fluids include viscosity (resistance to flow), density (mass per unit volume), pressure (force exerted per unit area), and buoyancy (upward force on a submerged object). Fluid mechanics studies the behavior of fluids, including how they move, interact with surfaces, and respond to forces. Bernoulli's principle, which explains the relationship between velocity, pressure, and elevation in a fluid, is a fundamental concept in fluid dynamics. In other words, we say that in solids the different volume elements have definite relative position and these elements change their relative position only when some external force is applied.

A nanofluid is a type of fluid that contains nanoparticles suspended in a base fluid, such as water or oil. These nanoparticles are typically on the nanometer scale (1-100 nanometers) and can be made from various materials like metals, oxides, or carbon-based materials. Usually, metals, metal oxides, carbide particles, or carbon-based nanotubes are employed to create the nanoparticles that are used in nanofluids. Nanofluids exhibit unique properties due to the presence of Zinc (Zn), Copper (Cu), Manganese (Mn), Iron (Fe), Titanium dioxide (Ti) and other metallic nanoparticles, such as enhanced thermal conductivity, improved heat transfer,

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and altered viscosity. These properties make nanofluids useful in various applications, including cooling systems, heat exchangers, electronics cooling, and advanced manufacturing processes. The study of nanofluids is a part of nanotechnology and has attracted attention for its potential to improve the efficiency of heat transfer systems and other industrial processes.

The concept of nanofluids was first introduced by Masuda *et al*[1]. While term "Nanofluid" was coined by Choi and Eastman[2] in their research paper 'enhancing thermal conductivity of fluids with nanoparticles" in these purposes using innovative nanofluids which are convention heat transfer fluid with the suspended metallic nanoparticles to address the limitations of lae of thermal conductivity in energy efficient heat transfer fluids. These nanofluids are expected to exhibit higher thermal conductivities compared to currently used fluids leading to significant reduction in exchanger pumping power. Chiam focuses on "stagnation-point flow towards a stretching sheet" with identical stretching and straining velocities exact solution for the nonlinear Navier-Stoke's equation include 2D and axisymmetric stagnation poiny flows (Hiemenz and Homann) another relevant solution in the 2D flow caused by a stretching plate presented by Crane commonly used in plastic film drawing[3]. Aluminum oxide (Al_2O_3) and Titanium oxide (TiO_2) are nanoparticles that are present in blood-based fluid are examined by Cui J[4]. Darcy described a traditional idea to represent flow via a porous medium. This idea, however, is valid for evaluations that have lower velocity and permeability. On the other hand, an extended Darcy-Forchheimer model is used to characterize diverse fluid flows where nanofluids flow more quickly than the usual Darcy theory is acceptable[5-8]. Yusof *et al.* in research paper used mathematical transformations to convert the governing equations into simpler ordinary differential equations[9]. They then solved these equations using Runge-Kutta-Fehlberg method with shooting technique in Maple software. The investigation looked at how various parameters, including unsteadiness, radiation, magnetic field, and prandtl number affect the heat transfer characteristics. The results showed that increasing the prandtl number and unsteadiness parameter led to higher heat transfer rates at the surface. On the other hand, an increase in the radiation and magnetic parameters resulted in a decrease in the heat transfer rate at the surface.

Sandeep *et al.*,[10] in "Unsteady boundary layer flow of thermophoretic MHD nanofluid past on a stretching sheet with space and time dependent internal heat source or sink". This study investigates the flow of a special type of fluid under various conditions, such as magneto hydrodynamics and thermophoresis. The fluid flows over a surface that changes with time and the surrounding medium is porous allowing fluid to flow through it. Additionally, there are space and time-dependent heat sources or sink present. To analyze the researchers used mathematical transformations to simplify the equations, turning them into ordinary differential equations. These equations were then solved numerically using shooting technique. The study explores how different parameters affect the fluid's velocity, temperature and concentration profiles, with previous studies showing a close relationship between the findings. Kalidas Das[11] he went through "Nanofluid flow over a non-liner permeable stretching sheet with partial slip". The numerical study investigated nanofluid flow over a non-linear stretching sheet at a prescribed surface temperature with partial slip. By performing a parametric study, the researchers explored the effects of governing parameters on fluid flow and heat transfer. The investigation revealed that the stream wise velocity of the nanofluid decreases with increasing slip parameter and non-linear stretching parameter. Additionally, an increase in the slip and stretching parameters leads to a thicker thermal boundary layer and the nanoparticle concentration increases with t values of both parameters.

Flow and heat transfer of a nanofluid over a nonlinearly stretching sheet. A numerical study by Rana and Bhargava[12]. This paper investigates the boundary layer flow resulting from stretching a flat surface in a nanofluid with Brownian motion and thermophoresis effects. The governing equations are transformed into ordinary differential equations using a similarity transformation and solved numerically with the 'Finite Element Method'. Different nanoparticles have varying heat transfer rates and can be used to control such stretching sheet problems. Douglas Ruth and Huiping MA[13] "examines the application of the averaging theorem to derive the macroscopic flow equation from the microscopic momentum equation in porous media. The Forchheimer equation arises from area integrals in the averaged equations, while manipulations of the microscopic inertial term to obtain the Forchheimer equation are not valid. Forchheimer effects arise from

alterations in velocity and pressure fields, leading to nonlinear dependence on velocity in the model equation. Most researchers contributed to the understanding of fluid movement and heat transfer by taking into account numerous physical phenomena like joule heating and viscous dissipation[14-19]. Thermal conduction in flowing liquids influences the rate of heat transfer, which has many uses in calculating the thermal demands for machinery and processes. Fundamental fluids characteristics are significantly enhanced by nanofluids, which incorporate nanoparticles[20-24]. The objective of incorporating the Forchheimer term into the study of nanofluids is to enhance the understanding and modelling of fluid flow through porous media when inertial effects become significant. The Forchheimer term accounts for non-linear effects that arise at higher flow velocities, which can be especially relevant when nanofluids are involved due to their potential to alter fluid behavior. The combination of nanofluids and the Forchheimer. We have incorporated Forchheimer term in the momentum equation, up to our knowledge in the before work which we have considered no one has included Forchheimer term with the Nanofluid flowing on a stretching sheet. As Forchheimer term has many applications so influenced by those applications we have considered this term where Forchheimer is related to the porous media and velocity gradient.

2.0 Mathematical Formulation

Let us consider the impacts of joule heating, magnetic, and viscous dissipation in the context of the continuous two-dimensional incompressible flow the fluid conducts electricity in unsteady state.

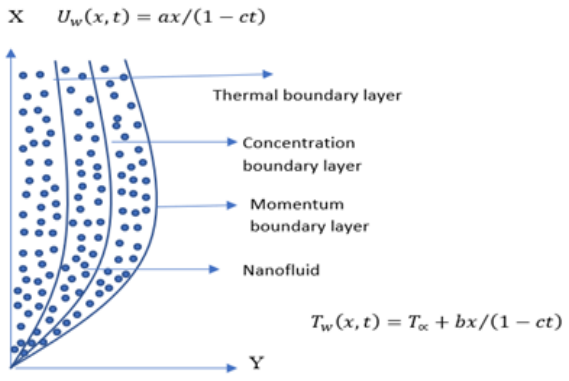


Figure 1. (a) Geometry.

Figure 1(a) depicts the physical representation of the problem under consideration. It is also assumed that a cartesian coordinate system exists, with the x-axis horizontal to the expanding region and the y-axis transverse to it. A uniform magnetic field is strongly applied to the perpendicular of the contracting sheet, where B0 is the constant magnetic strength.

Equations for the boundary layer selected from Saleem *et al.*[25]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\sigma}{\rho}B^2 u - Fu^2 \tag{2}$$

$$(\rho c)_f\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + (\rho c)_p\left\{D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Q_0}{\rho c_p}(T - T_\infty) - \frac{\sigma B_0{}^2 u^2}{\rho c_p} \tag{3}$$

$$D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} - \frac{\partial C}{\partial t} - u\frac{\partial C}{\partial x} - v\frac{\partial C}{\partial y} = 0 \tag{4}$$

Subject to the boundary condition: Saleem *et al.*[25]

$$u(x,0,t)=U_W,\ T(x,0,t) = T_W,\ C(x,0,t) = C_W,$$
$$v(x,0,t)= 0,\ u(x,\infty,t)= 0,\ C(x,\infty,t) = C_\infty,$$
$$T(x,\infty,t)=T_\infty, \tag{5}$$

Here, the parameters of the magnetic field and heat source are assumed to be $B=B_0/\sqrt{(1-ct)}$ and $Q=Q_0/\sqrt{(1-ct)}$, respectively, with B0 and Q0 as constants in order to get a similarity solution for the relevant problem.

By utilising the Rosseland approximation $q_r = -(4\sigma^*)/(3k^*)^* (\partial T4)/\partial z$ where k* is the Rosseland mean captivative constant and σ* denotes the content of the Stefan- Boltzman.

The classification of the similarity transformations is Saleem *et al.*[25]

$$\eta = y\left(\frac{U}{vx}\right)^{1/2}, \varphi = \sqrt{U_w vx} f(\eta),$$

$$\Phi(\eta) = \frac{c-c_\infty}{c_w-c_\infty}, \Theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}$$

(6)

Equation (1)'s fulfilling stream function is defined as

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \quad (8)$$

Equation (1), which is automatically fulfilled, using (6), (2-4) then converted to

$$f''' + ff'' - (f')^2[1 + Fr] - A\left[f' + \frac{\eta}{2}f''\right] - Mf' = 0$$

(9)

$$\frac{(1+R)}{P_r}\Theta'' + (f\Theta' - f'\Theta) - A(\Theta + \Theta'\eta 2^{-1}) + N_b\Theta'\Phi' + N_t(\Theta')^2 + E_c f''^2 + Jf'^2 + \gamma\theta = 0$$

(10)

$$\frac{1}{Sc}\emptyset'' - A(\eta 2^{-1}\emptyset' + \emptyset) + f'\emptyset - f\emptyset' + \frac{Nt}{Nb}\theta'' = 0$$

(11)

Reduced boundary circumstances include;

$$f = 0, f' = 1, \theta = \phi = 1 \text{ as } n \longrightarrow 0, \quad f' = \theta = \phi = 0 \text{ as } n \longrightarrow \infty \quad (12)$$

The following definition applies to the dimensionless parameters

$$E_c = \frac{u_w^2}{c_p(T_w - T_\infty)}, M = \frac{\sigma B_0^2}{\rho a}, Sc = \frac{\vartheta}{D_B}, \gamma = \frac{Q_0\theta(\eta)}{\rho c_p a},$$

$$A = \frac{c}{a}, \gamma = \frac{Q_0}{\rho c_p a}, N_b = \frac{(\rho c)_p D_B(C_w - C_\infty)}{\gamma(\rho c)_f}, N_t = \frac{(\rho c)_p D_T(T_w - T_\infty)}{\gamma(\rho c)_f T_\infty},$$

$$Fr = \frac{c_d u^2}{x\rho\sqrt{k}}, J = \frac{\sigma H^2 \vartheta}{\rho C_p (T_W - T_\infty)}$$

(13)

3.0 Shooting Technique

The shooting technique is a numerical method used to solve Ordinary Differential Equations (ODEs) or boundary value problems by transforming them into initial value problems. It involves "shooting" an initial guess for the unknown variables and adjusting it iteratively until the desired boundary conditions are satisfied. Here's a basic overview of how the shooting technique works. If you have a boundary value problem, transform it into an initial value problem by guessing one boundary condition and treating the other as an unknown parameter. Make an initial guess for the unknown parameter (s) and solve the resulting initial value problem using a numerical ODE solver (*e.g.,* Runge-Kutta methods) to obtain a solution. Compare the solution obtained from the initial value problem with the desired boundary condition that you didn't use for the initial guess. Once the answer has been achieved fits the required boundary condition within a given tolerance, adjust the estimated parameter and continue steps repeatedly. You have discovered a numerical solution to the initial boundary value problem after the solution converges to the required boundary condition. The success of the shooting technique depends on the initial guess and the behavior of the solution. It's a trial-and-error process that requires careful adjustments and convergence monitoring.

4.0 Results and Discussion

A is a non-dimensional parameter where it consists constants in ratio, so we can observe in Figure 1 little decrease in velocity as increase in unsteady parameter A. as we increase the unsteady parameter A, velocity decreases. Forchheimer term directly depends on the drag force and whenever drag force increases, velocity decreases exponentially, for different values to Forchheimer term. Figure 2 analyzes that as Forchheimer term increases velocity decreases gradually. Hartmann number is ratio of elecromagnetic force to the viscous force. Viscous force is inversely proportional to velocity, as increase in Hartmann number M, velocity should increase, but here velocity is gradually decreasing because of introducing the Forchheimer term, where it has drag force which retards the fluid flow. This is observed by giving different values for Hartmann number in Figure 3. This concludes that as increase in Hartmann number velocity decreases gradually. Since temperature is directly proportional to viscous force. Temperature decreases exponentially with the increase in Hartmann number seen in Figure 9. Also

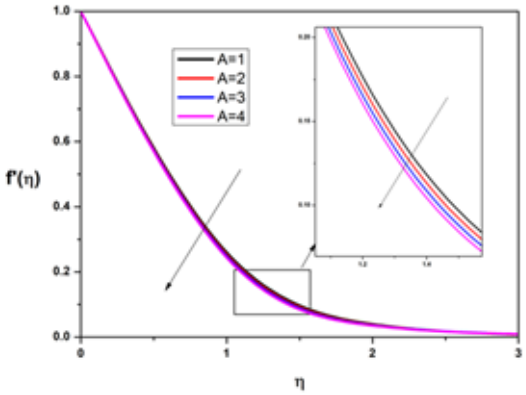


Figure 1. Variation of velocity f' (η) along Y-axis w.r.t. non-dimensional number η along X-axis in the presence of unsteady parameter A.

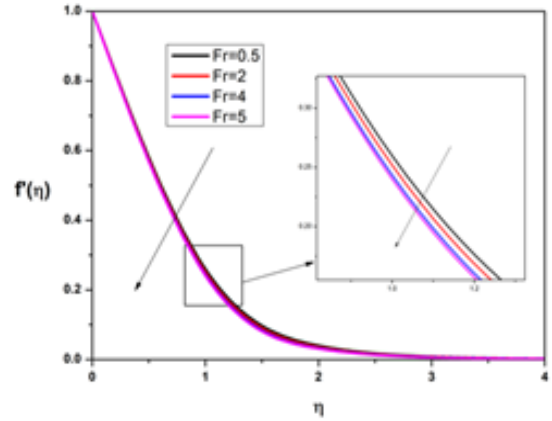


Figure 2. Variation of velocity f' (η) along Y-axis w.r.t. non-dimensional number η along X-axis in the presence of Forchheimerterm Fr.

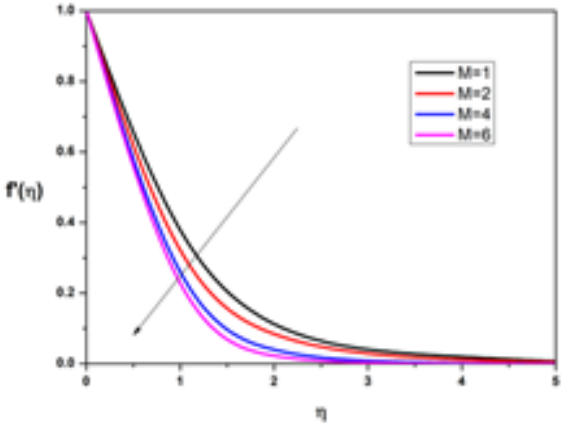


Figure 3. Variation of velocity f' (η) along Y-axis w.r.t. non-dimensional number η along X-axis in the presence of Hartmann number M

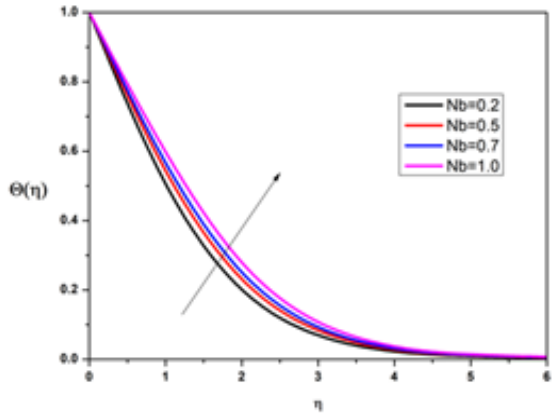


Figure 4. Variation of temperature Θ(η) along Y-axis w.r.t. non-dimensional number η along X-axis in the presence of Brownian motion constraint Nb.

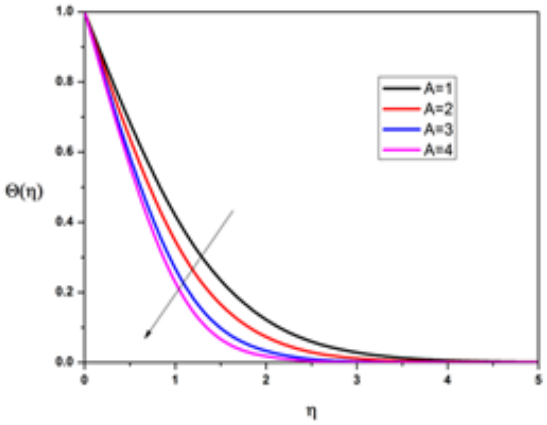


Figure 5. Variation of temperature Θ(η) along Y-axis w.r.t. non-dimensional number η along X-axis in the presence of unsteady parameter A.

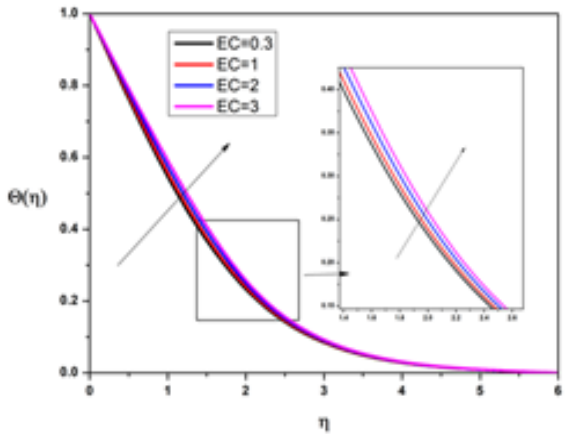


Figure 6. Variation of temperature Θ(η) along Y-axis w.r.t. non-dimensional number η along X-axis in the presence of Eckert numberEc.

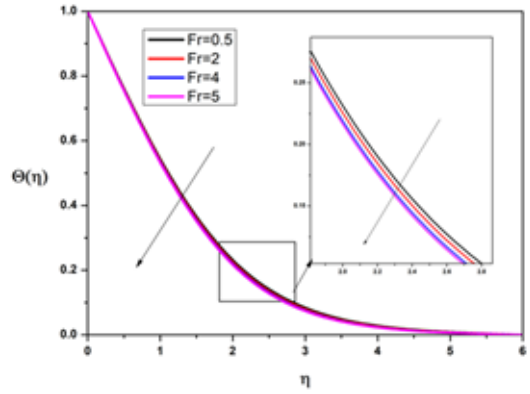


Figure 7. Variation of temperature Θ(η) along Y-axis w.r.t. non-dimensional number η along X-axis in the presence of Forchheimer term Fr.

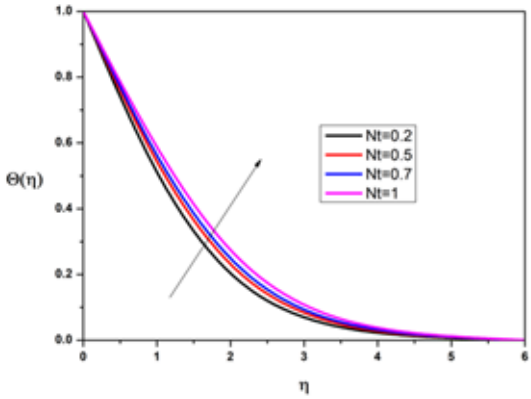


Figure 10. Variation of temperature Θ(η) along Y-axis w.r.t. non-dimensional number η along X-axis in the presence of thermophoresis constraint Nt

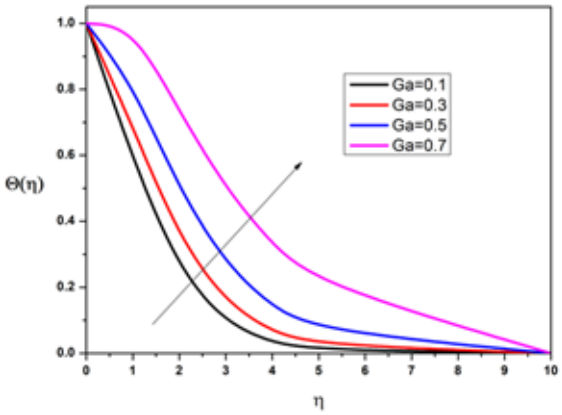


Figure 8. Variation of temperature Θ(η) along Y-axis w.r.t. non-dimensional number η along X-axis in the presence of Ga.

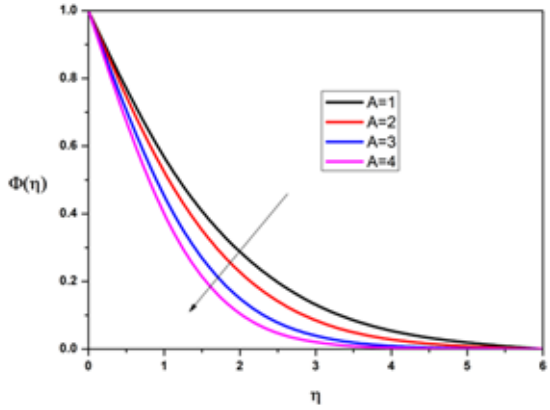


Figure 11. Variation of concentration Φ(η) along Y-axis w.r.t. non-dimensional number η along X-axis in the presence of unsteady parameter A.

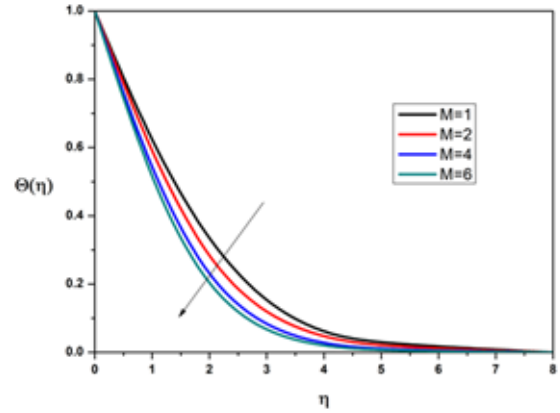


Figure 9. Variation of temperature Θ(η) along Y-axis w.r.t. non-dimensional number η along X-axis in the presence of Hartmann number M.

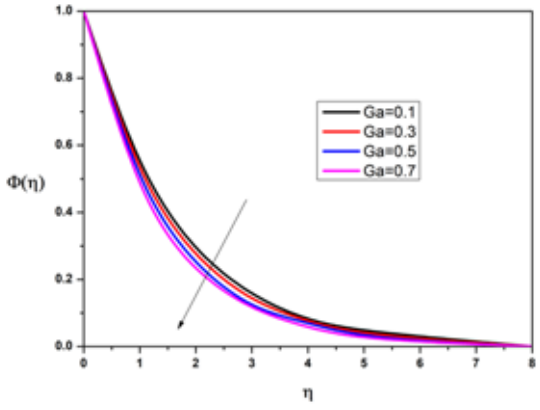


Figure 12. Variation of concentration Φ(η) along Y-axis w.r.t. non-dimensional number η along X-axis with Ga.

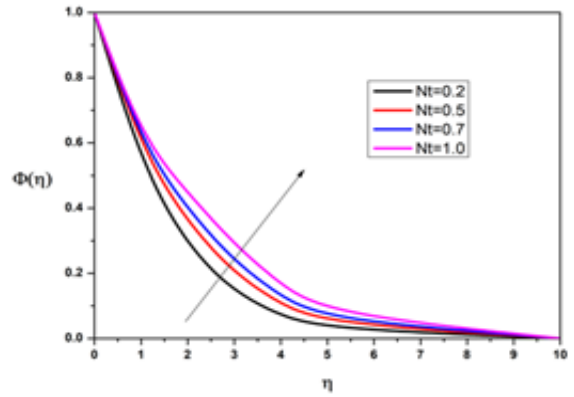


Figure 13. Variation of concentration Φ(η) along Y-axis w.r.t. non-dimensional number η along X-axis in the presence of Thermophoresis constraint Nt.

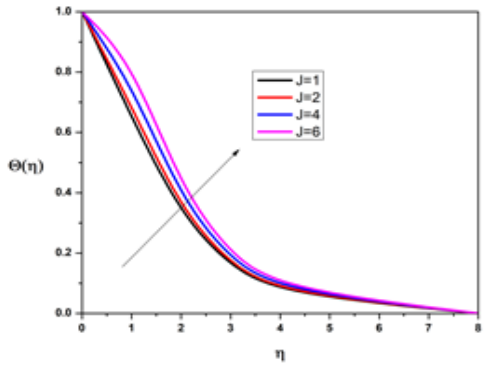


Figure 16. Variation of temperature Θ(η) along Y-axis w.r.t. non-dimensional number η along X-axis in the presence of joule heating effect J.

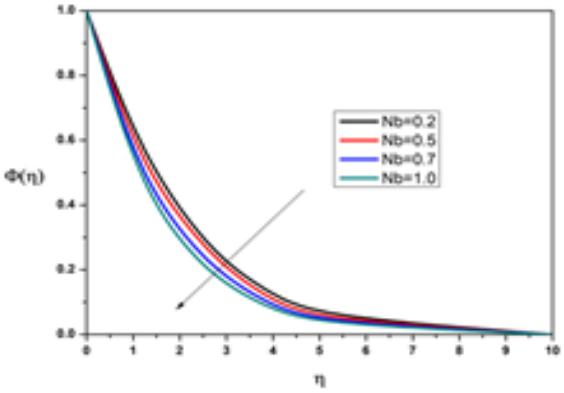


Figure 14. Variation of concentration Φ(η) along Y-axis w.r.t. non-dimensional number η along X-axis in the presence of Brownian motion constraint Nb.

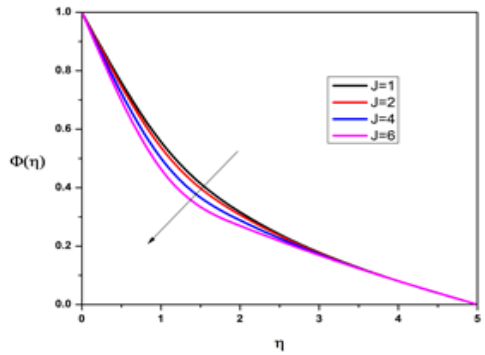


Figure 17. Variation of concentration Φ(η) along Y-axis w.r.t. non-dimensional number η along X-axis in the the presence of joule heating effect J.

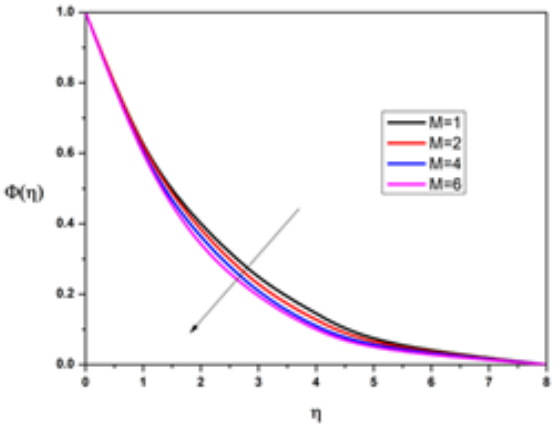


Figure 15. Variation of concentration Φ(η) along Y-axis w.r.t. non-dimensional number η along X-axis in the presence of Hartmann number M.

there is a decrease in concentration as Hartmann number increases from Figure 15.

Brownian motion is directly proportional to brownian diffusion. Brownian motion is directly proportional to temperature. Whenever brownian motion increases temperature increases. This can be observed in Figure 4 by giving different values to brownian motion. This shows the increase in temperature as the brownian motion increases. A is a dimensionless parameter i.e., unsteady parameter, it consists only constant in ratio. By the Figure 5 as unsteady parameter A increases temperature decreases a lot. Even though A has constant terms, temperature decreases, because temperature not only depends on thermopghoresis and brownian motion, but also depends on unsteady parameter A. This can be

observed by giving different values to unsteady parameter A. It concludes that as unsteady parameter increases, temperature gradually decreases.

Eckert number is a dimensionless number. It expressess the relationship between a flow's kinetic energy and the boundary layer enthalpy difference and is used to characterise the heat dissipation. Eckert number is inversely proportional to the heat dissipation. Whenever Eckert number increases heat loss decreases, this results increase in temperature. For different values of Eckert number, the temperature curve gradually increases as shown in Figure 6. The Forchheimer term is a concept in fluid dynamics that accounts for the effects of inertial resistance and non-linear flow behavior in porous media. In nanofluids, the Forchheimer term can be used to model the additional pressure drop experienced by the fluid as it flows through a porous media containing nanoparticles. Forchheimer term is depended on porosity of a fluid. Whenever porosity increases, temperature decreases, it shows, as Forchheimer term increases, temperature decreases. This is absorbed in Figure 7 by giving different values to the forchheimer term. This concludes that as Forchheimer term increases, tempearture decreases. is a dimensionless parameter, it consists only constant in ratio. By the Figure 8 it is noticed as increases temperature increases. Even though has constant terms, temperature increases, because temperature not only depends on thermopghoresis and brownian motion, but also depends on . This can be observed by giving different values to . It concludes that as increases.

The thermophoresis constraint has strong influence on temperature of the fluid. The temperature of the fluid increases with increase in thermophoresis. Temperature profile is same in both brownion motion and thermophoresis. It is observed from Figure 10 increasing temperature by giving different values for thermophoresis constraint. When N_t increseas temperature increases. A is a dimensionless parameter i.e., unsteady parameter, it consists only constant in ratio. By the graph as unsteady parameter A increases concentration decreases a lot. Even though A has constant terms, concentration decreases, because concentration not only depends on thermopghoresis and brownian motion, but also depends on unsteady parameter A. It concludes that as unsteady parameter increases, concentration gradually decreases which portrays in Figure 11. Ga(γ) is a dimensionless parameter, it consists only constant in ratio. By the graph as Ga(γ) increases concentration decreases. Even though Ga(γ) has constant terms, concentration decreases, because concentration not only depends on thermopghoresis and brownian motion, but also depends on Ga(γ). This can be observed by giving different values to Ga(γ), concentration gradually decreases in Figure 12. Thermoporosis parameter has strong influence on concentration of the fluid. Concentration profile with thermocolysis is reciprocal to the brownion motion. Whenever thermophoresis increases, concentration increases. This can be observed in Figure 13 by giving different values to thermophoresis. When N_t is least valued, thermophoresis constraint increases, concentration increases gradually. The concentration of the fluid is significantly influenced by the brownian motion parameter. As brownian motion increases, fluid concentration drops. Giving varied numbers to the

Table 1. The skin friction coefficients for a number of parameters

Pr	A	M	R	Ishak[26]	Saleem *et al.*[25]	Present Result
0.72	0	0	0	0.8086	0.80861	0.808612
1				1	1.00024	1.000241
3				1.9237	1.92361	1.923611
0.72	1	1	1	NA	0.70864	0.708642
1				NA	0.8678	0.86782
3				NA	1.6081	1.60814

Table 2. The heat transfer rate coefficient for a number of parameters

Sc	Pr	Ec	γ	R	F_r	J	Nusselt Number Saleem *et al.*,[25]	Nusselt Number (Present)	Sherwod Number Saleem *et al.*,[25]	Sherwood Number (Present)
0.5							0.14731	0.147311	0.20656	0.206566
1							0.14986	0.149863	-0.03305	-0.033051
1.5							0.1511	0.15112	-0.13568	-0.135685
	4						-0.68762	-0.687623	0.67507	0.675074
	8						-0.26558	-0.265581	1.04382	1.043822
	12						2.25205	2.252053	1.15988	1.15988
		0.5					-0.04382	-0.043822	0.22857	0.228573
		1					0.0859	0.08591	0.28849	0.288496
		1.5					0.21567	0.215674	0.34841	0.348414
			0.1				0.14757	0.147571	0.18064	0.180643
			0.2				0.10086	0.100863	0.19703	0.197032
			0.3				0.05304	0.053041	0.21353	0.213533
				0.5			0.11933	0.119335	0.22857	0.228571
				1			0.11035	0.110351	0.28849	0.288492
				1.5			0.09219	0.092196	0.34841	0.348413
					0.2		-	0.204575	-	0.354456
					0.4		-	0.200013	-	0.345568
					0.6		-	0.199355	-	0.333444
						0.1	-	0.251436	-	0.294578
						0.3	-	0.264587	-	0.306579
						0.5	-	0.273465	-	0.299945

brownian motion increasing order in Figure 14 will allow you to see this. The concentration decreases as the value of Nb rises. Figure 15 shows variation of concentration with magnetic field. Figure 16 depicts the impact of the joule heating effect. The graphical data show that while the joule heating increases, the internal temperature of the solution rises, yet the mass density decreases as shown in Figure 17. Because the joule heat parameter creates warmth in the fluid, as the amount of the joule parameter J grows, the internal temperature of the fluid rises and concentration profiles decline.

Table 1 compares the skin coefficients of friction obtained for different values of Pr, A, M, and R in the absence of nanofluid to Saleem *et al.*,[25] and Ishak[26]. The Nusselt number is compared in Table 2 for different amounts of Pr, Sc, Ec, and R. It also displays the current results for J and Fr. The result is relevant to the current work.

5.0 Conclusions

The effect of Forchheimerand joule heating term on nanofluid flow on a stretching sheet is studied and analyzed the flow with various factors such as temperature concentration and velocity with respect to non-dimensional parameters respectively are studied. Absolutely, when considering thermophoresis constraint, Brownian motion constraint, and increasing Eckert number, expect raise in temperature. These factors influence heat transfer and particle movement, leading to higher temperatures in the system.

- An increase in the unsteady parameter, Hartmann number, and Forchheimer term will generally lead to a decrease in temperature. These factors affect the fluid flow and heat transfer characteristics, resulting in cooler temperatures within the system.
- With the unsteady parameter, Hartmann number, and Forchheimer term increase, the velocity of the fluid tends to decrease. These factors can introduce additional resistance and constraints to the fluid flow, resulting in a reduction in its velocity.
- An increase in the unsteady parameter, Hartmann number, and the imposition of Brownian motion constraint, the concentration of the fluid is likely to decrease. These factors affect the movement and dispersion of particles, leading to a decrease in their concentration within the fluid.
- On increasing the thermophoresis constraint, you can expect the concentration of the fluid to increase. Thermophoresis influences the movement of particles in response to temperature gradients, causing particles to migrate towards regions of higher temperature, thus leading to an increase in their concentration in those areas.
- The study sheds light on the migration of viscous nanomaterials like metals, metal oxides, carbide particles, or carbon-based nanotubes.

6.0 Disclosure Statement

The authors declare no conflict of interest.

7.0 Author's Contribution

Sumithra R. performed the analysis, Dinesh P. A. conceived the study, Roopa K. R. and Sweeti Yadav performed the mathematical computations, Sadiya A. B. and Krishna Kanth D. worked on explanation of the findings, Sahana K. M. and Saritha C. prepared the draught paper.

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