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Numerical Study of Rayleigh-Benard and Marangoni Convection in the Presence of Magnetic Field and Heat Source/Sink

T. V. Karnakumar[1]* and Vijaya Kumar[2]

[1]KLE'S Nijalingappa College, Bengaluru - 560010, Karnataka, India; karnakumar663@gmail.com
[2]Department of Mathematics, Ramaiah Institute of Technology, MSR Nagar, MSRIT post, Bengaluru – 560054, Karnataka, India; vijaysowbhagya@gmail.com

Abstract

A numerical study of Rayleigh-Benard and Marangoni-Convection in a Newtonian liquid is studied numerically considering a combination of all possible boundary combinations such as rigid-rigid/free-free/rigid-free and isothermal/adiabatic to know the influence/effect of heat source/sink and externally applied magnetic field on the onset of convection. Initially Single-term Galerkin-technique is employed to obtain an estimate of the critical eigenvalue of the Rayleigh, Marangoni and wave numbers and then estimates are improved upon by the shooting -method, based on the Runge-Kutta-Fehlberg and Newton-Raphson methods. The effect of different parameters on the Rayleigh number and Marangoni numbers is tabulated. Moreover, the results detect that utilizing the internal heat source/sink and magnetic field have pronounced effects in determining the convection stability and instability thresholds. The results obtained demonstrates very clearly that the critical Rayleigh number & Marangoni number deviate in the same direction with respect to the parameter Chandrasekhar[1] number. It is also shown that the good agreement of the results obtained with those of Plattern and Legros[2] and Neild[3]. It is observed that The boundary combination RIRI & RIFA is more stable in respect of buoyancy and surface driven convection respectively, compared to other combinations

Keywords: *Shooting-method, Eigen-boundary Value Problem, Rayleigh -Benard, Marangoni- Convection*

1.0 Introduction

The influence of surface tension or buoyancy is the main mechanism for influencing a possible convective instability to a horizontal liquid layer heated at bottom and cooled from top. The RBC is due to buoyancy driven instability of convection and Marangoni-Benard convection is on account of surface-tension driven instability of convection. The convective instability from the combined influence of thermal buoyancy and surface-tension is the BMC. Theoretical study with linear analysis of Marangoni-Benard convection was started by Pearson[4] who considered non deformable case with an infinite liquid layer and the no-slip boundary condition case for the zero gravity at the bottom. He found that forces of surface tension can induce convection in the absence of buoyancy forces at which the Marangoni number exceeds a critical value. In the instability investigation of Marangoni-Benard, the instability of convection is influenced by temperature gradient which is linearly decreasing with height of liquid film. Sparrow *et al.*,[4] and Roberts[5] investigated the nonlinear distribution of temperature in a horizontal liquid layer for the thermal instability, which is induced by internal heat source.

**Author for correspondence*

Rudraiah and Sekhar[6] investigated the onset of convection in Boussinesq magnetic liquids with heat source and obtained that influence of heat source is to un-stabilize the system. The quadratic temperature profile state effect induced by heat source internally for B'enard-Marangoni convection was addressed first by Char and Chiang[7]. After, Wilson[8] studied the internal heat source effect on the start of instability of Marangoni-B'enard when there is a conducting lower boundary and when it is adiabatic to temperature perturbations. He obtained that the internal heat source is a controlling factor for the stability of liquid layer. Rudraiah and Siddheshwar[9] studied the onset of Marangoni convection influenced by non-uniform temperature gradient in a Boussinesq liquid having suspended particles and showed that the liquid layer having suspended particles heated at the bottom is more stable compared to the liquid not having suspended particles. Siddheshwar and Pranesh[10] analyzed the effect of magnetic field on the start of instability of surface tension and buoyancy forces in a Boussinesq liquid with suspended particles under 1g and g situations and obtained that the electrically conducting liquid surface having suspended particles heated at the bottom is more stable compared to an electrically conducting liquid not having suspended particles. Anwar *et al.*[11] investigated the influence of body force and surface tension driven convection flow with applied magnetic field and heat source in an electrically conducting liquid and they presented graphically in terms of isotherm and stream line plots, the influence of heat source along with combined influence of magnetic field and surface tension. Mahmud *et al.*[12] studied the influence of control in a micro polar liquid on the starting of BMC. They obtained that there is an increase in the due to controller gain parameter. Alloui *et al.*[13] analyzed the uncertainty of the basic state in a micropolar liquid for the onset of Benard-Marangoni convection. They discovered that the start of instability depends on the micro-rotation boundary condition and the parameters B, , K. Siddheshwar and Revathi[14] have employed shooting technique for good estimates of the eigenvalue in the RBMC with general boundary conditions on temperature and velocity. In their investigation they have established good estimate of eigenvalues. Siddheshwar *et al.*[15] analyzed the Rayleigh-B'enard magneto-convection in detail for all possible general boundary combinations and similarly for the Marangoni magneto-convection. The HORR technique is applied to get the eigenvalue for the issue. They found a good agreement results with that of Pranesh and Bawa[16] presented the combined effect of internal heat source and transverse magnetic field in a Nano fluid on the Rayleigh-B\'{e}nard convection. They showed that increase in H stabilizes whereas increase in internal heat source destabilizes the system. This paper is focused to study the problem numerically to obtain the good estimates of eigenvalues in respect of B-M-C and R-B-C in Newtonian fluids with general boundary combinations on temperature and velocity by employing shooting technique along with Runge-kutta-Felberg (RKF45) method. Less amount of details has been reported on the present problem explaining the Shooting-technique to get the eigenvalue under the influence of magnetic-field and internal heat source in the problem of convection. In the equations there is coupling between the temperature and velocity, also in the Boundary Conditions (BC's). This concept of coupled governing equations together with an eigenvalue to be determined makes this problem interesting.

1.1 Mathematical Formulation

Consider an infinite horizontal liquid layer of thickness d restricted between two plates. A Cartesian co-ordinate system is taken with the lower plate in the xy-plane and z-axis vertically upwards. The lower plate at z = 0 and the upper plate at z = d is considered subjected to temperature distribution $T_0 + \Delta T$ and T_0 respectively with an introduction of vertical magnetic field in existence of heat source/sink (Figure 1).

The system of equations for the problem considered are:

Equation of continuity: $\nabla.\vec{q} = 0$ (1)

Equation of linear momentum:

$$\rho_0\left[\frac{\partial\vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}\right] = \nabla\ p - \rho\vec{g} + \mu_f\nabla^2\vec{q} - {\mu_m}^2\,\sigma\,{H_0}^2\vec{q} \quad (2)$$

Energy equation: $\frac{\partial\vec{T}}{\partial t} + (\vec{q}.\nabla)T = \kappa\,\nabla^2 T$ (3)

Magnetic Induction equation:

$$\frac{\partial\vec{H}}{\partial t} = \nabla\times\left(\vec{q}\times\vec{H}\right) + \nu_m\nabla^2\vec{H} \quad (4)$$

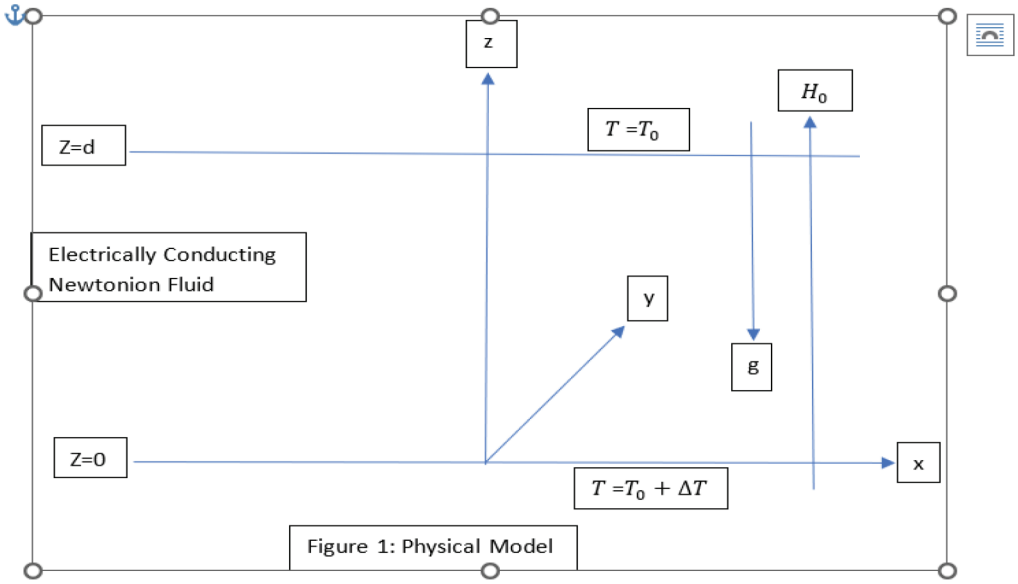


Figure 1: Physical Model

Equation of state: $\rho = \rho_0[1-\alpha(T-T_0)]$ (5)

Where q: (u,v,w)- velocity of the liquid, *p*- density, P- pressure, μ_f - fluid viscosity, *g*: (0, 0, - *g*)- gravity, μ_m – magnetic-permeability, *H*0- uniform transverse applied magnetic- field, p_0- reference density, K– thermal-diffusivity. To begin with let

$$\vec{q} = \vec{q}_b = 0 \tag{6}$$

$$T = T_b(z) = T_0 + \Delta T(1-z) \tag{7}$$

$$P = p_b(z) \tag{8}$$

$$\rho_b(z) = \rho_0[1-\alpha\Delta T(1-z)] \tag{9}$$

Where α is positive called the coefficient of thermal-expansion, a constant. Suppose that the system initially disturbed. Then the linearized equations are

$$p_0\frac{\partial(\nabla^2 w')}{\partial t} = p\alpha g T' + \mu_f\nabla^2(\nabla w') - \mu_m^2\sigma H_0^2\nabla^2 w' \tag{10}$$

$$-w'\left[\left(\frac{\Delta T\sqrt{R_I}}{d}\right)\frac{\cos\left(\sqrt{R_I}\left(1-\frac{z}{d}\right)\right)}{\sin\sqrt{R_I}}\right] = -\kappa\left(\frac{R_I\Delta T}{d^2\sin\sqrt{R_I}}\right)\sin\left(\sqrt{R_I}\left(1-\frac{z}{d}\right)\right) + \kappa\nabla^2 T' - \left[\frac{\partial\phi'}{\partial z}\frac{\partial T'}{\partial x} + \frac{\partial\phi'}{\partial x}\frac{\partial T'}{\partial z}\right] + \frac{Q_T\Delta T}{\sin\sqrt{R_I}}\left[\sin\left(\sqrt{R_I}\left(1-\frac{z}{d}\right)\right) + Q_T T'\right] \tag{11}$$

Where, $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and $\nabla^2 = \nabla_1^2 + D^2$

Non-dimensionalizing equations (10) and (11) using the following scaling:

$$T^* = \frac{T'}{\beta d}, \quad w^* = \frac{w'}{\kappa/d}, \quad t^* = \frac{t'}{d^2/\kappa} \text{ and } (x^*, y^*, z^*) = \frac{(x,y,z)}{d} \tag{12}$$

Resulting non-dimensional governing equations are:

$$\frac{1}{\mathrm{P}}\frac{\partial(\nabla^2 w^*)}{\partial t^*} = R\nabla_1^2 T^* + G(z)\nabla^4 W^* - Q\nabla^2 W^* \tag{13}$$

Where $\Pr = \frac{\gamma}{\kappa}$, $R = \frac{\alpha\beta g d^4}{\gamma\kappa}$, $G(z) = \frac{\mu_f d^2}{\rho_0\gamma}$, $Q = \frac{\mu_m^2\sigma H_0^2 d^2}{\rho_0\gamma}$. Which are respectively, the Prandtl number, the Rayleigh number and Chandrashekar number. R_I –Internal Rayleigh Number.

$$\frac{\partial(T^*)}{\partial t^*} - \nabla^2 T^* = W^*, \tag{14}$$

(13) and (14) is of the form

$$\lfloor W^*, T^* \rfloor = [W(z), T(z)]e^{[i\pi(lx+my)]} \tag{15}$$

Where l and m are the horizontal components of the wave number α α and $\alpha^2 = l^2 + m^2$, $W^*(z)$ and $T^*(z)$ are the amplitudes. Using equation (15) in (13) and (14) we have

$$\left(D^2 - \pi^2\alpha^2\right)^2 W - Q\left(D^2 - \pi^2\alpha^2\right)W - R\pi^2\alpha^2\theta = 0 \quad (16)$$

$$W + \left(D^2 - \pi^2\alpha^2\right)\theta = 0 \quad (17)$$

Where $D = \frac{d}{dz}$, The above system of equations is solved using the boundary condition

$$w = D^2 w = \theta = 0 \text{ , at } z = 0,1 \quad (18)$$

Table 1. Different boundary- combinations and corresponding trial - functions for Rayleigh-B *e′* nard convection

Case	Boundary combinations	BC's	Trial- function
1	R-I & R-I	w=Dw= θ=0, z=0 w=Dw=θ =0, z=1	$w = z^2(1-z)^2$ $\theta = z(1-z)$
2	R-A & R-I	w= Dw=$D\theta$ = 0, z=0 w= Dw=θ = 0, z=1	$w = z^4 - 2z^3 + z^2$, $\theta = z^2 - 2z$
3	R-I & F-I	w= Dw=θ = 0,z=0 w= D^2w=θ = 0, z=1	$w = 2z^4 - 5z^3 + 3z^2$, $\theta = z(1-z)$
4	R-A & F-I	w= Dw= $D\theta$ = 0, z=0 w= D^2w=θ = 0, z=1	$w = 2z^4 - 5z^3 + 3z^2$, $\theta = z^2 - 1$
5	R-A & R-A	w= Dw= $D\theta$ = 0, z=0 w= Dw= $D\theta$ = 0, z=1	$w = z^4 - 2z^3 + z^2$, $\theta = 1$
6	R-I & F-A	w=Dw=θ =0, z=0 w= D^2w= $D\theta$ = 0, z=1	$w = 2z^4 - 5z^3 + 3z^2$, $\theta = z^2 - 2z$
7	F-I & F-I	w= D^2w=θ = 0, z=0 w= D^2w=θ = 0, z=1	$w = z^4 - 2z^3 + z$, $\theta = z^2 - z$
8	F-A & F-I	w= D^2w= $D\theta$ = 0, z=0 w= D^2w=θ = 0, z=1	$w = z^4 - 2z^3 + z$, $\theta = z^2 - 1$
9	R-A & F-A	w= Dw= $D\theta$ = 0, z=0 w= D^2w= $D\theta$ = 0, z=1	$w = 2z^4 - 5z^3 + 3z^2$, $\theta = 1$
10	F-A & F-A	w= D^2w= $D\theta$ = 0, z=0 w= D^2w= $D\theta$ = 0, z=1	$w = z^4 - 2z^3 + z$, $\theta = 1$

R-I&R-I: Rigid – Isothermal & Rigid – Isothermal;
R-A&R-I: Rigid – Adiabatic & Rigid – Isothermal
R-I&F-I: Rigid – Isothermal & Free – Isothermal;
R-A&F-I: Rigid-Adiabatic & Free-Isothermal
R-A&R-A: Rigid – Adiabatic & Rigid – Adiabatic;
R-I&F-A: Rigid – Isothermal & Free - Adiabatic
F-I&F-I: Free – Isothermal & Free – Isothermal;
F-A&F-I: Free – Adiabatic & Free – Isothermal
R-A&F-A: Rigid – Adiabatic & Free – Adiabatic;
F-A&F-A: Free – Adiabatic & Free - Adiabatic

Table 2. Different boundary - combinations and corresponding trial-functions for Marangoni Convection

Case	Boundary combinations	Boundary condition	Trial function
1	R-I & F-A	w=0, Dw=θ = 0, z=0 w=0, D^2w+θ =Dθ =0, z=1	$w = z^2(1-z^2)$, $\theta = z\left(1-\frac{z}{2}\right)$
2	F-I & F-A	w= D^2w=θ =0, z=0 w= D^2w+θ =Dθ=0, z=1	$w = z^4 - 2z^3 + z$, $\theta = z^2 - 2z$
3	R-A & F-A	w= Dw=$D\theta$ = 0, z=0 w= D^2w+θ =Dθ=0, z=1	$w = z^2 - z^3$, $\theta = 1$
4	F-A & F-A	w= D^2w=Dθ=0, z=0	$w = z^4 - 2z^3 + z$, $\theta = 1$

Table 3. Comparison of and present values with those of Nield (2) for the ideal case

Boundary – combination	Nield (3) M_c .....a_c	Present Problem M_c .....a_c
R-I & F-A	79.61 1.992	79.5595 1.998
F-I & F-A	---- ---	48.5111 1.715
R-A & F-A	48.00 0.000	48.0000 0.000
F-A & F-A	--- ---	24.0000 0.000

Table 4. Comparison of M_c. and a_c present values with those of Plattern and Legros (1) for the ideal case

Boundary combination	Plattern and Legros (2) M_c .....a_c	Present problem M_c .....a_c
R-I & R-I	1707.762 3.12	1707.7522 3.1164
R-A & R-I (R-I & R-A)	1295.781 2.55	1295.80 2.5500
R-I & F-I (F-I & R-I)	1100.657 2.68	1100.6468 2.6822
R-A & F-I (F-I & R-A)	816.748 2.21	816.79 2.2148
R-A & R-A	720.000 0.00	720.00 0.0000
R-I & F-A (F-A & R-I)	669.001 2.05	669.00 2.0800
FIFI	657.511 2.22	657.5105 2.2214
F-A & F-I (F-I & F-A)	384.693 1.76	384.69 1.7592
R-A &F-A (F-A&R-A)	320.000 0.000	320.00 0.0000
F-A & F-A	120.000 0.000	120.000 0.0000

The BC's and the corresponding trial functions are given in Tables 1 and 2 respectively.

2.0 Estimation of the Eigenvalue Using the Single-Term Galerkin Method

Let $w \to w_i$, $\theta \to \theta_i$ multiply equation (16) by w_j and equation (17) by θ_j and integrate with respect to z from 0 to 1. On solving the resulting equation we obtain the following expression for the eigenvalue R in terms of Q and R_I.

$$R = \frac{\left\langle \left(D^2 w\right)\right\rangle + \left(2\pi^2\alpha^2 + Q\right)\left\langle \left(Dw\right)^2\right\rangle + \left(\pi^4\alpha^4 + Q\pi^2\alpha^2\right)\left\langle w^2\right\rangle + \left[\pi^2\alpha^2\left\langle \theta^2\right\rangle + \left\langle \left(D\theta\right)^2\right\rangle\right]}{\pi^2\alpha^2\left\langle w\theta\right\rangle\left\langle w\theta\right\rangle}$$

3.0 Marangoni Convection

If the fluid layer is confined between free surfaces, we get the velocity BC's in respect of surface-tension owing to the temperature [Pearson[4]]. By following the same procedure for obtaining an expression of Rayleigh number R, we can obtain an expression for the eigenvalue M (Marangoni number) with corresponding deformable boundary conditions is as follows

$$M = \frac{A_1 + \left(2\pi^2\alpha^2 + Q\right)A_2 + \left(\pi^4\alpha^4 + \pi^2\alpha^2 Q\right)A_3 - R\pi^2\alpha^2 A_4}{2\pi^2\alpha^2\left[\left(1+R_I\right)A_6 - A_5\right]} \quad (20)$$

where

$$A_1 = \left\langle \left(D^2 w\right)^2\right\rangle,\ A_2 = \left\langle \left(Dw\right)^2\right\rangle,\ A_3 = \left\langle \left(w^2\right)\right\rangle,$$

$$A_4 = \frac{\left\langle \left(w\theta\right)^2\right\rangle}{\left(\pi^2\alpha^2 - R_I\right)\left\langle \left(\theta\right)^2\right\rangle + \left\langle \left(D\theta\right)^2\right\rangle},$$

$$A_4 = \frac{\left\langle \left(w\theta\right)^2\right\rangle}{\left(\pi^2\alpha^2 - R_I\right)\left\langle \left(\theta\right)^2\right\rangle + \left\langle \left(D\theta\right)^2\right\rangle},$$

Table 5. Values of R_c and the corresponding a_c for the different values of Q and R_I on the stability of the Rayleigh - Be′nard convection

R-I&R-I Q	$R_I = -0.2$ M_c ..... a_c	$R_I = 0.0$ M_c ..... a_c	$R_I = 0.2$ M_c ..... a_c
0	1733.3048 2.8754	1707.7550 3.1164	1698.2728 3.0426
0.5	1756.3426 2.8773	1729.9845 3.1180	1720.3141 3.0442
1	1779.3802 2.8789	1752.2047 3.1196	1743.3531 3.0143
1.5	1802.4183 2.8808	1774.4144 3.1211	1766.3631 3.0150
2	1825.4562 2.8824	1796.6143 3.1227	1788.4037 3.0174
R-A&R-I/ R-I&R-A Q	$R_I = -0.2$ M_c ..... a_c	$R_I = 0.0$ M_c ..... a_c	$R_I = 0.2$ M_c ..... a_c
0	1333.8152 2.2798	1295.7806 2.5563	1273.8381 2.1391
0.5	1353.4433 2.2478	1312.6657 2.5556	1302.6949 2.1067
1	1372.8832 2.2471	1329.5394 2.5550	1315.4439 2.1061
1.5	1392.1478 2.2154	1346.4018 2.5544	1334.6313 2.3254
2	1411.7656 2.2148	1363.2530 2.5538	1352.5697 2.2930
R-I&F-I/ F-I&R-I Q	$R_I = -0.2$ M_c ..... a_c	$R_I = 0.0$ M_c ..... a_c	$R_I = 0.2$ M_c ..... a_c
0	1123.6283 2.6156	1100.6844 2.6697	1086.73352.6603
0.5	1147.6568 2.6235	1122.9041 2.6772	1110.24962.6050
1	1171.6711 2.5993	1145.0865 2.6844	1133.23232.5808
1.5	1194.7200 2.6065	1167.2332 2.6914	1156.31612.5880
2	1218.7535 2.6134	1189.3440 2.6983	1179.42912.5632
R-A&F-I/ F-I&R-A Q	$R_I = -0.2$ M_c ..... a_c	$R_I = 0.0$ M_c ..... a_c	$R_I = 0.2$ M_c ..... a_c
0	851.1822 1.9273	816.7491 2.2031	799.6699 2.1111
0.5	872.9324 1.9376	833.8915 2.2129	819.8043 2.0206
1	894.632 1.9041	850.9998 2.2170	840.0038 1.9930
1.5	916.289 1.8767	868.0759 2.2207	860.2464 1.9653
2	937.9282 1.8805	885.1211 2.2242	880.5198 1.9374
R-I&F-A/ FARI Q	$R_I = -0.2$ M_c ..... a_c	$R_I = 0.0$ M_c ..... a_c	$R_I = 0.2$ M_c ..... a_c
0	702.6918 1.8959	669.0038 2.0831	648.2986 1.8912
0.5	683.8066 2.0872	683.8066 2.0872	663.0329 1.9264
1	734.5132 1.9041	698.5962 2.0913	677.7999 1.8987
1.5	750.4035 1.9082	713.3734 2.0951	692.5460 1.9025
2	766.2809 1.9119	728.1389 2.0985	707.4577 1.9060

$$A_5 = \frac{\langle (w\theta) \rangle}{2\left[\left(\pi^2\alpha^2 - R_I\right)\langle(\theta)^2\rangle + \langle(D\theta)^2\rangle\right]},$$

$$A_6 = \frac{(24 - 6\pi^2\alpha^2)}{\pi^4\alpha^4}$$

Employing the shooting technique, the critical values of Marangoni number M and the wave number a for the respective cases are recorded in the Table 7. The values are covenant with Nield[3] which are presented in Table 3.

4.0 Result and Discussion

In this study, we have determined the influence of heat source and magnetic - field on Rayleigh -Benard and Marangoni

Table 6. Values of R_c, and the corresponding a_c for the different values of Q and R_I on the stability of the Rayleigh-Bnard convection

R-A&R-A Q	$R_I = -0.2$ M_c ..... a_c	$R_I = 0.0$ M_c ..... a_c	$R_I = 0.2$ M_c ..... a_c
0	881.6451 1.1573	720.4440 0.00031	153.2974 0.3848
0.5	892.6746 1.1554	728.5554 0.00031	157.0492 0.3848
1	903.6937 1.1535	737.1220 0.00031	160.1842 0.3848
1.5	914.7124 1.1520	745.6691 0.00031	163.2044 0.3848
2	925.7307 1.1501	754.2055 0.00031	165.5559 0.3848
F-I&F-I Q	$R_I = -0.2$ M_c ..... a_c	$R_I = 0.0$ M_c ..... a_c	$R_I = 0.2$ M_c ..... a_c
0	664.9703 2.2654	657.51862.2267	648.00992.2506
0.5	686.9795 2.2839	679.66462.2449	670.00702.2688
1	709.9917 2.3015	701.69642.2625	692.00282.2861
1.5	731.9920 2.3181	723.62292.2792	713.99782.3027
2	755.0056 2.3338	745.45122.2949	735.99192.3184
F-A&F-I/F-I&F-A Q	$R_I = -0.2$ M_c ..... a_c	$R_I = 0.0$ M_c ..... a_c	$R_I = 0.2$ M_c ..... a_c
0	394.7189 1.8686	384.70891.7470	378.8323 1.7492
0.5	409.5578 1.6930	399.43891.7596	393.1763 1.7611
1	425.6776 1.6735	414.11551.7712	407.4191 1.7728
1.5	441.5920 1.6534	428.74231.7822	412.9872 1.8148
2	456.8278 1.6641	443.32221.7929	429.4824 1.7938
R-A&F-A/F-A&R-A Q	$R_I = -0.2$ M_c ..... a_c	$R_I = 0.0$ M_c ..... a_c	$R_I = 0.2$ M_c ..... a_c
0	426.7652 1.0106	320.6751 0.00031	76.0519 0.3848
0.5	436.6972 1.0119	327.6135 0.00031	78.0663 0.3848
1	446.6239 1.0128	335.2163 0.00031	80.0732 0.3848
1.5	456.5464 1.0141	342.8197 0.00031	82.0730 0.3848
2	466.4653 1.0150	350.4853 0.00031	84.0662 0.3848
F-A&F-A Q	$R_I = -0.2$ M_c ..... a_c	$R_I = 0.0$ M_c ..... a_c	$R_I = 0.2$ M_c ..... a_c
0	174.2241 1.0662	120.0000 0.00031	29.5385 0.5105
0.5	182.3677 1.0411	126.0707 0.00031	30.9665 0.5105
1	190.4686 1.0156	132.1407 0.00031	34.4286 0.5105
1.5	198.5499 1.0524	138.2107 0.00031	35.9630 0.5105
2	206.6134 1.0577	144.2801 0.00031	37.8554 0.5105

convections under different boundary combinations. Employing Rayleigh-Ritz technique of single-term one can find R directly in terms of R, Q and a. One can see that Re is a linear -function of R and Q. From the Table values, we have R increases with increase in Q and R. depicts reverse case otherwise. This is one of the main effect of applied magnetic field which induce stiffness to the electrically-conducting fluid leading to the stability of the system, whereas the heat source leading to destabilization. It is clear from the tables that this result is prevalent for all boundaries mentioned. Comparison of the results of Rayleigh Bénard convection, we find that in general

$$R_c^{R-I\,\&R-I} > R_c^{R-A\,\&R-I} > R_c^{R-I\,\&F-I} > R_c^{R-A\,\&F-I} > R_c^{R-A\,\&R-A} > R_c^{R-I\,\&F-A} > R_c^{F-I\,\&F-I} > R_c^{R-A\,\&F-I} > R_c^{R-A\,\&F-A} > R_c^{F-A\,\&F-A}$$

Table 7. Values of M_c and the corresponding a_c for the different values of Q and R_I on the stability of the Bnard-Marangoni - Convection

R-I&F-A Q	$R_I = -0.2$ M_c ..... a_c	$R_I = 0.0$ M_c ..... a_c	$R_I = 0.2$ M_c ..... a_c
0	82.02952.0278	79.5595 1.9977	77.0500 1.9678
0.5	84.03002.4718	81.3295 2.4322	78.5943 2.3898
1	85.90592.4730	83.1481 2.4331	80.3548 2.3907
1.5	87.8132 2.0341	85.1736 2.0068	82.4921 1.9766
2	89.74332.0382	87.0470 2.0077	84.3078 1.9776
F-I&F-A Q	$R_I = -0.2$ M_c ..... a_c	$R_I = 0.0$ M_c ..... a_c	$R_I = 0.2$ M_c ..... a_c
0	49.99161.7429	48.5111 1.7156	46.9978 1.6864
0.5	51.82001.8280	50.3954 1.7281	48.7104 1.7768
1	53.75131.8400	52.1625 1.8152	50.5394 1.7885
1.5	55.67981.8513	54.0402 1.8262	52.3653 1.7991
2	57.60581.8588	55.9152 1.8337	54.1885 1.8064
R-A&F-A Q	$R_I = -0.2$ M_c ..... a_c	$R_I = 0.0$ M_c ..... a_c	$R_I = 0.2$ M_c ..... a_c
0	70.91221.2779	48.0000 0.0000	
0.5	70.32531.0144	48.8381 0.0000	
1	72.02381.0147	49.6727 0.0000	
1.5	73.72251.0147	50.5040 0.0000	
2	75.42161.0150	51.3324 0.0000	
F-A&F-A Q	$R_I = -0.2$ M_c ..... a_c	$R_I = 0.0$ M_c ..... a_c	$R_I = 0.2$ M_c ..... a_c
0	36.19240.9720	24.0000 0.0000	
0.5	37.82170.9258	25.2132 0.0000	
1	39.49810.9371	24.5659 0.0000	
1.5	41.17290.9427	27.6396 0.0000	
2	42.84620.9448	0.0000	

We now discuss the results obtained on the Marangoni - convection problem. From the Tables 3 and 7, we find that

$$M_c^{R-I\ \&F-A} > M_c^{F-I\ \&F-A} > M_c^{R-A\ \&F-A} > M_c^{F-A\ \&F-A}$$

We observed that our results in respect of Rayleigh Benard convection agree satisfactorily with Plattern and Legros[2] for a Newtonian fluid with Q = 0 which is presented in Table 4. In case of Marangoni Convection, it agrees. The results with regard to instability analysis of double diffusive Marangoni convection are presented in Table 3 and the tabulated results are compared with those of Nield[3].

5.0 Conclusion

The onset of Rayleigh-Bénard and Marangoni convection with heat source and applied magnetic field has been analyzed numerically, the values and comparison values are recorded in Tables 4-7 from which the following conclusions are drawn R_c takes higher values as Q takes higher values in all the cases. R_c has downward trend as R_t takes higher values, hence destabilization. Similar situation is observed in the case of Marangoni - convection

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