Journal of Mines, Metals and Fuels, *71(12D) : 105-109; 2023. DOI: 10.18311/jmmf/2023/47313*

Print ISSN : 0022-2755

Journal of Mines, Metals and Fuels

Contents available at: *www.informaticsjournals.com/index.php/jmmf*

Improved Exponential Synchronization of Neutral Delay Differential Systems with Time Varying Delays

P. Baskar[1*], G. Srinivasa[1], S. Padmanabhan[2] and Vijayakumar[3]

[1]Department of Mathematics, New Horizon College of Engineering, Bangalore – 560103, Karnataka, India; baskarp272@gmail.com, drsrinivasag@gmail.com

[2]Department of Mathematics, R.N.S Institute of Technology, Bangalore – 560098, Karnataka, India; padmanabhanrnsit@gmail.com

[3]Department of Mathematics, Ramaiah Institute of Technology, Bangalore – 560054, Karnataka, India; vijayakumar@msrit.edu

Abstract

The improved exponential synchronization of the neutral time delay differential system is examined in this research article. The augmented Lyapunov-Krasovskii Functional (LKF) with free matrix variables and Linear Matrix Inequalities (LMIs) are used to construct the stability requirements. To show that the suggested approach is effective, numerical examples are provided in terms of LMI.

Keywords: *Exponential Synchronization, (LMI) Linear Matrix Inequality, LKF (Lyapunov- Krasovski functional)*

1.0 Introduction

Time delay dynamical systems have attracted some fascinating attention in the research area during the last few decades. More generally, the synchronization of neutral systems with delays which makes a successful application in the research areas such as bio engineering, circuit theory, automatic control, population dynamics, and so on. The stability or synchronization of dynamical systems of independent delay, dependent delay criteria which attracts an interesting topic and there are more efforts are applied recently. The asymptotic synchronization of various systems was proposed by numerous scholars[1,2].

In addition, many researchers[3,4] found the sufficient conditions for delay dependent dynamical systems. Various researchers[5-9] shown the exponential stability of time delay systems with time delays. Researchers[10,11] describe the stabilities of fuzzy neural networks and neutral delay systems.

The exponential synchronization of time varying nonlinear uncertainties of neutral delay (time) differential system with was proved by Mr. Syed Ali by using the new way of approach generalized eigen value problem and he minimized the delay very effectively using an LMI approach[12-14]. The exponential synchronization of time varying neutral delay differential system using an LMI approach[15] is proved by the researcher. Umesha V. More specifically researchers Balasubramanian, Syed Ali, Saravanan, Mathiyalagan are showed necessary sufficient conditions of dynamical systems with delays and uncertainties.

The recent papers related to stability and asymptotic stability of competitive neural networks which makes the research on this field more interesting in this era[16-18]. Moreover, the improved exponential synchronization of

**Author for correspondence*

time delay neural network is an Open problem that makes us to more interest on it and makes us to get the stability on it. Using Jensen's inequality[19] and the LKF, we ensured the improved exponential stability and we discovered a new LMI condition in this paper.

Continuously, MATLAB Linear Matrix Inequality - control toolbox is used to find the maximum permitted length of delays .

2.0 Problem Statement and Preliminaries

The neutral time delay system is of the form

$$\dot{x} - A x(t) = B x(t-h) + C\dot{x}(t-h) \qquad (1)$$

with the initial condition

$$x(t_0+\theta) = \phi(\theta) \qquad \forall \theta \in (-\rho, 0),$$

where "*A, B, C* ∈ R ^{n*n}" be the constant and real matrices, "*x(t)* ∈ R^n " be the state vector and the positive time varying delay is h(t) Here let us execute the exponential synchronization of above system described by (1).

In this section we show about the definitions, lemmas and results related to LMI and stabilities of delay differential systems.

Lemma 2.1 (Schur complement[1])

Let E= E^T, F= F^T, G=G^T be the given matrices such that G > 0, which depends affinely

on x, then

$$\begin{pmatrix} F & E^T \\ E & -G \end{pmatrix} < 0 \leftrightarrow F + E^T G^{-1} E < 0$$

Lemma 2.2

For any two vectors y, $x \in R^n$ and a scalar $\in > 0$, Then

$$2x^T y \leq \in x^T x + \in^{-1} y^T y$$

Lemma 2.3

For a scalar *p*>0, a vector function $y:[0,\rho] \to R^n$ any constant matrix $G \in R^{n*n} > 0$

$G = G^T > 0$ and such that the integrations are well defined, then

$$\left(\int_0^\rho r(s)ds\right) G \left(\int_0^\rho r(s)ds\right)^T \leq \rho \int_0^\rho r^T(s) G r(s) ds$$

Definition 2.4

Let an equilibrium point of ($\dot{X}$ = f(X)) be x=0 is exponentially stable if the differentiable function

V: R^n → R and N >0, such that

(i) *V*(*X*)> 0.

(ii) $\dot{V}(X) \leq -\alpha V(X)$ then there exists N such that

$$\| x(t, \varphi \| \leq N e^{-\alpha t} \| x(0) \|$$

3.0 Main Result

Theorem 3.1

Dynamical System (1) is stable exponentially with a scalar h > 0 and convergence rate α, if there exist appropriate dimensions of $P_j > 0, j = 1,2,3.4 \in R^{n*n}$ of positive-definite matrices such that LMI in the following holds.

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} \\ * & \Theta_{22} & \Theta_{23} \\ * & * & \Theta_{33} \end{bmatrix} < 0$$

where,

$$\Theta_{11} = [(A^T P_1 + A P_1 + P_2 + A^T A P_3 + h A^T A P_4 - P_4 e^{-ah})]$$

$$\Theta_{12} = [(B^T P_1 + B^T A P_3 + h B^T A P_4 + P_4 e^{-2ah})]$$

$$\Theta_{13} = [(C^T P_1 + C^T A P_3 + hC^T A P_4)$$

$$\Theta_{21} = [(BP_1 + A^T B P_3 + hA^T B P_4 + P_4 e^{-\alpha h})$$

$$\Theta_{22} = -(-)P_2 e^{-\alpha h} + + B^T B P_3 + hB^T B P_4 + P_4 e^{-\alpha h}$$

$$\Theta_{23} = [(BC^T P_3 + BC^T h P_4)]$$

$$\Theta_{31} = [(CP_1 + A^T C P_3 + hA^T C P_4)]$$

$$\Theta_{32} = B^T C P_3 + B^T C h P_4$$

$$\Theta_{33} = C^T C P_3 + (1-d) P_3 e^{-\alpha h} + + C^T C h P_4$$

Proof:

This theorem can be proved by considering the Lyapunov functions are

$$V(X)=\sum_{i=1}^{4}V_i(t)$$

are as follows

We define the Lyapunov functions as follows

$$V_1(t)=x(t)P_1 * \mathrm{x}^T(t)$$

$$V_2(t)=\int_{t-h}^{t} e^{\alpha(s-t)}x(s)^{*}P_2\ \ x^T(s)ds$$

$$V_3(t)=\int_{t-h}^{t} e^{\alpha(s-t)}\dot{x}(s)^{*}P_3\ \ \dot{x}^T(s)ds$$

$$V_4(t)=\int_{t+\grave{e}}^{t}\int_{-h}^{0}\grave{e}e^{\alpha(s-t)\ *}\dot{x}^T(s)P_4\dot{x}(s)\ \mathrm{d}\ ds$$

Let us define the differentiations of the Lyapunov functions is as follows:

$$\dot{V}(X)=\sum_{i=1}^{4}\dot{V}_i(t)$$

$$\dot{V}_1=[\dot{x}^T(t)P_1\dot{x}(t)+\dot{x}^T(t)P_1x(t)]$$
$$=[\mathrm{A}P_1+A^TP_1]x(t)\mathrm{x}^T(t)+\mathrm{B}\ P_1\ \mathrm{x}(t-h)\mathrm{x}^T(t)$$
$$+\ \ CP_1\mathrm{x}^T(t)\dot{x}(t-h)+\mathrm{B}^TP_1\mathrm{x}(\mathrm{t})\ \mathrm{x}^T(t-h)$$
$$+\mathrm{C}^TP_1\ \dot{x}^T(\mathrm{t}-\ \ \mathrm{h})\ x(t)$$

$$\dot{V}_2=\ [\dot{x}^T(t)P_2x(t)-e^{-\alpha h}(1-\dot{d})(x^T(t-h)P_2x(t-h)]^{\cdot}-\alpha V_2$$

$$\dot{V}_3=[\mathrm{A}A^TP_3]x(t)x^T(t)+\mathrm{B}\ A^TP_3\,\mathrm{x}(t-h)\ x^T(t)$$
$$+A^TCP_3x^T(t)\dot{x}(t-h)+\mathrm{B}^T\mathrm{A}P_3\mathrm{x}(\mathrm{t})\ \mathrm{x}^T(t-h)$$
$$+\mathrm{B}^T\mathrm{B}P_3x^T(t-h)\ \mathrm{x}(\mathrm{t}-\mathrm{h})\ (t)$$
$$+\mathrm{B}^T\mathrm{C}P_3\,x^T(\dot{t}-h)x(\mathrm{t}-\mathrm{h})$$
$$+C^TAP_3\,\mathrm{x}(t)\dot{x}^T(t-h)$$
$$+\mathrm{C}^T\mathrm{B}P_3\dot{x}^T(\mathrm{t}-\mathrm{h})\ \ \mathrm{x}^T(t-h)$$
$$+\mathrm{C}^T\mathrm{C}P_3\dot{x}^T(t-h)\dot{x}^T(\mathrm{t}-\mathrm{h})\ (1-d)$$
$$e^{-\alpha h}\dot{x}^T(t-h)P_3\dot{x}(t-h)]-\cdot\ \alpha V_3$$

By combining all the differentials, we get

$$\dot{\boldsymbol{V}}(\boldsymbol{X})\le[A\ \dot{P}_1+A^T\,P_1+P_2+2\alpha P_1$$
$$+\dot{A}^T\,\mathrm{A}P_3+AA^TP_4h-e^{-\alpha h}P_4]x(t)\mathrm{x}^T(t)$$
$$+[\mathrm{B}^TP_1+\mathrm{B}^T\mathrm{A}P_3+B^TP_4hA+e^{-\alpha h}P_4]\mathrm{x}^T(t-h)\ \mathrm{x}(\mathrm{t})$$
$$+\left[\mathrm{C}^TP_1+\mathrm{C}^T\mathrm{A}P_3+C^TP_4hA\right]\dot{x}^T(\mathrm{t}-\mathrm{h})\ \mathrm{x}(t)$$
$$+[\dot{B}\dot{P}_1+\dot{A}^T\,\mathrm{B}P_3+AB^TP_4h-e^{-\alpha h}P_4]\mathrm{x}(\mathrm{t}-\mathrm{h})\ \mathrm{x}^T(t)$$
$$+[-e^{-\alpha h}\left(1-\dot{d}\right)\dot{P}_2+\dot{B}^T\,\mathrm{B}P_3+\dot{B}B^TP_4h-e^{-\alpha h}P_4\right]x^T(t-h)]^{\cdot}$$
$$+\left[\dot{C}^T\,\mathrm{B}P_3+BC^TP_4h\right]\dot{x}^T(t-h)\mathrm{x}(\mathrm{t}-\mathrm{h})$$
$$+\left[CP_1+\mathrm{A}^T\mathrm{C}P_3+A^TP_4hC\right]\dot{x}(\mathrm{t}-\mathrm{h})\ \mathrm{x}^T(t)$$
$$+\left[\dot{B}^T\,\mathrm{C}P_3+BC^TP_4h\right]\dot{x}^T(t-h)x(\mathrm{t}-\mathrm{h})$$
$$+\left[\dot{C}^T\,\mathrm{C}P_3+CC^TP_4h+e^{-\alpha h}(1-d)P_3\right]\dot{x}^T(\dot{t}-h)x(\mathrm{t}-\mathrm{h})$$
$$-\alpha V_2-\alpha V_3-\ \alpha V_4$$

The given derivative becomes

$$\dot{V}(X)+aV(.)\le AP_1+\mathrm{A}^TP_1+P_2$$
$$+2\alpha\dot{P}_1A^T\,\mathrm{A}P_3+AA^TP_4h-e^{-\alpha h}P_4]x(t)\mathrm{x}^T(t)$$
$$+[\mathrm{B}^TP_1+\mathrm{B}^T\mathrm{A}P_3+B^TP_4hA+e^{-\alpha h}P_4]\mathrm{x}^T(t-h)$$
$$\mathrm{x}(\mathrm{t})+\left[\mathrm{C}^TP_1+\mathrm{C}^T\mathrm{A}P_3+C^TP_4hA\right]\mathrm{x}(t)\dot{x}^T(\mathrm{t}-\mathrm{h})$$
$$+[BP_1+A^T\,\mathrm{B}P_3+AB^TP_4h-e^{-\alpha h}P_4]\mathrm{x}(\mathrm{t}-\mathrm{h})\ \mathrm{x}^T(t)$$
$$+[-e^{-\alpha h}(1-d)\left(B^T\,\mathrm{B}P_3+BB^TP_4h-e^{-\alpha h}P_4\right]x^T(t-h)]^{\cdot}$$
$$+\left[C^T\,\mathrm{B}P_3+BC^TP_4h\right]\ \dot{x}^T(t-h)$$
$$\mathrm{x}(\mathrm{t}-\mathrm{h})+\left[CP_1+\mathrm{A}^T\mathrm{C}P_3+A^TP_4hC\right]\dot{x}(\mathrm{t}-\mathrm{h})\ \mathrm{x}^T(t)$$
$$+\left[B^T\,\mathrm{C}P_3+BC^TP_4h\right]\dot{x}^T(t-h)x(\mathrm{t}-\mathrm{h})$$
$$+\left[C^T\,\mathrm{C}P_3+CC^TP_4h+e^{-\alpha h}(1-d)P_3\right]$$
$$\dot{x}(t-\mathrm{h})\dot{x}^T(t-h)=\xi^T\Theta\xi$$

where,

$$\xi = [x(t)\ x(t-h)\dot{x}(t-h)]$$

$$\Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} \\ * & \Theta_{22} & \Theta_{23} \\ * & * & \Theta_{33} \end{bmatrix}$$

By applying the previous lemmas with some effort

$$\theta < 0$$

Also

$$\dot{V}(X) \le -\alpha V(t, x(t))$$

where, $t \in R^+$

On Integrating, we have

$V(t, x(t)) \le V(\varnothing)e^{-\alpha t}$ where, $t \in R^+$

Further $\lambda_1 x(t,\ \varphi^2 \le V(X) \le e^{-\alpha t} V(\varphi) \le \lambda_2 e^{-\alpha t}\varphi^2$

Then $\| x(t,\ \varphi \| \le \sqrt{\frac{\chi_1}{\chi_2}}\ e^{-\alpha t} \| \varphi \|$ which concludes the proof of exponential stability by the Lyapunov stability theorem

By using the lemma 2.2, we achieve the following result as follows

$$\dot{V}(X) \le e^{\alpha t} V(t)$$

Remark 3.1. Here in Theorem 3.1 the LMI is solvable by using the MATLAB control LMI toolbox.

4.0 Numerical Examples

Example 4.1

The above System (1) of

$A = \begin{bmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{bmatrix}$; $B = \begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{bmatrix}$; C=0 and for with $(\alpha = 0)$, d=0.5 and h = 1.45 (Table 1)

Applying the above theorem, the possible and feasible solutions of the above LMI are

$$P_1 = \begin{bmatrix} 1.1543 & 1.5923 \\ 1.5923 & 1.6928 \end{bmatrix};\ P_2 = \begin{bmatrix} 8.4834 & 4.2292 \\ 4.2292 & 8.3735 \end{bmatrix};$$

$$P_3 = \begin{bmatrix} 1.4823 & -0.6323 \\ -0.6323 & 2.0694 \end{bmatrix};\ P_4 = \begin{bmatrix} 0.5305 & 0.4780 \\ 0.4780 & 0.2245 \end{bmatrix}$$

Table 1. Maximum delay rate

Methods		d	Maximum value h
[3]	0	0.5	1.4196
Theorem 3.1 (Our result)	0.5	0.5	**1.45**

Table 2. Maximum delay rate

Methods		d	Maximum value h
[3]	0.5	0.5	0.7308
Theorem 3.1 (Our result)	0.5	0.5	**1.6**

Example 4.2

The system (1) with the values from [12].

Consider the System (1) with

$A = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$; $B = \begin{bmatrix} 0 & 0.4 \\ 0.4 & 0 \end{bmatrix}$; $C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$ and for with $(\alpha = 0.5)$, d=0.5 and h = 1.6 (Table 2)

Applying the above theorem, the possible and feasible solutions of the above LMI are,

$$P_1 = \begin{bmatrix} 0.0614 & 1.1008 \\ 1.1008 & 0.1793 \end{bmatrix};\ P_2 = \begin{bmatrix} 5.6271 & 1.5173 \\ 1.5173 & 4.9301 \end{bmatrix};$$

$$P_3 = \begin{bmatrix} 0.9605 & -0.6009 \\ -0.6009 & 2.0881 \end{bmatrix};\ P_4 = \begin{bmatrix} 0.3789 & 0.5975 \\ 0.5975 & 0.7204 \end{bmatrix}$$

5.0 Applications

The neural networks has a successful application in the research areas such as stock market prediction, social media, bio engineering, deference, health care circuit theory, automatic control, aerospace, population dynamics, health care and so on. More interestingly our journal JMMF is publishing all application and concept related research articles effectively and it is very useful for all the researchers for good publication.

6.0 Conclusion

In this study, the necessary conditions for the exponential synchronisation of "time-delayed neutral systems are examined". The Schur complement lemma, Newton-Leibniz formula and two examples with comparisons are used to describe the adequate criteria in terms of LMI and showed our result using the MATLAB LMI toolbox.

7.0 References

1. Umesha V, Padmanabhan S, Baskar P, Ali MS. Exponential stability analysis for delay-differential systems of neutral type with an LMI approach. Khayyam J Math. 2019; 5(1):11-20.
2. Baskar P, Padmanabhan S, Ali MS. Finite-time H ∞ control for a class of Markovian jumping neural networks with distributed time varying delays-LMI approach. Acta Math Sci. 2018; 38(2):561-579. https://doi.org/10.1016/S0252-9602(18)30766-5
3. LMI based stability criterion for uncertain neutral-type neural networks with discrete and distributed delays control and cybernetics. 2020; 49(1):77-97.
4. Baskar P, Padmanabhan S, Syed Ali M. Novel delay-dependent stability condition for mixed delayed stochastic neural networks with leakage delay signals. Int J Comput Math. 2018; 96(6):1107-1120. https://doi.org/10.1080/00207160.2018.1439581
5. Boyd S, El Ghaoui L, Feron E, Balakrishnan V. Studies in applied and numerical mathematics linear matrix inequalities in system and control theory; 1994. https://doi.org/10.1137/1.9781611970777
6. Zou Z, Wang Y. New stability criterion for a class of linear systems with time-varying delay and nonlinear perturbations. IEE Proceedings - Control Theory and Applications. 2006; 153(5):623-626. https://doi.org/10.1049/ip-cta:20045258
7. Liu PL. Stability of continuous and discrete time-delay grey systems. Int J Syst Sci. 2001; 32(7):947-952. https://doi.org/10.1080/00207720118856
8. Liu PL. Exponential stability for linear time-delay systems with delay dependence. J Frank Inst. 2003; 340(6-7):481-488. https://doi.org/10.1016/j.jfranklin.2003.10.003
9. Liu PL. Robust exponential stability for uncertain time-varying delay systems with delay dependence. J Frank Inst. 2009; 346(10):958-968. https://doi.org/10.1016/j.jfranklin.2009.04.005
10. Rakkiyappan R, Balasubramaniam P. On exponential stability results for fuzzy impulsive neural networks. Fuzzy Set Systems. 2010; 161(13):1823-1835. https://doi.org/10.1016/j.fss.2009.12.016
11. Umesha V. LMI technique and proved the stability analysis of neutral delay differential system with time delays. Khayyam J Math. 2019; 5(1):11-20.
12. Ali MS. On exponential stability of neutral delay differential system with nonlinear uncertainties. Commun Nonlinear Sci Numer Simul. 2011; 17(6):2595-2601. https://doi.org/10.1016/j.cnsns.2011.09.019
13. Ali MS, Balasubramaniam P. Exponential stability of uncertain stochastic fuzzy BAM neural networks with time-varying delays. Neurocomputing. 2008; 72(4-6):1347-1354. https://doi.org/10.1016/j.neucom.2008.09.005
14. Park JH, Kwon O. Novel stability criterion of time delay systems with nonlinear uncertainties. Appl Math Lett. 2005; 18(6):683-688. https://doi.org/10.1016/j.aml.2004.04.013
15. Zuo Z, Wang Y. New stability criterion for a class of linear systems with time-varying delay and nonlinear perturbations. IEE Proc-Contr Theor Appl. 2006; 153(5):623-626. https://doi.org/10.1049/ip-cta:20045258
16. Robust Stability Of Recurrent Neural Networks With Time-Varying Delays (Baskar and P, Padmanabhan 2021) Dynamics of Continuous, Discrete and Impulsive Systems SeriesB: Applications and Algorithms 2022, 29(4), pp. 285-295
17. Shao S, Du B. Global asymptotic stability of competitive neural networks with reaction-diffusion terms and mixed delays. Symmetry. 2022; 14(11):2224–2224. https://doi.org/10.1016/j.ifacol.2022.07.306
18. Pan H, Zhang W, Yu L. Exponential stability of switched neural networks with partial state reset and time-varying delays. 2022; 10(20):3870.
19. Parada-Mayorga A, Wang Z, Gama F, Ribeiro A. Stability of aggregation graph neural networks. IEEE Trans Signal Inf Process Netw. 2023; 9:850-864. https://doi.org/10.1109/TSIPN.2023.3341408