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Soft Graph on A Minimal Edge Dominating Set

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Abstract

Mining is the process of extracting valuable minerals or other geological materials from the earth. Mining plays an important role in supplying raw materials for various industries, including construction, energy and manufacturing. Methods of mining range from surface mining, which extracts minerals from the earth's surface, to underground mining, which involves tunneling into the earth to access deposits. When finding paths from any point in the mining area to a mining location several challenges can be encountered. One of them is that the mining area may have challenging terrain, such as mountains, dense forests, or water bodies, making pathfinding complex and resource-intensive. Graphs play a crucial role in finding a path from a point to a mining location.To find a better path from any point to a mining location we introduce a new soft graph in this paper. Let G=(V,E) be a simple connected graph and A be a minimal edge dominating set. Let a subset R of A×E be an arbitrary relation from A to E. Let a mapping F:A→P(V) can be defined as F(xy)={x,y,z∈V/xyRxz} and a mapping K:A→P(E) can be defined as K(xy)={yz∈E/z≠x and y,z∈F(y)}, where x,y,z∈V(G), xy∈A and P is a power set. The pair (F, A) is a soft set over V and (K, A) is a soft set over E. Then (F(xy), K(xy)) is a sub graph of G*, ∀xy∈A.*

Keywords: *Minimal Edge Dominating Number, Minimal Edge Dominating Set, Soft Graph, Soft Set*

1.0 Introduction

In terms of graph theory, there are numerous types of graphs, and each type is connected to a unique attribute. Uncertainty is one of these traits. A kind of graph that deals with uncertainty is the soft graph. Terms "Soft graph" and "Soft set" were first used by researcher Molodtsov, in 1999 [1]. Maji *et al.*,[2] later examined certain soft graph operations and applications. The domination set in soft graphs was defined by Gillbert Rani *et al.*,[3] in 2015. In the paper "Edge domination in trees", Mitchell *et al.*,[4] introduced the concept of edge domination, in 1977. Soft graphs employing soft sets were first introduced by Rajesh Thumbaraka *et al.*,[5] in 2014. Palani *et al.*,[6] introduced many soft graphs of particular graphs in 2021.

This paper defines a new soft graph using a minimal edge dominating set. On the basis of it, we discuss about some fundamental soft graph features which helps in finding a better path from any key access point to mining location in the mining area.

2.0 Preliminaries

Following fundamental definitions are stated in order to better understand the concept.

Definition 2.1 [1]**:** Let U be an initial universe set and E be a set of parameters. Let $A \subseteq E$, a pair (F, E) is called a soft set over U if and only if F is a mapping of E into the set of all subsets of the set U i.e., $F : A \to P(U)$.

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Definition 2.2 [5]: Let $G=(V,E)$ be a simple connected graph, A be any nonempty set. Let R be an arbitrary relation between elements of A and elements of V. That is $R\subseteq A\times V$. A set valued function $F:A\to P(V)$ can be defined as $F(x)=\{y\in V/xRy\}$. The pair (F,A) is a soft set over V.

Definition 2.3 [6]: A set D is said to be a dominating set of a soft graph (F,A) if for every $x\in A$, every vertex of $F(x)$ in $V-D$ is adjacent to at least one vertex in D.

Definition 2.4 [7]: A subset K of $E(G)$ is called an edge dominating set of a graph G if every edge not in K is adjacent to some edge in K.

Definition 2.5 [7]: An edge dominating set K is called a minimal edge dominating set if no proper subset of K is a dominating set of G. The edge domination number (γ_e) of a graph G is the cardinality of a minimal edge dominating set of G.

Definition 2.6 [8]: A $4-$tuple $G=(G^*,F,K,A)$ is called a soft graph of G, if it satisfies the following properties:

i. $G^*=(V,E)$ is a simple graph
ii. A is a non-empty set of parameters
iii. (F,A) is a soft set over V
iv. (K,A) is a soft set over E

$H(a)=(F(a),K(a))$ is a sub graph of $G^*,\forall a\in A.$

3.0 Soft Graph on A Minimal Edge Dominating Set

Definition: Let $G^*=(V,E)$ be a simple connected graph and A be a minimal edge dominating set. Let a subset R of $A\times E$ be an arbitrary relation from A to E. Let a mapping $F:A\to P(V)$ can be defined as $F(xy)=\{y,z\in V/xxyRxz\}$ and a mapping $K:A\to P(E)$ can be defined as $K(xy)=\{yz\in E/z\neq x \text{ and } y,z\in F(xy)\}$ where $x,y,z\in V(G)$ $xy\in A$ and P is a power set.

A $4-$tuple $G=(G^*,F,K,A)$ is called a **soft graph on a minimal edge dominating set** of G, if it satisfies the following properties:

(i) $G^*=(V,E)$ is a simple connected graph
(ii) A is a minimal edge dominating set
(iii) (F,A) is a soft set over V
(iv) (K,A) is a soft set over E
(v) H(xy)=(F(xy), K(xy))is a sub graph of $G^*,\forall x\in A.$

3.1 Results and Discussions

Theorem 3.1: For any $x_1x_2\in A$,

(i) if $G=C_3$, then $H(x_1x_2)=P_3$.
(ii) If $G=C_n(n>3)$, then $H(x_1x_2)=2K_2$.

Proof:
Let $x_1x_2\in A$.

(i) Suppose $G=C_3$. Then $\gamma_e=1$ and $H(e)=C_3-e$, which is P_3.
(ii) Suppose $G=C_n(n>3)$, then for each $x_1x_2\in A$, $F(x_1x_2)=\{x_1,x_2,x_3,x_4\}$ and $\langle F\rangle$ is P_4. Further, $K(x_1x_2)=\{x_1x_3,x_2x_4\}$ and $H(x_1x_2)=\langle F\rangle-\{x_1x_2\}=2K_2$.

Theorem 3.2: For any $x_1x_2\in A$, if $G=K_n(n>2)$, then $H(x_1x_2)=K_{2,n-2}$.

Proof:
Let $x_1x_2\in A$, and $G=K_n$ with $n>2$.

Then $F(x_1x_2)=V(K_n)$ and $K(x1x2)=\{x_1x_i,x_2x_j/x_ix_iRx_1x_2,$ where $i,j=3,4,...,n$ and $i\neq j\}$. So, $V(H(x_1x_2))=V(K_n)$ and $E(H(x_1x_2))=K(x_1x_2)$.

In $H(x_1x_2)$, $x_1\notin N(x_2)$ and $x_2\notin N(x_1)$ and $V(H(x_1x_2))$ can be partitioned into two subsets V_1 and V_2 such that $V_1=\{x_1,x_2\}$ and $V_2=\{x_3,x_4,...,x_n\}$. And since $G=K_n$, $N(x_1)=\{x_3,x_4,...,x_n\}$ and $N(x_2)=\{x_3,x_4,...,x_n\}$, $H(x_1x_2)$ is $K_{2,n-2}$.

Note: If $G=K_n$, then $|E(H(x_1x_2))|=|E(K_n)|-|E(K_{n-2})+\{x_1x_2\}|$.

Theorem 3.3: For any graph G and $x_1x_2\in A$, $H(x_1x_2)$ is a complete bipartite graph if and only if $N(x_1)=F(x_1x_2)-x_1$ and $N(x_2)=F(x_1x_2)-x_2$.

Proof:

Let G be any connected graph with n vertices.

For any $x_1x_2 \in A$, suppose $H(x_1x_2)$ is a complete bipartite graph. Then $x_1, x_2 \in V_1$ and $x_3, x_4, ..., x_m \in V_2$, $m \le n$. and also $F(x_1x_2) = \{x_1, x_2, ..., x_m\}$ and $K(x_1x_2) = \{x_ix_j / x_ix_jRx_1x_2\}$ where $i = 1,2$ and $3 \le j \le m\}$ and $N(x_1) = N(x_2) = V(V_2)$. Since $x_1x_2 \in E(G)$ in G, $N(x_1) = F(x_1x_2) - x_1$ and $N(x_2) = F(x_1x_2) - x_2$.

Conversely,

Suppose $N(x_1) = F(x_1x_2) - x_1$ and in G. Then $E(H(x_1x_2)) = \{x_ix_j / x_ix_jRx_1x_2\}$ where $i = 1,2$ and $3 \le j \le m\}$. Since $N(x_1) = F(x_1x_2) - x_1, x_1x_j \in E(H(x_1x_2))$ for $3 \le j \le m$. Similarly, $N(x_2) = F(x_1x_2) - x_2, x_2x_j \in E(H(x_1x_2))$ for $3 \le j \le m$. By the definition of $H(x_1x_2)$, $x_1x_2 \in G$ and $x_1x_2 \notin H(x_1x_2)$. This implies, $V(H(x_1x_2))$ is partitioned into two subsets V_1 and V_2 such that $x_1, x_2 \in V_1$ and $x_3, x_4, ..., x_m \in V_2$. For any two vertices $a, b \in V_2, ab \notin E(H(x_1x_2))$ because $ab \notin N(x_1x_2)$. Then $K(x_1x_2) = \{x_1x_j, x_2x_j\}_{j=3}^{m}$. Hence, $H(x_1x_2)$ is a complete bipartite graph.

Theorem 3.4: For any graph G, $x_1x_2 \in A$ and $x_ix_j \in N(x_1x_2), H(x_1x_2) = C_4$ if and only if $|N(x_ix_j) \cap K(x_1x_2)| = 2$ where $i = 1,2$ and $j = 3,4,...,n$.

Proof:

Let $x_1, x_2 \in A$ and $H(x_1x_2) = C_4$. Then $F(x_1x_2) = \{x_1, x_2, x_3, x_4\}$ and $K(x_1x_2) = \{x_ix_j / x_ix_jRx_1x_2\}$ where $i = 1,2$ and $j = 3,4,...,n\}$. In $H(x_1x_2)$, since $\deg(x_k) = 2$ for each $x_k \in F(x_1x_2), (1 \le k \le 4), N(x_ix_j) = 2$. Then in G, $|N(x_ix_j)| \ge 2$. This implies $|N(x_ix_j) \cap K(x_1x_2)| = 2$.

Conversely,

Suppose for any $x_ix_j \in N(x_1x_2)$, $|N(x_ix_j) \cap K(x_1x_2)| = 2$.

Then $E(H(x_1x_2)) = \{x_ix_j / x_ix_jRx_1x_2\}$ where $i = 1,2$ and $j = 3,4,...,n\}$.

Since $x_1x_2 \in E(G), x_1x_2 \notin E(H(x_1x_2))$ and since $x_ix_j \in N(x_1x_2)$, we have the following cases:

Case (i): For $j = 3$, then $|N(x_ix_3) \cap K(x_1x_2)| = 1$, a contradiction.

Case(ii): For $j = 3,4$, then $|N(x_ix_j) \cap K(x_1x_2)| = 2$. Then $F(x_1x_2) = \{x_1, x_2, x_3, x_4\}$ and $K(x_1x_2) = \{x_1x_3, x_1x_4, x_2x_3, x_2x_4\}$. Hence $H(x_1x_2) = C_4$.

Case(iii): For $j = 3,4,...,n$, if $|N(x_ix_j) \cap K(x_1x_2)| = 2$, then there exists at least one $x_j \in F(x_1x_2)$ such that either $x_1x_j \in K(x_1x_2)$ or $x_2x_j \in K(x_1x_2)$, a contradiction since $|N(x_ix_j) \cap K(x_1x_2)| = 2$ for $i = 1,2$. This implies $x_1x_j, x_2x_j \in K(x_1x_2)$. Hence, for any $x_ix_j \in N(x_1x_2)$, if $|N(x_ix_j) \cap K(x_1x_2)| = 2$, then $H(x_1x_2) = C_4$.

Theorem 3.5: For any minimal edge dominating set A, if $H(x_1x_2)$ is a cycle then the cycle is only C_4.

Proof:

Let A be any minimal edge dominating set and $H(x_1x_2)$ be a cycle graph.

Consider $x_1, x_2 \in A$. Assume that $H(x_1x_2) \ne C_4$. Then we have the following cases:

Case(i):If $H(x_1x_2) = C_3$, then $x_1x_2 \in E(H(x_1x_2))$. By the definition of $H(x_1x_2), x_1x_2 \in E(G)$ and $x_1x_2 \notin E(H(x_1x_2))$. Hence $H(x_1x_2) \ne C_3$.

Case(ii): If $H(x_1x_2) = C_n, n > 4$, then there exists at least one $x_3 \in F(x_1x_2)$ such that

$d(x_i, x_3) \geq 2, (i = 1,2)$, a contradiction to the definition of $F(x_1x_2)$.

Hence, $H(x_1x_2) = C_4$.

Theorem 3.6: For any graph G and $x_1x_2 \in A, H(x_1x_2)$ has a cycle C_4 with at least one pendant edge if and only if the following conditions satisfies:

(i) there exist two vertices $x_3, x_4 \in F(x_1x_2)$ such that $|N(x_j) \cap \{x_1, x_2\}| = 2$ for $j = 3,4.$

(ii) there exists at least one $x_j \in F(x_1x_2)$ such that $|N(x_j) \cap \{x_1, x_2\}| = 1$ for $4 < j \leq n$.

Proof:

Let G be a connected graph with n vertices.

Let for any $x_1x_2 \in A, H(x_1x_2)$ has a cycle C_4 with at least one pendant edge. Then $F(x_1x_2) = \{x_1, x_2, ..., x_m\}$ and $K(x_1x_2) = \{x_ix_j / x_ix_j R x_1x_2, \text{where i} = 1,2 \text{ and j} = 3,4,...,\text{m}\}$, $m \leq n$. Then $|F(x_1x_2) - \{x_1, x_2\}| > 2$.

(i) There exists $x_3, x_4 \in F(x_1x_2)$ such that $x_1x_3, x_1x_4, x_2x_3, x_2x_4 \in K(x_1x_2)$ such that $x_1, x_2 \in N(x_3)$ and $x_1, x_2 \in N(x_4)$. Then $N(x_3) \cap \{x_1, x_2\} = \{x_1, x_2\}$ and $N(x_4) \cap \{x_1, x_2\} = \{x_1, x_2\}$. This means that $|N(x_j) \cap \{x_1, x_2\}| = 2$ for $j = 3,4$.

(ii) Since $H(x_1x_2)$ has at least one pendant edge, there exists at least one $x_j \in F(x_1x_2)$ such that either $x_1x_j \in K(x_1x_2)$ or $x_2x_j \in K(x_1x_2)$ and from (i), $4 < j \leq m$. Suppose $x_1x_j \in K(x_1x_2)$. Then $x_1 \in N(x_j)$ and $x_2 \notin N(x_j)$.

Similarly, if $x_2x_j \in K(x_1x_2)$, then $x_1 \notin N(x_j)$ and $x_2 \in N(x_j)$. This implies $N(x_j) \cap \{x_1, x_2\} = x_1$ or x_2. Hence, $|N(x_j) \cap \{x_1, x_2\}| = 1$ for $4 < j \leq m (m \leq n)$.

Conversely,

Suppose there exists two vertices $x_3, x_4 \in F(x_1x_2)$ such that $|N(x_j) \cap \{x_1, x_2\}| = 2$ for $j = 3,4$

and there exists at least one $x_j \in F(x_1x_2)$ such that $|N(x_j) \cap \{x_1, x_2\}| = 1$ for $4 < j \leq n$.

Then $\{x_1, x_2\} = N(x_3) \cap N(x_4)$ and also $x_3, x_4 \in N(x_i)(i = 1,2)$. Then $H(x_1x_2)$ contains a cycle C_4.

Since there exists at least one $x_j \in F(x_1x_2)$ with $|N(x_j) \cap \{x_1, x_2\}| = 1$ for $4 < j \leq m$, either $x_1 \in N(x_j)$ or $x_2 \in N(x_j)$. This implies, either x_1x_j is a pendant edge or x_2x_j is a pendant edge.

Hence, $H(x_1x_2)$ contains a cycle C_4 with at least one pendant edge.

Theorem 3.7: For any $x_1x_2 \in A$, if $G = T$, a tree, then $H(x_1x_2)$ is a disconnected graph.

Proof:

Let $x_1x_2 \in A$ and $G = T$, a tree with n vertices.

Since every $x_ix_j \in E(T)$ for $i \neq j$, is a bridge, x_1x_2 is also a bridge. Hence $H(x_1x_2)$ is a disconnected graph.

Theorem 3.8: For any graph G, if $x_1x_2 \in A$ is a pendant edge, then $H(x_1x_2)$ is a disconnected graph with at least one isolated vertex.

Proof:

Let G be any connected graph and $x_1x_2 \in A$.

Suppose $x_1x_2 \in E(G)$ is a pendant edge. Then either x_1 or x_2 is a pendant vertex.

Suppose $V(G) = \{x_1, x_2\}$ and $A = E(G) = \{x_1x_2\}$. Then $N(x_2) = x_1$, $N(x_1) = x_2$ and $N(x_1) \cap [F(x_1x_2) - \{x_1x_2\}] = \phi$ and $N(x_2) \cap [F(x_1x_2) - \{x_1x_2\}] = \phi$. Then $H(x_1x_2)$ is a disconnected graph with two isolated vertices.

Suppose $x_1x_2 \in V(G)$ and $x_1x_2 \in E(G)$ Then either $N(x_2) = x_1$ or $N(x_1) = x_2$ in G. This implies,

either $N(x_1) \cap [F(x_1x_2) - \{x_1x_2\}] = \phi$ or $N(x_2) \cap [F(x_1x_2) - \{x_1x_2\}] = \phi$. Hence $H(x_1x_2)$ is a disconnected graph with one isolated vertex.

Proposition 3.8.1: If $G = K_{1,n}$, then $H(x_1x_2)$ has one isolate.

Proposition 3.8.2: If $G = K_2 = P_2$ then $H(x_1x_2)$ is a null graph with two isolates (or $H(x_1x_2) = \overline{K_2}$).

Proposition 3.8.3: For any $x_1x_2 \in A$ (i) if $G = P_3$ then $H(x_1x_2) = K_1 \cup K_2$ (ii) If $G = P_n (n > 3)$ then either $H(x_1x_2) = K_1 \cup K_2$ or $H(x_1x_2) = 2K_2$.

Proof:
Let $x_1x_2 \in A$

(i) Suppose $G = P_3$ Then $x_1x_2 \in E(G)$ is a pendant edge and therefore either x_1 or x_2 is a pendant vertex. So, $H(x_1x_2) = P_3 - \{x_1x_2\}$, is a disconnected graph with x_1 or x_2 as isolated vertex and either x_1x_3 or x_2x_3 as K_2. Hence, $H(x_1x_2) = K_1 \cup K_2$.

(ii) Suppose $G = P_n (n > 3)$.

Case(i): If $x_1x_2 \in E(G)$ is a pendant edge, then either x_1 or x_2 is a pendant vertex.

If x_1 is a pendant vertex, $N(x_1) \cap [F(x_1x_2) - \{x_1x_2\}] = \phi$ and $N(x_2) \cap [F(x_1x_2) - \{x_1x_2\}] = x_3$ for some $x_3 \in F(x_1x_2)$. Then, $H(x_1x_2)$ is a disconnected graph with x_1 as isolated vertex and x_2x_3 as K_2 and vice versa. Hence, $H(x_1x_2) = K_1 \cup K_2$

Case(ii): If $x_1x_2 \in E(G)$ is not a pendant edge, then there exists $x_3, x_4 \in F(x_1x_2)$ such that $N(x_2) \cap [F(x_1x_2) - \{x_1x_2\}] = x_3$ and $N(x_2) \cap [F(x_1x_2) - \{x_1x_2\}] = x_4$. Then $H(x_1x_2)$ is a disconnected graph with two components, each of which is K_2. Hence, $H(x_1x_2) = 2K_2$

Note: For any minimal edge dominating set, if $H(x_1x_2)$ is a disconnected graph then it has only 2 components.

Theorem 3.9: For any graph G with n vertices and $x_1x_2 \in A$, $H(x_1x_2)$ is a disconnected graph with every component is either isolated vertex or $K_{1,m} (m \le n)$ if and only if $N(x_1) \cap N(x_2) = \phi$.

Proof:
Let G be connected graph with n vertices and $x_1x_2 \in A$. Suppose $H(x_1x_2)$ is disconnected graph with every component is either isolated vertex or $K_{1,m} (m \le n)$. Assume that $N(x_1) \cap N(x_2) = \phi$. Then there exists at least one $x_3 \in F(x_1x_2)$ such that $x_3 \in N(x_1) \cap N(x_2)$. This implies that, there exists a path between x_1 and x_2 passing through x_3. Then $H(x_1x_2)$ is connected, a contradiction. Hence $N(x_1) \cap N(x_2) = \phi$.

Conversely,
Suppose $N(x_1) \cap N(x_2) = \phi$. Then there is no path joining x_1 and x_2. So, $H(x_1x_2)$ is disconnected. Then $x_1x_2 \in E(G)$ is a bridge and therefore there exists only two components $C_1, C_2 \in H(x_1x_2)$. If x_1x_2 is a pendant edge then by Theorem 8, $H(x_1x_2)$ is a disconnected graph with at least one isolated vertex.

Suppose $x_1x_2 \in E(G)$ is not a pendant edge. Then any component $C_i \in H(x_1x_2)$ does not contain isolated vertex for $i = 1,2$ and so each C_i is a connected graph with either $V(C_i) = \{x_1, N(x_1) - x_2\}$ or $V(C_i) = \{x_2, N(x_2) - x_1\}$. Since $N(x_1) \cap N(x_2) = \phi$, for each $x_j \in N(x_1) - x_2, \deg(x_1) = j$ and $\deg(x_j) = 1$ in $H(x_1x_2)$ for $3 \le j \le n$. Similarly, for each $x_k \in N(x_2) - x_1, \deg(x_2) = k$ and $\deg(x_k) = 1$ in $H(x_1x_2)$ for $3 \le k \le n, (j \ne k)$.

Hence, each $C_i \in H(x_1x_2)$ is $K_{1,m}$ for $i = 1,2$ and $m \le n$.

Corollary 3.9.1: For any graph G and $x_1x_2 \in A$, $H(x_1x_2)$ is connected if and only if $N(x_1) \cap N(x_2) \ne \phi$

Corollary 3.9.2: For any $x_1x_2 \in A$ if $G = K_{m,n}$ then $H(x_1x_2)$ is a disconnected graph.

Theorem 3.10: For any graph G and $x_1x_2 \in A$, $H(x_1x_2)$ is a tree if and only if there exists one $x_3 \in F(x_1x_2) - \{x_1, x_2\}$ such that $x_3 \in N(x_1) \cap N(x_2)$ and for any $x_i \in F(x_1x_2) - \{x_1, x_2\}$ either $x_1 \in N(x_i)$ or $x_2 \in N(x_i)$ where $4 \le i \le n$.

Proof:
Let G be a connected graph n vertices and $x_1x_2 \in A$

Consider $F(x_1x_2) = F(x_1x_2) - \{x_1, x_2\}$.

Suppose $x_3 \in N(x_1) \cap N(x_2)$ for $x_3 \in F(x_1x_2)$ and/or for any $x_i \in F(x_1x_2)$ either $x_i \in N(x_1)$ or $x_i \in N(x_2)$ with $4 \le i \le m (m \le n)$. Then $N(x_1) \cap N(x_2) \ne \phi$ and so $H(x_1x_2)$ is a connected graph. And each x_i is a pendant vertex adjacent either to x_1 or to x_2. This implies that $H(x_1x_2)$ is acyclic. Hence $H(x_1x_2)$ is a tree.

Conversely,
Suppose $H(x_1x_2)$ is a tree with $|F(x_1x_2)| = m (3 \le m \le n)$

Suppose $m = 3$. Then $H(x_1x_2) = P_3$ and there exists $x_3 \in F(x_1x_2)$ such that $x_3 \in N(x_1) \cap N(x_2)$.

Suppose $m > 3$ If there exists some $x_i, x_j \in F(x_1x_2) - \{x_1, x_2\}$ such that $x_i, x_j \in N(x_1) \cap N(x_2)$ then there are two paths between x_1 and x_2, a contradiction. Hence, there exists one $x_3 \in F(x_1x_2) - \{x_1, x_2\}$ such that $x_3 \in N(x_1) \cap N(x_2)$. Also, since $H(x_1x_2)$ is a tree and $m > 3$, there exists at least one vertex $x_i \in F(x_1x_2) - \{x_1, x_2\}$ such that either $x_1 \in N(x_i)$ or $x_2 \in N(x_i)$ where $4 \le i \le m$.

Note: (i) For any G, $H(x_1x_2) \ne P_2$ for $n > 5$.
(ii) For any G, if $H(x_1x_2)$ is a tree, then maximum level of the tree is 2.

Theorem 3.11: For any graph G and $x_1x_2 \in A$, if $K(x_1x_2)$ is an independent edge set, then $H(x_1x_2) = 2K_2$.

Proof:
Let G be a connected graph with vertices.

Suppose $x_1x_2 \in A$ and $K(x_1x_2)$ be an independent edge set. Then for each $x_ix_j \in K(x_1x_2)$, $N(x_ix_j) \notin K(x_1x_2)$, $3 \le i \ne j \le n$. Therefore $|N(x_1x_2)| = 2$ Then $H(x_1x_2)$ is disconnected with two components each of which is K_2 Hence, $H(x_1x_2) = 2K_2$.

4.0 Application

Consider some city or town. Connectivity between the areas is a big issue. For instances inappropriate road construction/damaged roads leads to traffic congestion and consumes more time and in turn fuel. To overcome these factors, we have constructed an example in this paper.

Let G be a graph representing the map of some city which has majorly weaker/unconditional roads.

Consider

$V = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}\}$

is the set of areas in the city and the edges represents the connectivity between the areas. Let A be a set of major roads connecting the areas. Let $F(xy)$ represents the areas covered by the road *xy* and *K(xy)* represents the roads adjacent to the road xy, where *xy*∈*A*

In the graph G of Figure 1, let $A = \{x_2x_3, x_5x_6, x_7x_8, x_9x_{10}, x_{14}x_{12}\}$, a minimal edge dominating set of G.

For $x_2x_3 \in A$, $F(x_2x_3) = \{x_1, x_2, x_3, x_4, x_8, x_9, x_{10}\}$ and $K(x_2x_3) = \{x_1x_2, x_2x_8, x_3x_4, x_3x_9, x_3x_{10}\}$.

For $x_5x_6 \in A$, $F(x_5x_6) = \{x_4, x_5, x_6x_{10}, x_{11}, x_{12}\}$ and . $K(x_5x_6) = \{x_4x_5, x_5x_{10}, x_5x_{11}, x_6x_{11}, x_6x_{12}\}$

For $x_7x_8 \in A$, $F(x_7x_8) = \{x_1, x_2, x_7, x_8, x_{13}\}$ and $K(x_7x_8) = \{x_1x_7, x_2x_8, x_7x_{13}, x_2x_{13}\}$.

For $x_9x_{10} \in A$,

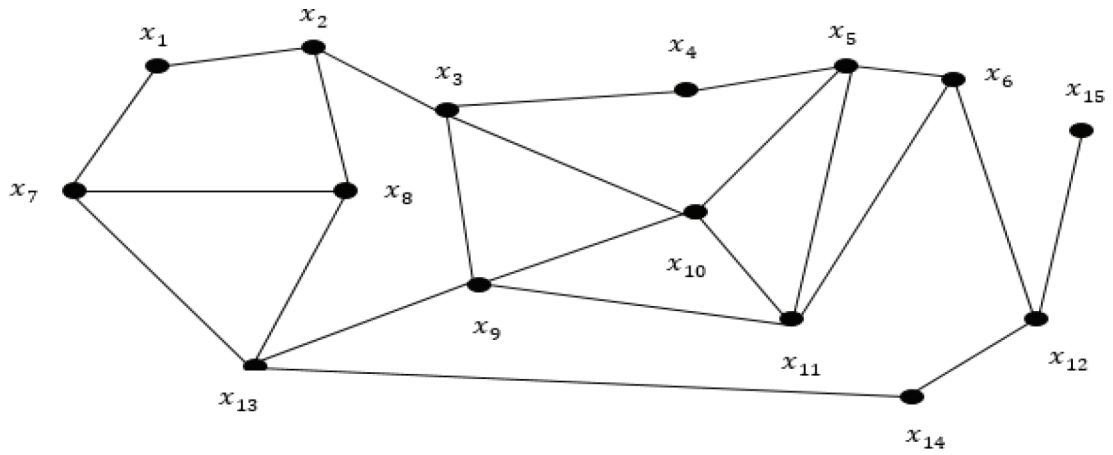


Figure 1. Graph G

$F(x_9x_{10}) = \{x_3, x_5, x_9, x_{10}, x_{11}, x_{13}\}$ *and*
$K(x_9x_{10}) = \{x_3x_9, x_9x_{11}, x_9x_{13}, x_3x_{10}, x_5x_{10}, x_{10}x_{11}\}$.

For $x_{14}x_{12} \in A$, $F(x_{14}x_{12}) = \{x_6, x_{12}, x_{13}, x_{14}, x_{15}\}$ and

$$K(x_{14}x_{12}) = \{x_6x_{12}, x_{12}x_{15}, x_{13}x_{14}\}.$$

Corresponding to each element of A we have the subgraphs $H(x_2x_3)$, $H(x_5x_6)$, $H(x_7x_8)$, $H(x_9x_{10})$ and $H(x_{14}x_{12})$ of G which represents the areas that are connected to the major roads x_2x_3, x_5x_6, x_7x_8, x_9x_{10} and $x_{14}x_{12}$ respectively.

Here, $H(x_2x_3)$ in Figure 2 and $H(x_{14}x_{12})$in Figure 6 are the disconnected graphs. So x_2x_3 and $x_{14}x_{12}$ are the only roads which covers all the areas of $F(x_2x_3)$ and $F(x_{14}x_{12})$ respectively.

The subgraph $H(x_5x_6)$ in Figure 3, x_5x_6 is the road which covers all the areas of $F(x_5x_6)$. The sub graph $H(x_7x_8)$ in Figure 4 is a path P_5. To reach the area from x_1 to x_8, we have to travel through x_7 and x_8 It consumes more time and in turn fuel. Instead, if the road between the areas x_7 and x_8 is well maintained it reduces time and in turn fuel consumption.

Similarly, $H(x_9x_{10})$ in Figure 5, if the road between x_9 and $x_{10}$is maintained in good condition, it saves time and in turn fuel consumption.

It is hard to reconstruct all roads at a time, which leads to financial burden to the Government. Thus, by identifying the roads which covers maximum areas and maintaining them, helps in reduction of travelling time

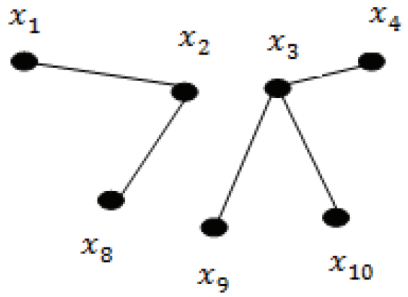


Figure 2. $H(x_2x_3)$

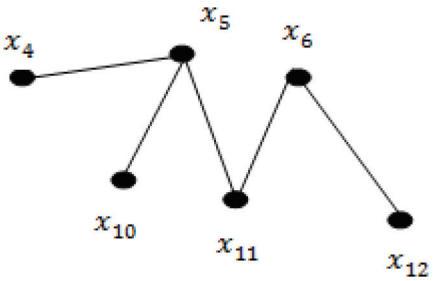


Figure 3. $H(x_5x_6)$

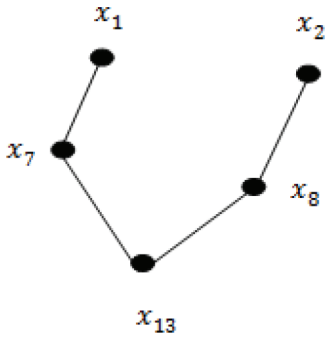


Figure 4. $H(x_7x_8)$

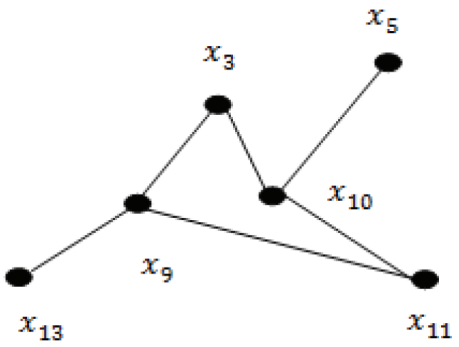


Figure 5. $H(x_9x_{10})$

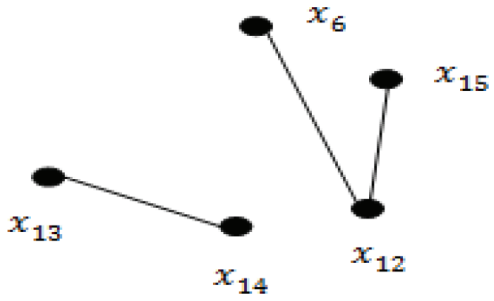


Figure 6. $H(x_{14}x_{12})$

and in turn fuel consumption. This can be done effectively by considering the minimal edge dominating set, which gives the roads which covers all areas. And the soft graph corresponding to an edge in minimal edge dominating set gives the areas covered by each edge of edge dominating set.

Hence, the maintenance of edges of the minimal edge dominating set is most essential. The soft graph can be used to find a better path from any key access point to the mining location in the mining area.

5.0 Conclusion

Soft graph is one of the most useful typesof graph theory which solves real world problems, mainly on uncertainty, in various fields which includes business, social science and physical science etc. In this paper, we have introduced the new soft graph by taking the dominating edges as new parameters and the results on this soft graph are established. The necessity of defining the new soft graph is illustrated with an application.

6.0 References

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