Journal of Mines, Metals and Fuels, *71(12D) : 128-131; 2023. DOI: 10.18311/jmmf/2023/47301*

Print ISSN : 0022-2755

Journal of Mines, Metals and Fuels

Contents available at: *www.informaticsjournals.com/index.php/jmmf*

A Strive to Construct a Function to Obtain the Irrational and Transcendental Real Numbers and its Dominant Role in Signal Processing

K. Sowmiya

Assistant Professor, Department of Mathematics, Dayananda Sagar College of Engineering, Shavige Malleshwara Hills, Kumaraswamy Layout, Bangalore - 560050, Karnataka, India

Abstract

A transcendental real number is a number which will not be the root of any integer polynomial, must also be irrational number, whereas not all irrational numbers are transcendental. In this paper, we construct a mathematical function to obtain the irrational and transcendental real numbers. And also, we elucidate its importance in signal processing.

Keywords: *Irrational, Transcendental Numbers and Signal Processing*

1.0 Introduction

Hippasus, one of the Pythagoras students discovered irrational number while finding the fraction value of √2. As the value of √2 cannot be written as fraction, hence it became the irrational number. Later, German mathematician F. Lindemann, found that the famous number pi is transcendental in 1882 and it can not be expressed as the finite series of arithmetic numbers or algebraic operations. Without pi circle cannot be obtained. Euler's Number 'e' is another crucial transcendental number which plays an inevitable role in mathematics. In the realm of mathematics and its application in various fields, the exploration of irrational and transcendental real numbers holds paramount significance. These numbers, often elusive and enigmatic, defy simple representation as fractions or roots of integers. Instead, they reveal themselves through infinite decimal expansions that neither repeat nor terminate. All trigonometric functions are Irrational which are basically used to find the height, distance and also in the major calculations of physics. Despite their mysterious nature, irrational and transcendental numbers play a dominant role in signal processing, a critical domain with applications spanning from telecommunications to the mining industry.

Signal processing entails the manipulation and analysis of signals with the objective of extracting valuable information or enhancing particular features. These signals, which can be audio, visual, or even seismic waves in mining operations, are inherently analog in nature, and their processing often relies on mathematical techniques that involve the use of real numbers. However, the complexity of real-world signals often necessitates the utilization of irrational and transcendental numbers to accurately model and analyze them. In this context, understanding the properties and applications of these numbers becomes indispensable for signal processing engineers and researchers striving to advance technology across various industries.

This paper delves into the significance of irrational and transcendental real numbers in signal processing, particularly highlighting their vital role in the mining

**Author for correspondence*

industry. In this paper we have constructed the function and proved those functions will give irrational and transcendental numbers by the mathematical way of contradiction. Also, the necessary role of irrational numbers in signal processing and mining is briefed in the paper.

1.1 The Mysterious Numbers

Irrational Numbers: Irrational numbers cannot be expressed as the ratio of two integers. (i.e.) The real numbers which are not rational numbers is called irrational numbers.

Algebraic number: These numbers are not a root of a polynomial with rational co. efficient.

Transcendental Numbers: It cannot be expressed as the root of any algebraic equations with rational coefficients. In other words, it is a complex number which is not algebraic.

2.0 Construction of Function to Obtain the Irrational and Transcendental numbers

In this we prove the functions for Irrational and transcendental numbers.

Theorem 1: We show that $f(x)=\frac{\sqrt[n]{x}}{x}$ is Irrational, where x is prime and $n \geq 2$.

Proof: We know that $\sqrt[n]{x}$ is irrational for all prime x and $n \geq 2$

We prove this theorem by contradiction

Suppose $f(x)=\sqrt[n]{x}/x$ is rational,

Let $\sqrt[n]{x}/x = a/b$ where a, b has no common factors (i.e.) g.c.d. $(a, b) = 1$.

Now, $\frac{\sqrt[n]{x}}{x}=\frac{a}{b}$

Table 1. Transcendental Numbers

Proved as Transcendental Numbers	Unproved
The number e	Euler's Constant Gamma γ
The number pi(π)	Catalan's Constant
The number Log 2	Champerwone Constant
The number $e^{\sqrt{2}}$	Zeta (3)
The Hilbert's number $2^{\sqrt{2}}$	Log pi

$$\Rightarrow \left(\frac{\sqrt[n]{x}}{x}\right)^n = \left(\frac{a}{b}\right)^n$$

$$\Rightarrow x^{1-n} = \left(\frac{a}{b}\right)^n$$

$$\Rightarrow \frac{1}{x^{n-1}} = \left(\frac{a}{b}\right)^n$$

$$\Rightarrow 1 = x^{n-1}\left(\frac{a}{b}\right)^n$$

$$\Rightarrow 1 = x^{n-1}\left(\frac{a^n}{b^n}\right)$$

Since a and b have no common factors a^n *and* b^n cannot have the common factors, therefore the value of $x^{n-1}(a/b)^n$ cannot be equal to 1. Thus, we arrive at a contradiction to our assumption that $f(x)=\sqrt[n]{x}/x$ is irrational.

Hence $f(x)=\sqrt[n]{x}/x$ is Irrational.

Theorem 2: We prove that *log π* is Trancendental Number

Proof: To prove *log π* as transcendental, we have to prove it is not root of any algebraic equations with rational coefficients.

Let us prove by contradiction.

Assume that log pi is a root of algebraic equation $f(x)=a_0x^n+a_1x^{n-1}+a_2x^{n-2}+.....+a_{n-1}x+a_n$ with rational coefficients where all a_i's are not zero, $i=0,1,2,\ldots\ldots n$.

Then

$$f(\log\pi)=a_0(\log\pi)^n+a_1(\log\pi)^{n-1}+a_2(\log\pi)^{n-2}$$
$$+.....+a_{n-1}(\log\pi)+a_n=0$$
$$\Rightarrow a_0(\log\pi)^n+a_1(\log\pi)^{n-1}+a_2(\log\pi)^{n-2}$$
$$+.....+a_{n-1}(\log\pi)+a_n=0$$

By taking out $\log\pi$ common, we get

$$\Rightarrow \log\pi\{a_0(\log\pi)^{n-1}+a_1(\log\pi)^{n-2}+.....+a_{n-1}\}+a_n=0$$
$$\Rightarrow a_0(\log\pi)^{n-1}+a_1(\log\pi)^{n-2}+.....+a_{n-2}(\log\pi)$$
$$+a_{n-1}=-a_n/\log\pi$$
$$\Rightarrow \log\pi\{a_0(\log\pi)^{n-2}+a_1(\log\pi)^{n-3}+.....$$
$$+a_{n-2}\}+a_{n-1}=-a_n/\log\pi$$
$$\Rightarrow a_0(\log\pi)^{n-2}+a_1(\log\pi)^{n-3}+.....+a_{n-2}$$
$$=-a_n/(\log\pi)^2-a_{n-1}/\log\pi$$

Repeating the process n times, we get

$$\log\pi\{a_0\}+a_1=-a_n/(\log\pi)^{n-1}-a_{n-1}/(\log\pi)^{n-2}$$
$$-a_{n-2}/(\log\pi)^{n-3}-\ldots\ldots-a_2/(\log\pi)^1$$
$$\Rightarrow\log\pi\{a_0\}=-a_n/(\log\pi)^{n-1}-a_{n-1}/(\log\pi)^{n-2}-$$
$$a_{n-2}/(\log\pi)^{n-3}-\ldots\ldots-a_2/(\log\pi)^1$$
$$\Rightarrow a_0=-a_n/(\log\pi)^n-a_{n-1}/(\log\pi)^{n-1}-$$
$$a_{n-2}/(\log\pi)^{n-2}-\ldots\ldots-a_2/(\log\pi)^2-a_1/(\log\pi)$$
$$\Rightarrow a_0=-\{a_n/(\log\pi)^n+a_{n-1}/(\log\pi)^{n-1}$$
$$+a_{n-2}/(\log\pi)^{n-2}+\ldots\ldots+a_2/(\log\pi)^2+a_1/(\log\pi)$$
$$\Rightarrow 0=-\{a_n/(\log\pi)^n+a_{n-1}/(\log\pi)^{n-1}+$$
$$a_{n-2}/(\log\pi)^{n-2}+\ldots\ldots+a_2/(\log\pi)^2+a_1/(\log\pi)+a_0\}$$

Since $\log\pi$ cannot be zero, this implies all a_i's are zero, $i=1,2,\ldots\ldots n$, hence also we get a_0=0 which is a contradiction to our assumption.

Hence $\log\pi$ is a transcendental number.

Generalization of the above theorem 2: Prove that $g(x)=\log x$ is transcendental, if *x* is transcendental.

Proof: It follows the same lines as in the proof of Theorem 2 by replacing $\log\pi$ *as* log x

3.0 Dominant Role of Irrational Numbers in Signal Processing

The term signal processing has a broad scope of meanings and encompasses a diverse array of applications. When two sinusoids with different frequencies are summed in the continuous time domain, the resulting signal becomes periodic only if the ratio of their frequencies is a rational number. If both frequencies are integers or have finite decimal representations, their sum will always yield a periodic signal. However, if one of the frequencies is irrational, the combined signal will not exhibit periodic behavior. This principle holds true in theory, although real-world factors like measurement precision and digital signal processing considerations may introduce complications[1-5].

In the discrete time domain, in computer simulation, an irrational number must be truncated in the number of decimals so it is also seen as a rational number. It means that, digitally, if we are summing two discrete sinusoids of any frequency because of the impossibility of truly representing an irrational number, we can never get an aperiodic signal from their sum. However, there are signals that never precisely repeat over any reasonable correlation length of any filters in the system. Therefore, they might as well be considered to have a normalized frequency in the discrete-time domain[6-13].

In the realm of signal processing within the mining industry, irrational numbers manifest in various facets. For instance, in seismic exploration, which aids in identifying subsurface structures and potential mineral deposits, signal processing techniques involve the use of Fourier transforms where trigonometric functions yield irrational results. Additionally, in processing data from Ground-Penetrating Radar (GPR) or electromagnetic surveys used for geological mapping, algorithms employ complex numbers, including irrational components, to analyze and interpret signals reflected from underground formations. Moreover, in the realm of vibration analysis for monitoring equipment health and detecting structural anomalies in mines, Fourier analysis and wavelet transforms are utilized, often involving irrational numbers in the mathematical operations. Furthermore, in machine learning applications for predictive maintenance or ore grade estimation, algorithms may utilize neural networks or support vector machines, which rely on complex mathematical operations potentially involving irrational numbers. Hence, in signal processing applications throughout the mining industry, irrational numbers play a significant role in extracting valuable insights from various types of data collected in exploration, monitoring, and operational contexts[13-19].

4.0 Conclusion

The motive of the paper is to construct the functions of irrational and transcendental numbers, which gives some insights of the numbers and helps to investigate more about it, can develop the interest to relate it with applications of different fields. The exploration of irrational and transcendental real numbers unveils a fascinating intersection between abstract mathematical concepts and practical applications in signal processing, with profound implications for industries like mining. By embracing the complexity inherent in these numbers, signal processing engineers and researchers can develop more sophisticated algorithms to extract meaningful information from intricate signals encountered in real-world scenarios. As technology continues to evolve, the role of these numbers is poised

to become even more pronounced, driving innovation and efficiency in signal processing methodologies across diverse industrial sectors.

5.0 References

1. Koblitz N. A course in number theory and cryptography. 2nd ed. New York: Springer-Verlag; 2000.
2. Hsiung CY. Elementary theory of numbers. New Delhi: Allied Publishers Limited; 1995.
3. Hasselblatt B, Katok A. A first course in dynamics with a panorama of recent developments. Cambridge/New York: Cambridge University Press; 2003.
4. Byrne CL. Signal processing: a mathematical approach. 2nd ed. Boca Raton: CRC Press, Taylor & Francis Group; 2015.
5. Allouche JP, Shallit J. Automatic sequences: theory, applications, generalizations. Cambridge: Cambridge University Press; 2003.
6. Baker A. Transcendental number theory. Paperback ed. Cambridge: Cambridge University Press; 1990.
7. Bourbaki N. Elements of the history of mathematics. Berlin: Springer; 1994.
8. Bugeaud Y. Distribution modulo one and Diophantine approximation. Cambridge Tracts in Mathematics. Vol. 193. Cambridge: Cambridge University Press; 2012.
9. Burger EB, Tubbs R. Making transcendence transparent: an intuitive approach to classical transcendental number theory. Berlin: Springer; 2004.
10. Chudnovsky GV. Contributions to the theory of transcendental numbers. Providence: American Mathematical Society; 1984.
11. Calude CS. Information and randomness: an algorithmic perspective. Texts in Theoretical Computer Science. 2nd rev. and ext. ed. Berlin: Springer; 2002.
12. Hardy GH. An introduction to the theory of numbers. 5th ed. Oxford: Clarendon Press; 1979.
13. Higgins PM. Number story. New York: Copernicus Books; 2008.
14. Kempner AJ. On transcendental numbers. Trans Am Math Soc. 1916; 17(4):476–82. doi:10.2307/1988833.
15. Le Lionnais F. Les nombres remarquables. Paris: Hermann; 1979.
16. LeVeque WJ. Topics in number theory. New York: Dover; 2002.
17. Loxton JH. Automata and transcendence. In: Baker A, editor. New advances in transcendence theory. Cambridge: Cambridge University Press; 1988.
18. Baker A. Transcendental number theory. Cambridge: Cambridge University Press; 2022.
19. Lenart I. Periodic, irrational and transcendental numbers in factorial series. J Appl Math Phys. 2022; 10(2):558–75.