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Combined Impact of Temperature-Dependent Heat Source and Gravity Fluctuation on the Onset of Darcy-Brinkman Reaction-Convection in an Anisotropic Porous Layer

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Abstract

This paper investigates the combined influence of temperature-dependent heat sources and gravitational fluctuations on the onset of double-diffusive reaction-convection in an anisotropic porous medium. The momentum equation is modelled using the Darcy-Brinkman approach, and linear stability analysis is conducted using normal mode analysis to formulate the eigenvalue problem. Four gravity functions linear, parabolic, cubic and exponential are analysed and the critical Rayleigh numbers for stationary and oscillatory modes are derived using a single-term Galerkin approximation. The results reveal that temperature-dependent heat sources, gravity variations, anisotropy and the Damköhler number significantly affect convection instability thresholds. Notably, exponential gravity fluctuations provide the highest system stability, delaying the onset of convection. These findings offer valuable insights into the stability behaviour of anisotropic porous systems, with applications in optimising geothermal energy systems, industrial heat management, porous media-based chemical reactors, groundwater filtration, subsurface energy storage and thermal insulation systems.

Major Findings: *The findings revealed that temperature-dependent heat sources and exponential gravity fluctuations delay convection onset, enhancing system stability. Linear, parabolic, and cubic gravity variations promote instability, reducing stability thresholds. Anisotropy amplifies system sensitivity to governing parameters, while the Damköhler number significantly influences instability. Exponential gravity fluctuations emerge as the most stabilizing factor.*

Keywords: *Heat Source, Mechanical Anisotropy, Thermal Anisotropy, Variable Gravity*

1.0 Introduction

Double-diffusive convective motion in a porous bed has numerous applications, ranging from solidifying binary mixtures to transporting soil-based solutes saturated with water. This phenomenon plays a significant role in fields such as oceanography, electrochemistry, geophysical systems, the production of high-purity pharmaceuticals, the storage and transport of liquid gases and moisture transport in fibre insulation. Understanding thermo-double-diffusive convection is crucial in designing and optimising engineering systems, including geothermal energy applications, chemical reactors, industrial heat management and groundwater filtration systems. These systems often rely on the efficient transport of heat and solutes within porous media, making insights into convection stability critical for improving performance and energy efficiency. The study of double-diffusive convection in porous media has been extensively explored, with notable contributions from researchers

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such as Ingham and Pop[1], Nield and Bejan[2], Vafai [3], Vadasz [4], Steinberg and Brand[5] and Gangadharaiah[6-8] who have all done exceptional documentation of their work.

Fluid-containing dissolved minerals flow vertically because of the buoyancy force. The quantity of solute content of the fluid might be non-preserved since that can disintegrate or precipitate on a porous matrix during this procedure. This occurrence is influenced by pressure, temperature along with local rock chemistry. As an outcome, the impact on convective instability as an outcome of these processes is critical. However, this is not widely recognised or has garnered little attention. Certain initiatives have been made to address this problem by referring to the presentation of the impact of a biochemical process on the commencement of convection. Steinberg and Brand[9] produced great work to determine the impression of chemical reactions on the convective motion of porous media. The impact of exothermic reactions on porous medium stability was observed in Gatica *et al*,.[10] and Viljoen *et al*,.[11] Thermally generated fronts of retaliation in considered media were examined by Jupp and Woods[12] and Malashetty and Gaikwad[13] examined chemically induced binary liquids mix that are unstable with rapid chemical reactions.

Alex and Patil[14] examined the gravity fluctuation with internal heating of the configuration that is on the porous bed and they showed that a delay in the gravity factor increases the consistency of the configuration. Using the regular perturbation technique, the impact of linear gravity fluctuation with throughflow and internal heating in a porous bed configuration was researched by Suma *et al*.,[15] and Gangadharaiah *et al*[16]. The impacts of a gravity fluctuation on porous layers were examined by Nagarathnamma *et al*.,[17] using the Galerkin approach. The magnetic field in addition to throughflow effects on porous bed configuration is studied by Yadav[18]. Mahajan and Tripathi[19] looked into how gravity fluctuation affected the consistency of a thermo-solutal convective flow inside a situation in which convective motion arises from non-uniformity in thermophoresis number. Very recently, the penetrative solutal convective motion with varying gravity and throughflow inside a fluid layer configuration has been analysed by Gangadharaiah *et al*., [20] Recent studies have explored thermal convective motion within fluid-saturated porous layers with varying gravity. These investigations consider how gravity variations affect heat transfer along with fluid flow with applications in geophysics, environmental science, and space exploration. References (Dhananjay Yadav[21,22], Gangadharaiah *et al*.,[23-25] Samrat *et al*.,[26] Suma and Gangadharaiah[27], Gangadharaiah and Suma[28], Gangadharaiah[29,30] and Gangadharaiah and Anand[31]) contain detailed findings and methodologies on this topic.

In this work, the study focuses on thermo-solutal convection inside a fluid-saturated anisotropic porous bed along with varying gravity, considering a temperature-dependent heat source and reaction. The research examines the impact of different gravity functions (linear, parabolic, cubic and exponential), as well as temperature-dependent heat source, reaction and anisotropic parameters on system stability. The Galerkin technique is used to analyse the linear stability threshold through an eigenvalue problem resolution.

2.0 Mathematical Formulation

The simplified configuration of thermal reaction convection, displayed in Figure 1, is an infinite anisotropic porous bed bounded through the surfaces at z = 0 to z = d with a temperature-dependent heat source Q and a gravity fluctuation $g(z) = (1+\delta h(z))g$.

- The internal heat source is assumed to vary linearly with temperature, while gravity acts downwards and changes with height.
- The momentum equation governing the flow is formulated using the Darcy-Brinkman model, which accounts for viscous and permeability effects.
- The porous medium under consideration exhibits anisotropic properties, influencing thermal and mechanical responses.

For flow governing equations are considered as below (Gaikwad and Dhanraj[32], Pritchard and Richardson[33]):

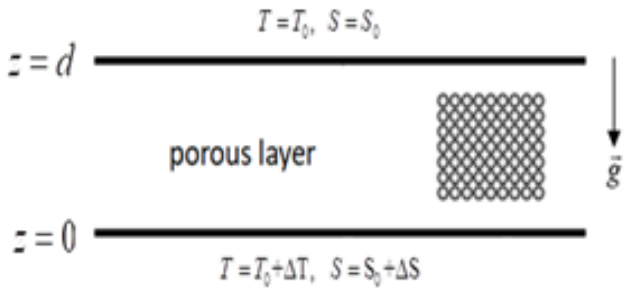


Figure 1. Physical configuration.

$$\nabla \cdot \vec{q} = 0 \tag{1}$$

$$\nabla p + \frac{\mu}{K}\vec{q} - \mu_e \nabla^2 q - \rho\vec{g} = 0 \tag{2}$$

$$\begin{aligned}&(\rho c)_m \frac{\partial T}{\partial t} + (\rho c)_f (\vec{q}\cdot\nabla)T = \\ &(\rho c)_m \nabla\cdot(\kappa_T \cdot \nabla T) + Q(T - T_0)\end{aligned} \tag{3}$$

$$\varepsilon\frac{\partial S}{\partial t} + (q\cdot\nabla)S = \varepsilon\kappa_S\nabla^2 S + k\left(S_{eq}(T) - S\right) \tag{4}$$

$$\rho = \rho_0\left[1 - \beta_T(T - T_0) + \beta_S(S - S_0)\right], \tag{5}$$

the fluid which is in its basic state is thought that be quiescent of form.

$$\begin{aligned}&\vec{q}_b = 0,\ p = p_b(z), T = T_b(z),\\ &S = S_b(z),\ \rho = \rho_b(z)\end{aligned} \tag{6}$$

Then the basic state solutions are

$$\kappa_T\frac{d^2(T_b - T_0)}{dz^2} + Q(T_b - T_0) = 0, \tag{7}$$

$$K_S\frac{d^2 S_b}{dz^2} + D\frac{d^2 T_b}{dz^2} = 0, \tag{8}$$

$$T_b = \frac{\sin\sqrt{R_i}(1-z)}{\sin\sqrt{R_i}}, \tag{9}$$

$$\begin{aligned}S_b = &\frac{S_r L_e \sin\left(\sqrt{R_i}(1-z)\right)}{\sin\sqrt{R_i}} - \\ &(S_r L_e + 1)(1-z)\end{aligned} \tag{10}$$

To examine the stability of a basic state, introduce small perturbations and linearise the governing equations.

$$\begin{aligned}&\vec{q} = q_b + q',\ T = T_b(z) + T',\\ &p = p_b(z) + p', S = S_b(z) + S',\\ &\rho = \rho_b(z) + \rho',\end{aligned} \tag{11}$$

Replacing Equation (11) into Equations (1)-(5) utilising basic state solutions yields perturbation equations describing that system's linearised behaviour.

$$\nabla\cdot\vec{q}' = 0, \tag{12}$$

$$\begin{aligned}&\nabla p' + \frac{\mu}{K}\vec{q}' - \mu_e\nabla^2\vec{q}' + \\ &\rho_0\left(\beta_T T' - \beta_S S'\right)g\hat{k} = 0\end{aligned} \tag{13}$$

$$\begin{aligned}&(\rho c)_m\frac{\partial T'}{\partial t} + (\rho c)f\left((q'\cdot\nabla)T' - w'\frac{\Delta T}{d}\right)\\ &= (\rho c)_m\nabla\cdot\left(\kappa_T.\nabla T'\right) + Q\left(T' - T_0'\right)\end{aligned} \tag{14}$$

$$\begin{aligned}&\varepsilon\frac{\partial S'}{\partial t} + (q'\cdot\nabla)S' - w'\frac{\Delta S}{d} =\\ &\varepsilon K_S\nabla^2 S' + K\left(S_{eq}(T') - S'\right)\end{aligned} \tag{15}$$

By applying Curl two times on equation (13), we remove p then use the scaling

$$\begin{aligned}&(x,y,z) = (x^*, y^*, z^*)d, t = \frac{d^2}{K_{T_z}}t^*,\\ &\vec{q}' = \frac{\varepsilon K_{T_Z}}{d}\vec{q}^{\,*}, T' = (\Delta T)T^*,\\ &S' = (\Delta S)S^*,\end{aligned} \tag{16}$$

To attain non-dimensionalise, Equations (13)-(15) which in the following form (on dropping asterisks for simplicity)

$$\begin{aligned}&\left(\nabla_1^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)w - D_a\nabla^4 w -\\ &R\nabla_1^2 TG(z) + Rs\nabla_1^2 SG(z) = 0,\end{aligned} \tag{17}$$

$$\begin{aligned}&\frac{\partial T}{\partial t} + \varepsilon_n(\vec{q}\cdot\nabla)T - RiT - \varepsilon_n\frac{dT_b}{dz}w =\\ &\eta\nabla_1^2 T + \frac{\partial^2 T}{\partial z^2},\end{aligned} \tag{18}$$

$$\frac{\partial S}{\partial t}+(\vec{q}\cdot\nabla)S-\frac{dS_b}{dz}w=\frac{1}{Le}\nabla^2 S+\chi(T-S), \tag{19}$$

Where

$R=\dfrac{\alpha_0 g\beta_T \Delta T K_z d}{\mu\varepsilon\kappa_{Tz}}$, is a thermal Rayleigh number

$Rs=\dfrac{\alpha_0 g\beta_S \Delta S K_z d}{\mu\varepsilon\kappa_{Tz}}$, is a solute Rayleigh number

$Le=\dfrac{\kappa_{Tz}}{\kappa_S}$,is a Lewis number,

$\xi=\dfrac{K_h}{K_v}$,is a mechanical anisotropy parameter

$\eta=\dfrac{k_h}{k_v}$,is a thermal anisotropy parameter

$\varepsilon_n=\dfrac{\varepsilon(\rho c)_f}{(\rho c)_m}$,is a normalised porosity parameter

$G(z)=1+\delta h(z)$ is a variable gravity function.

The equation for normal mode is given by

$$(W,\theta,\phi)=\{\hat{W}(z),\hat{\theta}(z),\hat{\phi}(z)\}\exp[i(lx+my)+\sigma t] \tag{20}$$

On application of Equation (20) into Equations. (17) – (19), we can write:

$$\left(\frac{D^2}{\xi}-a^2-Da(D^2-a^2)^2\right)\hat{W}+\left(R\hat{\theta}-Rs\,\hat{\phi}\right)a^2G(z)=0 \tag{21}$$

$$\{\omega-(D^2-\eta a^2)+R_i\}\hat{\theta}-\varepsilon_n\frac{dT_b}{dz}\hat{W}=0 \tag{22}$$

$$\{\omega-Le^{-1}(D^2-a^2)+\chi\}\hat{\phi}-\chi\hat{\theta}-\frac{dS_b}{dz}\hat{W}=0 \tag{23}$$

Along with boundary conditions

$$\hat{W}=\hat{\theta}=\hat{\phi}=0 \quad \text{at z = 0,1} \tag{24}$$

3.0 Technique of the Solution

Galerkin-type weighted residuals approach is employed to determine a numerical solution, in which three variables, $\hat{W}$, $\hat{\theta}$ and $\hat{\phi}$ are considered as

$$\hat{W}=\sum_{i=1}^{N}A_i\hat{W}_i,\quad \hat{\theta}=\sum_{i=1}^{N}B_i\hat{\theta}_i \quad \text{and} \quad \hat{\phi}=\sum_{i=1}^{N}C_i\hat{\phi}_i$$

$$n=1,2,3.... \tag{25}$$

whereas *Ai,Bi* and *C* are unidentified constants. The base functions $\hat{W}_i,\hat{\theta}_i$ and $\hat{\phi}_i$ are expected as

$$\hat{W}=\hat{\theta}_i=\hat{\phi}_i=s\,\text{in}(i\pi z) \tag{26}$$

Employing Equation (26) into Equations (21)–(23) also integrates each equation taking.z = 0 to z =1we get,

$$\begin{bmatrix}P_1 & RP_2 & P_3\\ P_4 & P_5 & P_6\\ P_7 & P_8 & P_9\end{bmatrix}\begin{bmatrix}\omega_0\\ \theta_0\\ \phi_0\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \tag{27}$$

Therefore, from the determinant of the coefficient matrix, we gain a critical Rayleigh number in terms of other numbers as

$$R=\frac{(\delta_1^2+Da\delta^4)(\omega+\delta_2^2+R_i)}{a^2\varepsilon_n\int_0^1 G(z)dz}+\frac{Rs}{\varepsilon_n}\frac{(\omega-\varepsilon_n\chi+\delta_2^2+R_i)}{(\omega+\chi+Le^{-1}\delta^2)} \tag{28}$$

Where $\delta^2=\pi^2+a^2$, $\delta_1^2=\pi^2k^{-1}+a^2$, $\delta_2^2=\pi^2+\eta a^2$.

3.1 Marginal Stationary Convection

In the stationary mode, we take ω= 0, in Equation (28) to get the Rayleigh number which is given by

$$Ra_T^{St} = \frac{\left(\pi^2+\eta a^2+R_i\right)}{a^2\varepsilon_n\int_0^1 G(z)dz}$$
$$\left\{a^2+\frac{\pi^2}{k}+Da(\pi^2+a^2)^2\right\} \qquad (29)$$
$$+\frac{L_e Ra\left(\pi^2+\eta a^2+\varepsilon_n\chi+R_i\right)}{\varepsilon_n\left(Le\chi+\pi^2+a^2\right)}$$

When Darcy number is not present, i.e. Da = 0 Eq. reduces to

$$Ra_T^{St} = \frac{\left(\pi^2+\eta a^2+R_i\right)}{a^2\varepsilon_n\int_0^1 G(z)dz}\left(a^2+\frac{\pi^2}{k}\right)+ \qquad (30)$$
$$\frac{L_e Ra\left(\pi^2+\eta a^2+\varepsilon_n\chi+R_i\right)}{\varepsilon_n\left(Le^{-1}\chi+\pi^2+a^2\right)}$$

3.2 Marginal Oscillatory Convection

Regarding the Oscillatory mode, put $\omega = i\omega_i$ in equation (28) along with that collect terms for the real part to gain the Rayleigh number which is given by

$$Ra_T^{Osc} = \frac{\left[\delta_1^2+D_a\delta^4\right]\left[\delta_2^2+R_i\right]}{a^2\varepsilon_n\int_0^1 G(z)dz}+\frac{Ra_S}{\varepsilon_n} \qquad (31)$$
$$\frac{\omega_i^2+\left[-\varepsilon_n\chi+\delta_2^2+R_i\right]\left[Le^{-1}\delta^2+\chi\right]}{\left[\chi+Le^{-1}\delta^2\right]^2+\omega_i^2}$$

where,

$$\omega_i^2 = \frac{Ra_s a^2\int_0^1 G(z)\,dz}{\delta_1^2+D_a\delta^4}\left[\delta_2^2-L_e^{-1}\delta^2-\chi\left(1+\varepsilon_n\right)\right]$$
$$-\left(\chi+L_e^{-1}\delta^2\right)^2$$

4.0 Results and Discussion

The outcomes for distinct gravity fluctuation and reaction on heat source parameter, the gravity number, the thermal anisotropy parameters, Damkohler number, Darcy number, and mechanical anisotropy parameter on oscillatory and stationary convection have been plotted graphically and their instabilities have been explored in this paper. We considered four types of distinct gravity fluctuation (linear, parabolic, cubic, and exponential) as follows:

$$(i)\,h(z)=-z,\ (ii)\,h(z)=-z^2,$$

$$(iii)\,h(z)=-z^3,\text{and}$$

$$(iv)\,h(z)=-\left(e^z-1\right).$$

The results for the single diffusive and isotropic porous layer were validated by comparing them with those from Rionero and Straughan[34] in Table 1.

Figure 2, depicts the oscillatory along with stationary mode neutral stability curves for every four cases of gravity functions with fixed parameters. Da = 0.001, ε_n = 1, η =0.5 L_e = 20, ξ=0.5, χ = 3, ω = 1, δ = 0.5 From these figures, It is obvious that stability curves will shift downwards for the stationary mode and shift upwards for

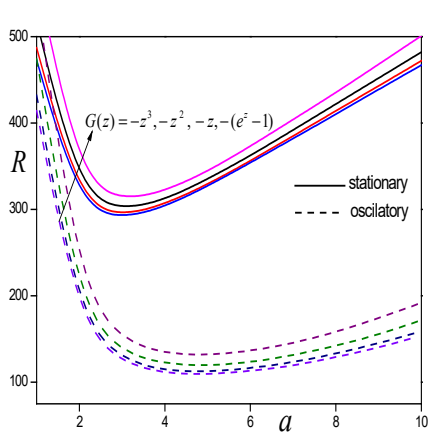


Figure 2. Impact of R against a for four types of gravity fluctuation with fixed parameters.

oscillatory mode further, it is noticed that arrangement is most steady for exponential type gravity fluctuation as well as most unsteady for cubic type gravity fluctuation function. The exponential gravity field offers a more gradual change in the gravitational force with height, which contributes to a more stable convection process, as it avoids the abrupt changes seen in linear, parabolic and cubic gravity profiles.

Figure 3, shows the impact of R^c for both oscillatory as well as stationary modes against gravity parameter $Da = 0.01$, $\varepsilon_n = 1$, $\eta = 0.5$, $L_e = 20$, $\xi = 0.5$, $\chi = 3$, $\omega = 1$, Results reveal that for convective motion due to gravity, the parameter rises. R^c decreases. As such occurred due to a rise in the numerical value of δ rises the gravity when it comes to a growing gravitational field, therefore convective motion starts quickly. The stationary mode has a more stabilising impact as related to the oscillatory mode. Furthermore, an exponential gravitational field with greater stability when associated with the residual types of gravity fluctuation.

Figure 4, exhibits, R^c as a function of wave number for $Da = 0.01$ and 0.001, respectively with four types of gravity functions by fixing other parameters. $\varepsilon_n = 1$, $\eta = 0.5$, $L_e = 20$, $\xi = 0.5$, $\chi = 3$, $\omega = 1$, $\delta = 0.5$. From these figures, we can discern a growth in the Darcy number. Da leads to an increase in the critical Rayleigh number. This phenomenon can be explained by the fact that, at higher values of Darcy number, the condition for the stability

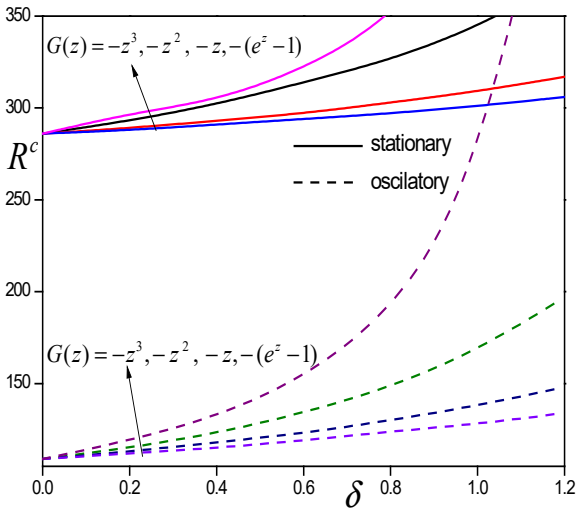


Figure 3. Plot of R^c verses δ for four types of gravity fluctuation with fixed parameters.

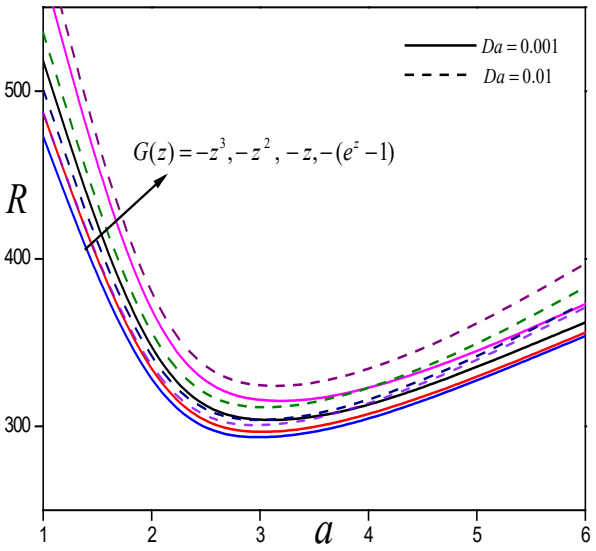


Figure 4. Plot of R verses a with two values of Da for four types of gravity fluctuation with fixed parameters.

of convection becomes more dependent on the average properties of the medium, consequently causing an elevation in the critical Rayleigh number. Furthermore, data additionally illustrates that the system exhibits greater stability when subjected to exponential gravity fluctuations in comparison to other types of fluctuation patterns.

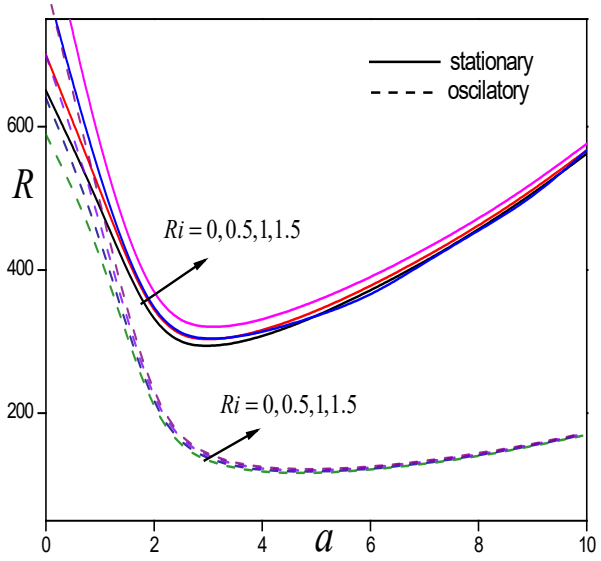


Figure 5. Impact of R against a for various values of Ri for linear-type gravity fluctuation with fixed parameters.

The variation of temperature-dependent heat source parameters, *Ri* on the stability of the structure is exposed in Figures 5 - 8 for linear, parabolic, cubic and exponential types of gravity fluctuation respectively with $Da = 0.001$, $\varepsilon_n = 1$, $\eta = 0.5$, $L_e = 20$, $\xi = 0.5$, $\chi = 3$, $\omega = 1$, $\delta = 0.5$. These figures indicate that the thermal Rayleigh number also increases with an increase in internal heat source parameters. This behaviour suggests that in presence of the internal heat source contributes to the stabilisation of the system. Furthermore, the data reveals that the system is more stable when subjected to an exponential gravitational field compared to other types of gravity fluctuations. Additionally, the stationary mode has a more significant impact on stabilising the system compared to the oscillatory mode.

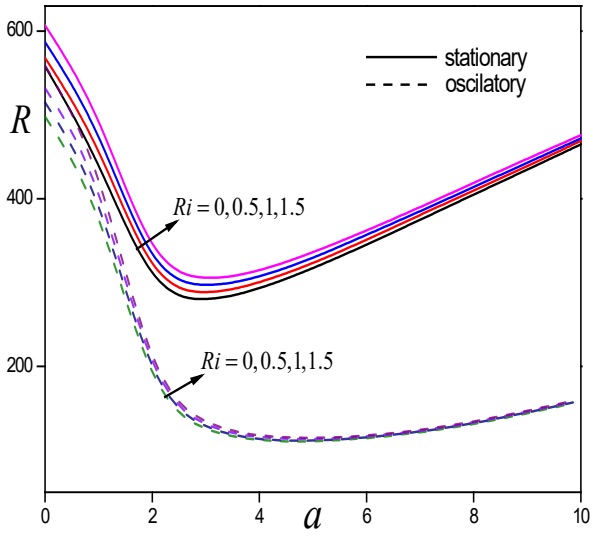


Figure 6. Plot of *R* verses *a* for various values of *Ri* for parabolic-type gravity fluctuation with fixed parameters.

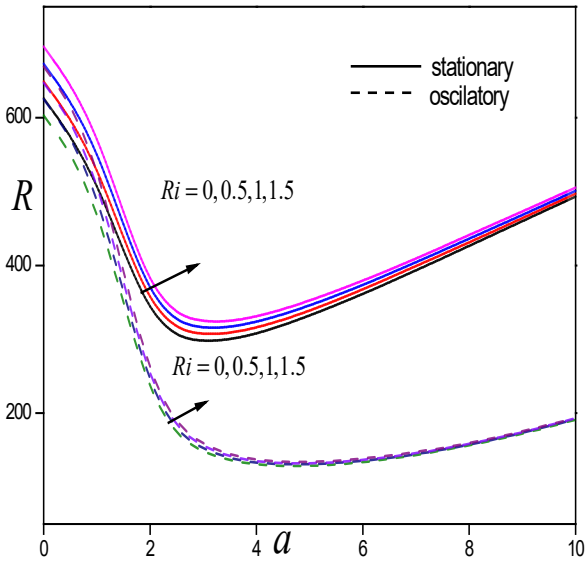


Figure 8. Impact of *R* against *a* for various values of *Ri* for exponential-type gravity fluctuation with fixed parameters.

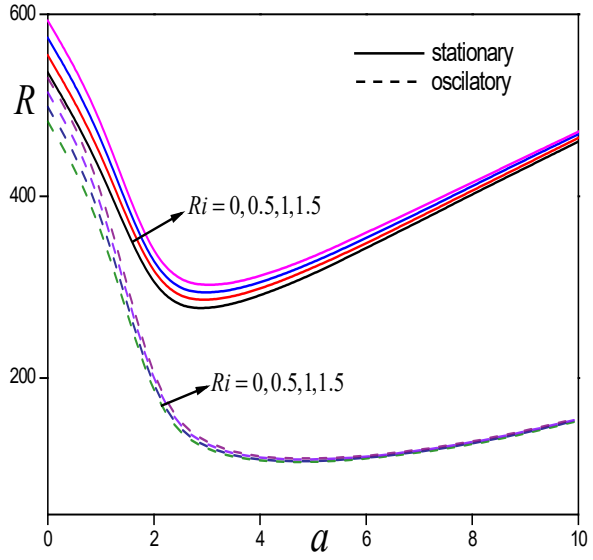


Figure 7. Impact of *R* verses *a* for various numerical values of *Ri* for cubic-type gravity fluctuation with fixed parameters.

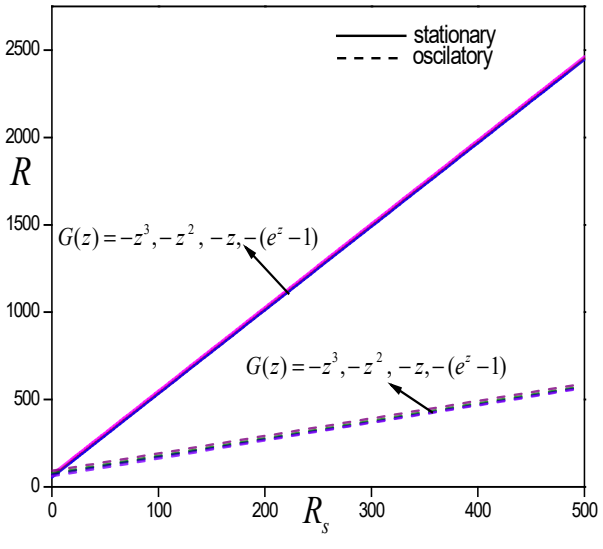


Figure 9. Impact of *R* against *Rs* for various gravity fluctuations with fixed parameters.

The impact of *Rs* with critical Rayleigh number for all four cases of gravity functions by fixing parameters *Da* = *0.001*, ε_n = *1*, η =*0.5* L_e = *20*, ξ=*0.5*, χ = *3*, ω = *1*, δ = *0.5* is displayed in Figure 9. It's noteworthy that as the solute Rayleigh number increases, the critical Rayleigh number rises for both the stationary and oscillatory modes. This observation proves the effect of stabilisation in gravitational fields of all four types on the onset of convective motion.

Figures 10-13 demonstrate the alterations in the critical Rayleigh number about the wave number for various Damkohler number values while keeping other

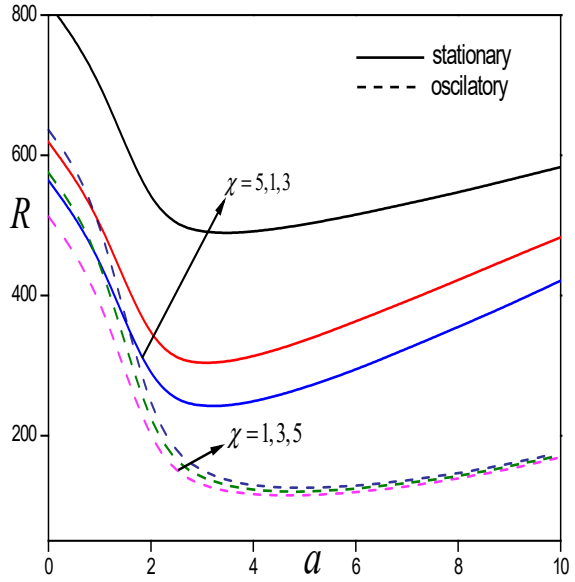


Figure 10. Impact of *R* verses *a* for various values of χ for linear-type gravity fluctuation with fixed parameters.

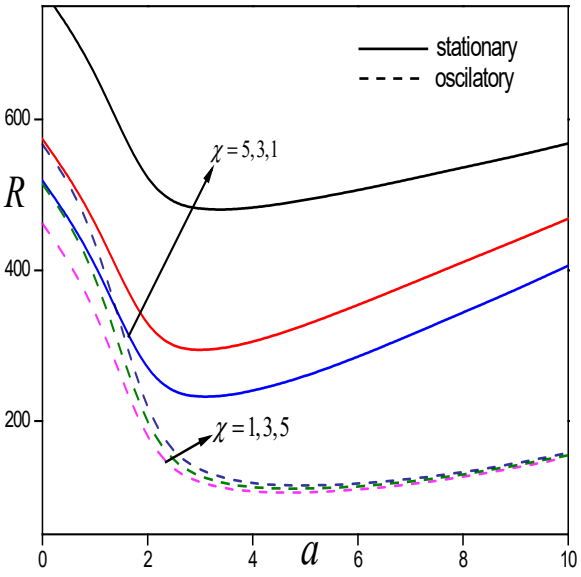


Figure 12. Impact *R* against *a* for various values of χ for the parabolic type of gravity fluctuation with fixed parameters.

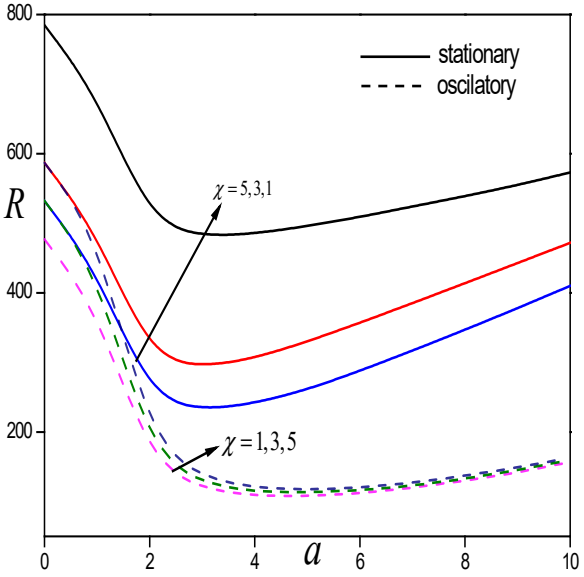


Figure 11. Impact of *R* against *a* for various values of χ for cubic-type gravity fluctuation with fixed parameters.

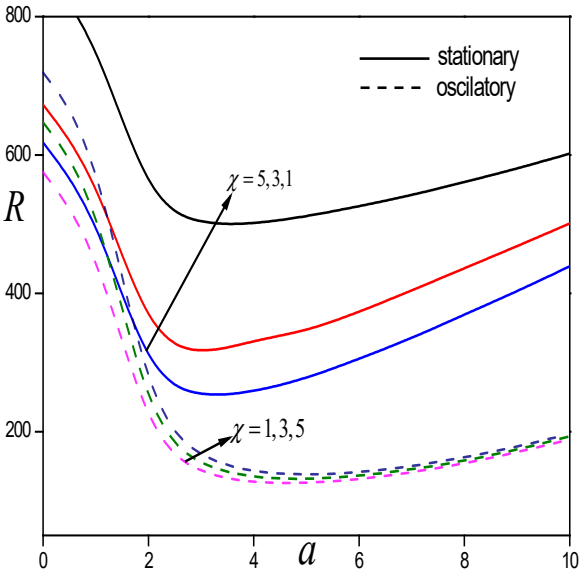


Figure 13. Impact of *R* against *a* for various values of χ for exponential-type gravity fluctuation with fixed parameters.

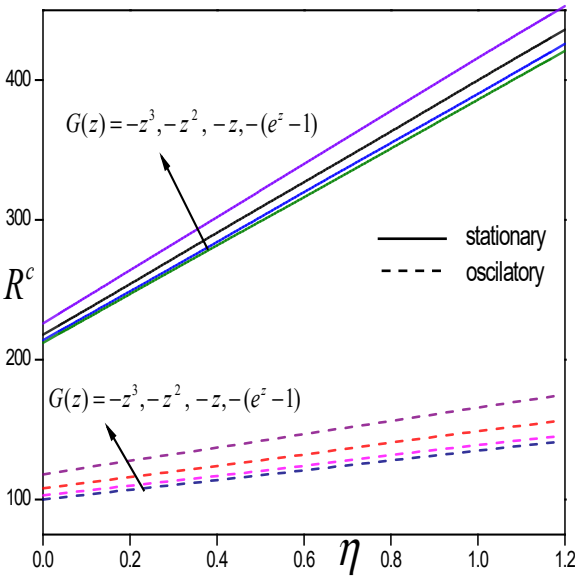


Figure 14. Plot of R^c verses η for different gravity fluctuations with fixed parameters.

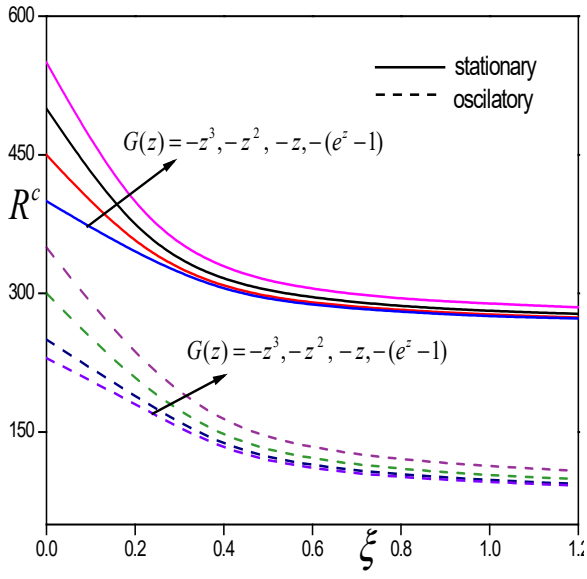


Figure 15. Plot of R^c verses ξ for various gravity fluctuations with fixed parameters.

parameters constant. Notably, the critical Rayleigh number in which stationary modes increase as the Damkohler number rises, suggests that the Damkohler number tends to maintain the stability of convective motion in stationary modes. Conversely, for oscillatory modes critical Rayleigh number decreases with an upsurge in the Damkohler number, representing that the influence of chemical reaction parameters disrupts the system in oscillatory mode. However, in oscillatory modes, the same increase in the Damköhler number leads to a destabilising effect, as the reaction rate becomes large enough to drive the system into an unstable oscillatory state. These findings are consistent with the physical reality of reaction-driven convection, where chemical reactions can either stabilise or destabilise fluid motion depending on the convection mode and the reaction rate.

The impact of thermal anisotropic parameter η with other parameters $Da = 0.01$, $\varepsilon_n = 1$, $L_e = 20$, $\xi = 0.5$, $\chi = 3$, $\omega = 1$, $\delta = 0.5$ is exhibited in Figure 14. It is observed that the R^c increases with increasing η, the thermal anisotropic parameter *has* a stabilising result on the configuration.

Figure 15 describes the variation of R^c with mechanical anisotropic parameter ξ for all four cases of gravity functions by fixing another parameter $Da = 0.001$, $\varepsilon_n = 1$, $c = 0.5$, $L_e = 20$, $\chi = 3$, $\omega = 1$, $\delta = 0.5$. Well, it is detected that. R^c increases with the decreasing ξ Using mechanical methods that conductive solution converts more stable in a porous medium, hence higher values of ξ has a destabilising impact on the configuration.

5.0 Conclusion

In this paper, the study focuses on the stability of thermo-solutal convective motion in a fluid-saturated anisotropic porous medium while being affected by a temperature dependent heat source, chemical reaction effects and gravity fluctuations, as described by the Darcy–Brinkman model. Four different gravity functions, namely linear parabolic cubic and exponential are examined. The findings suggest that the system tends to exhibit stationary thermal convective motion rather than oscillatory behaviour. The existence of a chemical reaction contributes to the overall stability of the system, favouring the stationary mode. Moreover, several parameters, including the temperature-dependent heat source, thermal anisotropy, gravity, Darcy number, and solute Rayleigh number, promote system stability. Notably, the mechanical anisotropy parameter as well as the Damkohler number play a significant role in determining the threshold for convective motion

Table 1. Comparison of R^c and a_c with λ for $\zeta = \eta = 1$ and Rs = 0 for (i) h(z) = -z (ii) h(z) = -z^2 (iv) h(z) = -(e^z-1)^2

h(z)δ		Present study		Rionero and Straughan[34]	
		R^c	a_c^2	R^c	a_c^2
Case(i)	0	39.47	9.87	39.47	9.87
	1	77.08	10.20	77.02	10.20
	1.5	132.02	12.21	132.02	12.31
	1.8	189.90	17.19	189.90	17.19
	1.9	212.28	19.47	212.28	19.47
Case(ii)	0	39.47	9.87	39.47	9.87
	0.2	41.83	9.87	41.83	9.87
	0.4	44.45	9.88	44.45	9.88
	0.6	47.38	9.91	47.38	9.91
	0.8	50.68	9.96	50.68	9.96
	1	54.39	10.03	54.39	10.03
Case(iv)	0	39.47	9.87	39.47	9.87
	0.1	42.33	9.87	42.33	9.87
	0.2	45.60	9.88	45.60	9.88
	0.3	49.39	9.90	49.39	9.90
	0.4	53.82	9.941	53.828	9.94
	0.5	59.05	10.00	59.053	10.00

instability. Additionally, the study indicates that the system is further stable when subjected to exponential gravity fluctuations compared to the linear, parabolic and cubic gravity functions.

6.0 References

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