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Thermal and Radiative Analysis on Dynamics of Ternary Hybrid Nanofluid Accompanied by Significance of Viscous Dissipation and Magnetic Field

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Abstract

This work is aimed at examining the thermal and radiative analysis of ternary hybrid nanofluid dynamics accompanied by the significance of viscous dissipation and magnetism. The physical model was based on the mixture of Cu, TiO_2, and SiO_2 Ternary Hybrid Nanoparticles (THN) which are suspended in H2O past a stretchable sheet. The dynamics of fluid past a stretchable sheet is a notable analysis for thermal and momentum boundary layers. It finds applications in various technological fields and industries. A system of Partial Differential Equations (PDEs) was employed to explore the model equations. These PDEs were changed into total differential equations (ODEs) by implementing acceptable transformation. The Runge Kutta technique alongwith shooting technique was applied to solve the system of equations. The analysis described in this study explained that ternary hybrid nanoparticles have high performance of radiative and thermal processes when compared with hybrid nanofluids. An ascending value of magnetism was noticed to decline the motion of the fluid due to the creation of Lorentz force. A comparison was conducted with previous work and was in close agreement.

Major Findings: *This study examines the behaviour of a two-dimensional, incompressible, laminar ternary hybrid nanofluid (Cu-TiO_2-SiO_2/H_2O) flowing over a stretching surface, highlighting its superior thermal performance compared to conventional hybrid nanofluids. The findings indicate that an increase in the Eckert number and thermal Grashof number enhances the velocity, whereas the magnetic parameter has an inverse effect. Further, temperature rises with increasing values of the magnetic parameter, thermal radiation and Eckert number.*

Keywords: *Eckert Number, Magnetic Parameter, Porosity Parameter, Thermal Grash of Number, Ternary Hybrid Nanofluid, Thermal Radiation*

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1.0 Introduction

Recent studies have garnered significant attention toward the heat transfer analysis of hybrid nanofluids through numerical techniques. The evaluation of thermal performance, including the incorporation of radiative effects, is critical for enhancing the efficiency of heat transfer processes. This research has substantial implications for applications in pharmaceutical industries, thermal management systems, and advanced engineering technologies. Alqawasmi *et al*., examined numerical simulation toward ternary hybrid nanofluid movement[1]. Ashish Paul *et al*., examine the interplay of non-linear mixed convection, spatially varying heat source/sink, and various other complex factors on the flow and heat or mass transfer behaviors of Casson ternary hybrid nanofluid over a rotating stretched disk[2]. Raza *et al*., investigated nanolayer effect on movement over tri-hybrid nanofluid base on nanoparticles morphology along with gyrotactic microorganisms[3]. The mechanisms of radiation on the flow of dissipative ternary hybrid nanofluid in rotating disk was investigated by Shamshuddin *et al*[4]. Nanofluids explains synthesis approach by utilizing nanometalic as a result of nano-scale formation and thermal design. A comparative analysis of tri-hybrid nanofluids by examining by impact of thermal radiation together with Cattaneo-christov heat fluxrole was studied by Noreen *et al*[5]. Choudhary *et al*., focused on mixture of CNT-Fe_3O_4-Gr and MgO-Au-Cu kerosene oil-based ternary hybrid nanoparticles through a bidirectional stretching sheet[6]. Rajamani *et al*., examined MHD pulsating channel with Joule heating along with thermal radiation impacts[7]. Nabwey *et al*., was analysed heat transmission on MHD dynamics of Carreau tri-hybrid nanofluid exponentially stretched past a curved surface ponentially stretched[8]. Yu *et al*., also observed the optimisation of heat transmission with viscous ternary nanofluid dynamics in a stretching and shrinking thin needle was discussed extensively[9].

Thermal radiation significantly influences energy exchange mechanisms by altering the temperature of bodies. Its effect on a moving fluid is characterized by changes in the material's internal energy. Thermal radiation is particularly critical in high-temperature environments, such as in heat exchangers and combustion chambers, making it essential in industrial processes and thermal engineering applications where elevated temperatures prevail. Alsagri *et al*., examined the significance over a thermal radiation, magnetized rotating dynamics of a hybrid nanofluid on three-dimensionally[10]. Ramzan *et al*., recently considered thermal and theoretical analysis of ternary hybrid nanofluids dynamics past a multiple geometries non-isothermal and non-isosolutal mechanisms[11]. Goud *et al*., focused on the effect of ternary hybrid nanofluidthe thermal analysis in a dovetail fin through examining generation of heat internally studied[12]. Alrihieli *et al*., explored radiative MHD nanofluid dynamics based on a linearly stretching sheet by considering viscous dissipation alongside convective heating[13]. The significance of activation energy, viscous dissipation and joule heating on nanofluid dynamics caused through MHD overa vertically surface was examined by Jayanthi *et al*[14]. Yaseen *et al*., examined the role of thermal radiation and heat generation on hybrid and mono hybrid nanofluid dynamics through double parallel plates[15]. Harish Babu *et al*., look over the thermal dynamics of radiative nonlinear stretchable sheet on tri-hybrid nanofluid[16]. Alao *et al*., investigates the heat transfer and fluid dynamics of an MHD dissipative ternary fluid flow through a permeable channel, using numerical and neural network approaches[17]. Manjunath *et al*., analyzed convective heat transfer in ternary nanofluid flow past a stretching sheet[18].

Magnetohydrodynamics finds numerous usefulness in industries like heat exchangers, micro-electronics and the modelling of combusting. In recent time, nanotechnology has been explained as the approach for thermal augumentation. Asghar *et al*., examined MHD simulation of nanofluid dynamics by considering viscous dissipation[19]. Fatunmbi *et al*., recently examined MHD micropolar nanofluid dynamics past an upright elongating sheet with temperature-dependent viscosity[20]. Waqas *et al*., go through the significance of past rotating disk, radiative dynamics and MHD hybrid nanofluid[21]. The dynamics of micropolar magnetic viscous nanofluid dynamics on penetrable inclined surface was studied by Eid *et al*.[22]. Priyadharshini *et al*., considered machine learning prediction scheme in analysing ternary hybrid nanofluid dynamics and some studies presents a mathematical model analysis to investigate the effects of relevant thermophysical parameters on the transport phenomena of a micropolar nanofluid over an inclined, exponentially

stretching surface, and reports the key findings on the rates of heat exchange, couple stress, wall shear stress, and Sherwood number by Tadesse Walelign[23,24]. Nayak *et al.*, examined MHD 3D dynamics and heat transmission analysis of nanofluid with the role of viscous dissipation and thermal radiation in a shrinking surface[25]. Asjad *et al.*, studied dynamics of bioconvection of MHD viscous nanofluid with activation energy and chemical reaction[26]. Idowu *et al.*, discussed about the effect of MHD by varying viscosity along with thermal conductivity when double non-newtonian fluids move simultaneously[27]. Biswas *et al.*, modelled Prandtl-nanofluid dynamics utilizing exponentially vertical surface with chemical reaction[28]. Kommaddi *et al.*, presents an analytical solution for the unsteady MHD free convective double diffusive heat and mass transfer rotating flow of a viscous incompressible non-newtonian Jeffrey fluid past an inclined porous plate in the presence of Hall current, radiation absorption, and Dufour effects[29]. Lakshmi *et al.*, work compares chemically reacted nanofluids (SWCNT and MWCNT) using sophisticated modelling of Cattaneo-Christov heat fluxes[30]. Xiaofang Zhao *et al.*, analyzes the unsteady free convective flow of a nanofluid due to an inclined surface, considering the effects of thermos-diffusion, chemical reaction, radiation, and heat sources, and presents the results using a dimensionless model and perturbation technique[31]. Venakt Rao Kanuri *et al.*, focusses on Poiseuille flow, which is significant in several contexts including human blood circulation, crude oil transport, and industrial fluid dynamics[32]. The research explores fluid flow dynamics in an inclined channel, specifically examining the Poiseuille flow of a second-grade blood nanofluid, while making comparative analyses with different types of nanofluids.

The important significance of ternary hybrid nanofluids has frequently been overlooked in the many studies that have examined the dynamics of nanofluids in the literature. Compared to conventional nanofluids, ternary hybrid nanofluids-which are composed of a blend of three different nanoparticles-offer improved heat transfer characteristics and greater thermal conductivity. To the best of our knowledge, the combined thermal and radiative effects have not yet been extensively investigated in ternary hybrid nanofluid research. In this work, the effects of magnetic fields and viscous dissipation are taken into account while we explore the thermal and radiative behavior of a ternary hybrid nanofluid made of Cu, TiO_2, SiO_2, and H_2O. There doesn't seem to be any sort of investigation of this kind in the literature this moment.

Ternary hybrid nanofluids are significant in view of their potential to improve the transmission of heat and cooling systems. These fluids are more effective than conventional or binary nanoparticle systems as they combine multiple nanoparticles that offer enhanced thermal conductivity, increased heat transfer capabilities, and higher stability. Because of these characteristics, ternary hybrid nanofluids are very beneficial in a variety of industrial settings, notably energy systems that need sophisticated cooling systems.

The findings of this study are extremely pertinent to engineering applications, specifically in the optimization of heat transfer mechanisms for metallurgical processes, heat exchangers, and cooling systems. We extensively investigate the flow of ternary hybrid nanofluids using numerical simulations, providing comprehensive graphical representations to clarify the impact of important flow parameters. The study also offers thorough tabulations of engineering quantities, which adds insightful information for applications in reality. This article introduces a novel theoretical tri-hybrid nanofluid model designed to enhance heat transfer efficiency and examines the heat transfer characteristics of a ternary hybrid nanofluid ($Cu\text{-}TiO_2\text{-}SiO_2/H^2O$) flowing over a inclined stretching surface while analysing the influence of key parameters, including the Thermal Grash of number $(0.5 \le Gr \le 2.0)$, inclined angle parameter $(0 \le \alpha \le \pi/2)$, porosity parameter $(0.5 \le K \le 2.0)$, thermal radiation $(1 \le R \le 4)$, heat source/sink parameter $(-1 \le \Delta_f \le -4)$, Eckert number $(1 \le Ec \le 4)$ within specified ranges. The combination of Cu, TiO_2, and SiO_2 in a water-based fluid aids in the decomposition of harmful substances, environmental purification, and various cooling applications. The governing boundary-layer partial differential equations describing the physical flow are transformed into ordinary differential equations using a similarity transformation. To obtain numerical solutions, a highly efficient BVP5C code is implemented in MATLAB and its accuracy is validated through comparisons with previously published results under limiting conditions. The graphical illustrations highlight the effects of crucial parameters, and the computational outcomes are systematically compared with existing

literature. These insights contribute to a deeper understanding of boundary layer flow behaviour, offering valuable implications for researchers and engineers in diverse fields. The optimization process presented in this study offers valuable insights into the manufacturing of polymer sheets, glass fibre, petroleum products, plastic films, heat exchangers, and electronic devices. Therefore, the findings are suggested for application in the advancement of industrial equipment and systems.

2.0 Mathematical Analysis

A steady, incompressible, laminar flow of ternary hybrid nanofluids which contains Copper Cu, Titanium dioxide TiO_2 and Silicon dioxide SiO_2 nanoparticles. These nanoparticles were considered in combination with water H_2O. Thermal and radiative analysis over a moving surface with it speed to be and free stream temperature was considered. A transverse magnetic field was impose in a perpendicular direction. The heat transmission analysis was elucidated with the viscous dissipation and heat generation on the ternary hybrid nanofluids. Figure 1 shows the physical configuration. The surmise of boundary layer is substantial and relative governing equations are Priyadharshini *et al*[23].

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{thnf}}{\rho_{thnf}}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_{thnf}}{\rho_{thnf}}B_0^2u+[g\beta_t(T-T_\infty)]\cos(\alpha)-\frac{\mu_{thnf}}{\rho_{thnf}k}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{thnf}}{(\rho c_p)_{thnf}}\frac{\partial^2 T}{\partial y^2}+\frac{Q_c}{(\rho c_p)_{thnf}}(T-T_\infty)-\frac{1}{(\rho C_p)_{thnf}}\frac{\partial q_r}{\partial y}+\frac{\mu_{thnf}}{(\rho C_p)_{thnf}}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

Corresponding B.C.'s are:

$$\text{At } y=0\ u=U_w, v=0, k_{thnf}\frac{\partial T}{\partial y}=-h_t(T_w-T_\infty) \tag{4}$$

$$\text{As } y\to\infty, u\to 0, T\to T_\infty \tag{5}$$

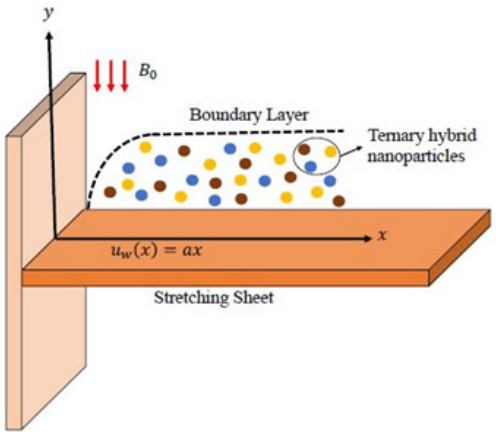


Figure 1. Flow geometry of the problem.

The flow encountered a radiative heat flux which is explained through the utilization of the Rosseland diffusion approximation as used in Alao *et al.*[17], such that:

$$q_r=-\frac{4\sigma_s}{3ke}\frac{\partial T^4}{\partial y} \tag{6}$$

With the use of Rosseland approximation, the thermal and radiative analysis of the study have considered an optically thin fluids. Consider that temperature difference existing in the flow is small such that Equation (6) is linearized by simplifying by utilizing Taylor's series in and avoiding the higher order to obtain:

$$T^4\cong 4T_\infty^3T-3T_\infty^4 \tag{7}$$

The energy equation becomes:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{thnf}}{(\rho c_p)_{thnf}}\frac{\partial^2 T}{\partial y^2}+\frac{Q_c}{(\rho c_p)_{thnf}}(T-T_\infty)+\frac{16\sigma_s T_\infty^3}{3ke(\rho C_p)_{thnf}}\frac{\partial^2 T}{\partial y^2}+\frac{\mu_{thnf}}{(\rho C_p)_{thnf}}\left(\frac{\partial u}{\partial y}\right)^2 \tag{8}$$

The similarity variables employed in this paper are defined as:

$$u=axf'(\eta)\ ,\ v=-(av)^{\frac{1}{2}}f(\eta)\ ,\ \theta=\frac{T-T_\infty}{T_w-T_\infty}\ ,\ \eta=\left(\frac{a}{v}\right)^{\frac{1}{2}}y \tag{9}$$

Employing the equation (9) above on the governing equations to obtain:

$$A_1 f'''+A_2(ff''-f'^2+Gr\cos(\alpha)\theta)-(A_3M+A_1K_0)f'=0 \tag{10}$$

$$(A_4+R)\theta''+\Pr\Delta_f\theta+A_1Ec\Pr f''^2+A_5\Pr f\theta'=0 \tag{11}$$

Where

$$A_1=\frac{\mu_{thnf}}{\mu_f},A_2=\frac{\rho_{thnf}}{\rho_f},A_3=\frac{\sigma_{thnf}}{\sigma_f},A_4=\frac{k_{thnf}}{k_f},A_5=\frac{(\rho C_p)_{thnf}}{(\rho C_p)_f}.$$

Subject to:

$$f'(0)=1,f(0)=0,f'(\infty)=0 \tag{12}$$

$$\theta'(0)=-\frac{\gamma}{A_4}(1-\theta(0)),\theta(\infty)=0. \tag{13}$$

$$M=\frac{\sigma_f B_0^2}{a\rho_f},Gr=\frac{g\beta_t(T_w-T_\infty)}{a^2x},K=\frac{\nu_f}{aK_0},$$

$$R=\frac{16\sigma_s T_\infty^3}{3kek_f},\Pr=\frac{\nu_f\left(\rho C_p\right)_f}{k_f},Ec=\frac{(ax)^2}{(c_p)_f(T_w-T_\infty)}$$

$$\Delta_f=\frac{Q_c}{(\rho c_p)_f a}. \tag{14}$$

Table 1. Ternary hybrid nanofluids correlations[23]

Properties	Ternary Hybrid Nanofluid
Viscosity	$\mu_{thnf}=\frac{\mu_f}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}(1-\phi_3)^{2.5}}$
Thermal conductivity	$\frac{k_{hnf}}{k_{nf}}=\frac{k_2+2k_{nf}-2\phi_2(k_{nf}-k_2)}{k_2+2k_{nf}+\phi_2(k_{nf}-k_2)}$, $\frac{k_{nf}}{k_f}=\frac{k_3+2k_f-2\phi_3(k_f-k_3)}{k_3+2k_f+\phi_3(k_f-k_3)}$ $\frac{k_{thnf}}{k_{hnf}}=\frac{k_1+2k_{hnf}-2\phi_1(k_{hnf}-k_1)}{k_1+2k_{hnf}+\phi_1(k_{hnf}-k_1)}$
Electrical conductivity	$\frac{\sigma_{thnf}}{\sigma_{hnf}}=\frac{(1+2\phi_1)\sigma_1+(1-2\phi_1)\sigma_{hnf}}{(1-\phi_1)\sigma_1+(1+\phi_1)\sigma_{hnf}}$, $\sigma_{hnf}=\sigma_{nf}\cdot\frac{(1+2\phi_2)\sigma_2+(1-2\phi_2)\sigma_{nf}}{(1-\phi_2)\sigma_2+(1+\phi_2)\sigma_{nf}}$ $\frac{\sigma_{nf}}{\sigma_f}=\frac{(1+2\phi_3)\sigma_3+(1-2\phi_3)\sigma_f}{(1-\phi_3)\sigma_3+(1+\phi_3)\sigma_f}$
Density	$\rho_{thnf}=(1-\phi_1)\left[(1-\phi_2)\left[(1-\phi_3)+\phi_3\frac{\rho_3}{\rho_f}\right]+\phi_2\frac{\rho_2}{\rho_f}\right]+\phi_1\frac{\rho_1}{\rho_f}$
Heat capacity	$(\rho c_p)_{thnf}=(1-\phi_1)\left[(1-\phi_2)\left[(1-\phi_3)+\phi_3\frac{(\rho c_p)_3}{(\rho c_p)_f}\right]+\phi_2\frac{(\rho c_p)_2}{(\rho c_p)_f}\right]+\phi_1\frac{(\rho c_p)_1}{(\rho c_p)_f}$

Table 2. Thermophysical properties of Cu,TiO_2, SiO_2 and H_2O [23]

Properties	H_2O	TiO_2	Cu	SiO_2
k(w/mk)	0.6071	8.953	400	1.4013
c_p (J/kgk)	0.4179	686.2	385	765
ρ(kg/m^3)	0.997	4250	8933	2270
σ(s/m)	$(5.5)(10^{-6})$	$(2.4)(10^6)$	1.67	$(3.5)(10^6)$

The above are the magnetic parameter, thermal Grashof number, permeability, thermal radiation, Prandtl number, heat generation and Eckert number respectively. The quantities of engineering interest for momentum and thermal boundary layer are described as follows:

$$C_{fx} = \frac{(\mu)_{thnf}}{\mu_f a x^2}\left(\frac{\partial u}{\partial y}\right)_{y=0} \text{ and}$$

$$Nu_x = -\frac{x k_{thnf}}{k_f (T_w - T_\infty)}\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

By implementing the similarity variables to obtain:

$$C_{fx} = \frac{\mu_{thnf}}{\mu_f}\frac{f''(0)}{\sqrt{\mathrm{Re}_x}} \text{ and } Nu_x = \frac{k_{thnf}}{k_f}\theta'(0)\sqrt{\mathrm{Re}_x}$$

3.0 Methodology

Runge-Kutta along with shooting method was applied to solve the nonlinear governing ODEs in Equations (10)-(11) pertaining to (12) and (13). It would be impractical to solve the BVP for even a very large finite interval because it cannot be solved on an infinite interval. The infinite boundary condition will be applied in this work at a finite point η at ∞ = 6. To integrate f'' (0), and θ' (0) which is an IVP, we require a value for f''(0)=y_3 (0)=β_1, θ' (0)=y_7 (0)= β_2, β_1=1.5 and β_2=2.0 are guess based on the shooting approach to the IVP. Steps have been carefully planned, shoots are refined gradually, and calculations are done with 15 decimal places to prevent rounding off errors. To implement this technique, convert the nonlinear ODE into first order ODEs as follows:

$$f = y_1, \frac{df}{d\eta} = \frac{dy_1}{d\eta} = y_2, \frac{d^2 f}{d\eta^2} = \frac{d}{d\eta}\left(\frac{dy_1}{d\eta}\right) =$$

$$\frac{d}{d\eta}\left(\frac{dy_1}{d\eta}\right) = \frac{dy_2}{d\eta} = y_3, \frac{d^3 f}{d\eta^3} = \frac{d}{d\eta}\left(\frac{dy_2}{d\eta}\right) = \frac{dy_3}{d\eta}$$

$$\theta = y_4, \frac{d\theta}{d\eta} = \frac{dy_4}{d\eta} = y_5, \frac{d^2\theta}{d\eta^2} = \frac{d}{d\eta}\left(\frac{dy_4}{d\eta}\right) = \frac{dy_5}{d\eta} \quad (15)$$

Substituting Equations (14) and (15) into the transformed equations to obtain:

$$A_1\frac{dy_3}{d\eta} + A_2(y_1 y_3 - y_2^2 + Gr\cos(\alpha)y_4) - (A_3 M + A_1 K_0)y_2 = 0. \quad (16)$$

$$(A_4 + R)\frac{dy_5}{d\eta} + \mathrm{Pr}\,\Delta_f y_4 + A_1 Ec\,\mathrm{Pr}\,y_3^2 + A_5\,\mathrm{Pr}\,y_1 y_5 = 0. \quad (17)$$

Subject to:

$$y_1(0) = 1, y_2(0) = 0, y_2(\infty) = 0 \quad (18)$$

$$y_5(0) = -\gamma A_4(1 - y_4(0)), y_4(\infty) = 0 \quad (19)$$

Simplifying Equations (16) and (17) to obtain:

$$\frac{dy_3}{d\eta} = \frac{(A_3 M + A_1 K_0)y_2 + A_2\left(y_2^2 - y_1 y_3 - Gr\cos(\alpha)y_4\right)}{A_1} \quad (20)$$

$$\frac{dy_5}{d\eta} = \frac{-\left(\mathrm{Pr}\,\Delta_f y_4 + A_1 Ec\,\mathrm{Pr}\,y_3^2 + A_5\,\mathrm{Pr}\,y_1 y_5\right)}{(A_4 + R)}. \quad (21)$$

The results are obtained by iteratively adjusting the scale factor to a specified value through an approximate solution method. Initial assumptions are systematically refined, and the boundary layer thickness is rigorously incorporated into the analysis to ensure accuracy.

4.0 Results and Discussion

This study have discussed the thermal and radiative analysis of THN (Cu -TiO_2 - SiO_2/H_2O) dynamics with impact of viscous dissapation and magnetic field. To explain the physical influence of pertinent dynamic parameters on velocity and temperature, The numerical scheme called Runge-Kutta technique along with the shooting method was utilized on the equations. The control parameters have been chosen as: Pr = 6.2, R = 0.5, Δ_f = 0.2, Gr = 2.0, M = 1.0, K_0 = 0.3, Ec = 0.1. The Prandtl number was chosen to be 6.2 based on the type of hybrid nanofluid considered in this study while other parameters values are chosen based on experimental computations.

The significance of magnetic parameter on the velocity of trihybrid nanofluid (Cu -TiO_2 - SiO_2/H_2O) flow when $\phi_1 = \phi_2 = \phi_3 = 0.3$ is displayed in Figure 2. An ascending

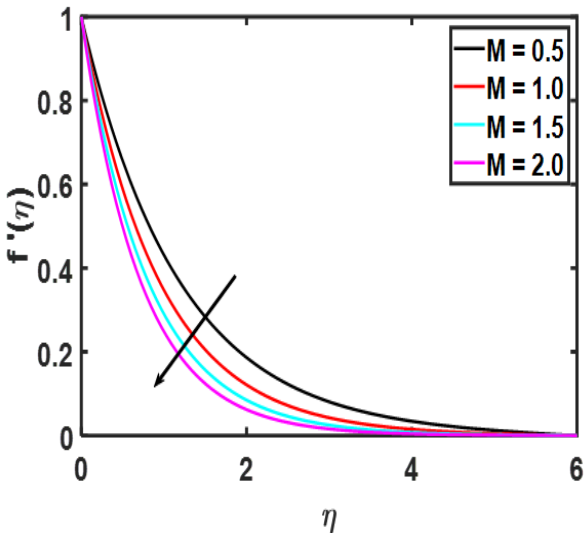


Figure 2. velocity distribution of M.

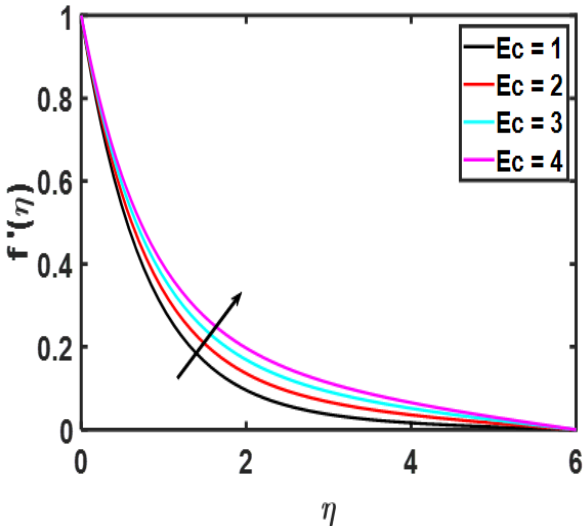


Figure 4. Velocity distribution of Ec.

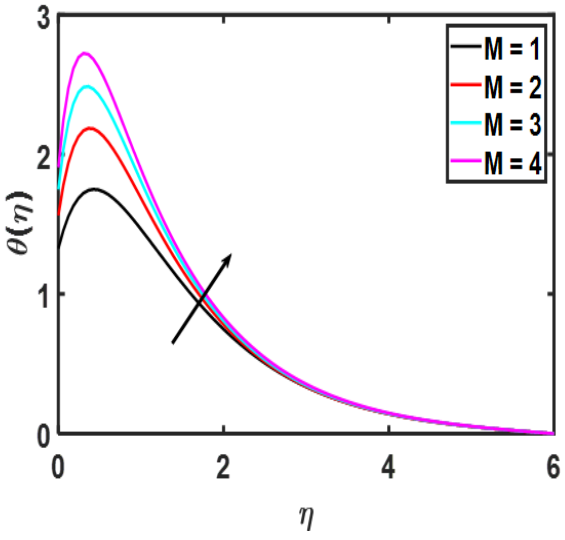


Figure 3. Temperature distribution of M.

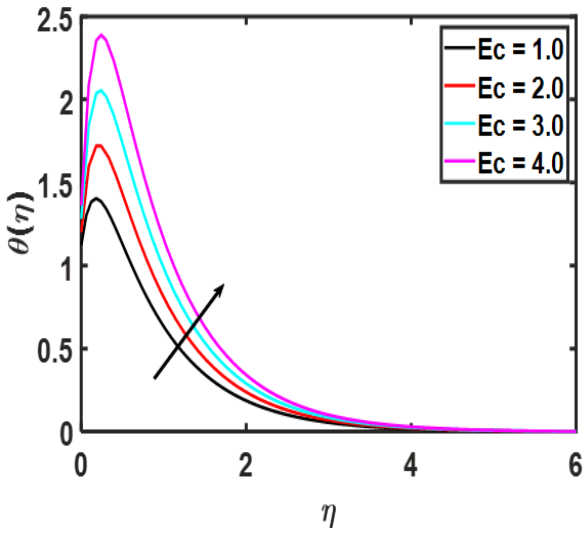


Figure 5. Temperature distribution of Ec.

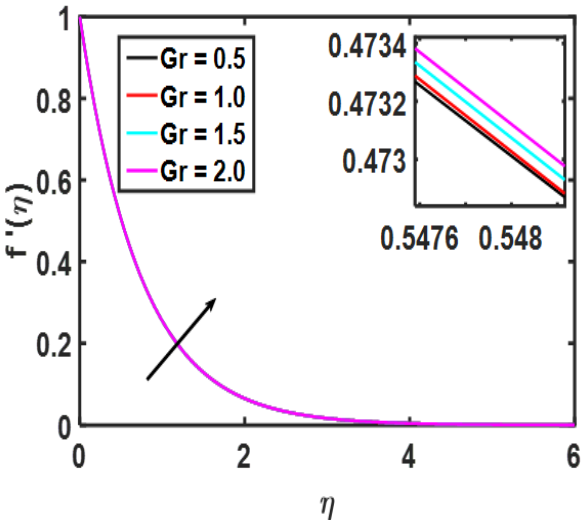


Figure 6. Velocity distribution of Gr.

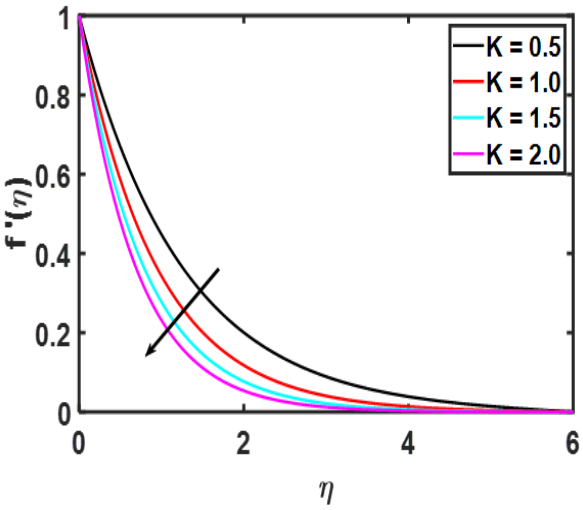


Figure 8. Velocity distribution of K.

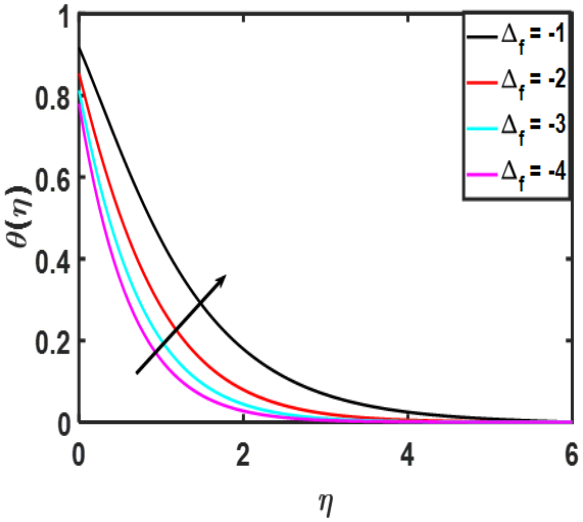


Figure 7. Temperature distribution of heat source/sink.

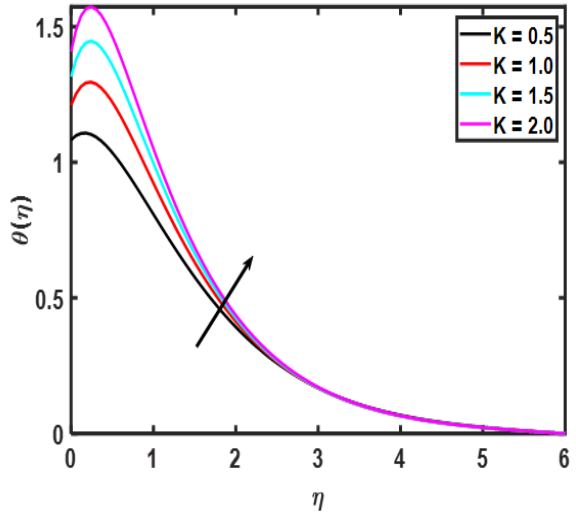


Figure 9. Temperature distribution of K.

magnetic parameter was identified that decreases the thickness of the boundary layer. Lorentz force arises in the momentum equation and has the propensity to slow down the velocity of tri-nanocomposite particles. From Figure 3, as the magnetic field strength increases, the external force acting perpendicularly on the flowing fluid becomes more pronounced. Consequently, the Lorentz force, originating from the magnetic field, hinders and disrupts the movement of fluid particles.

Figures 4 and 5 explain the effect of Eckert number ($1 \leq Ec \leq 4$) on the velocity and temperature profiles of trihybrid nanofluid flow. It shows that an ascending of Ec shows an increment in the profile of velocity along temperature. The dissipative viscous transforms the kinetic into internal energy based on work done in anticipating viscous liquid stresses. Based on this fact, ascending Eckert number raises the temperature and the entire boundary layer thickness.

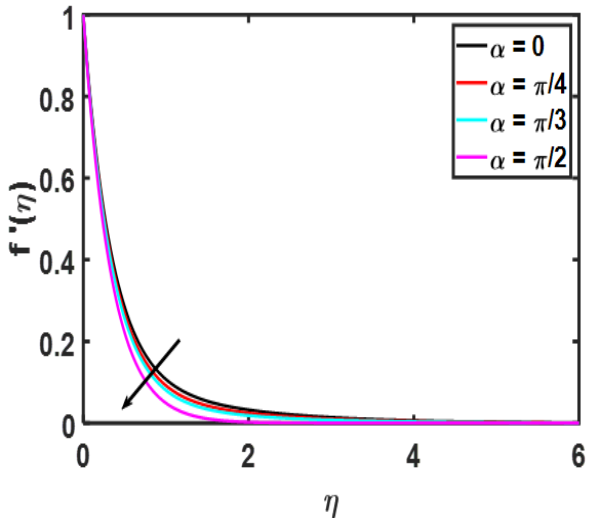


Figure 10. Velocity distribution of inclined angle α.

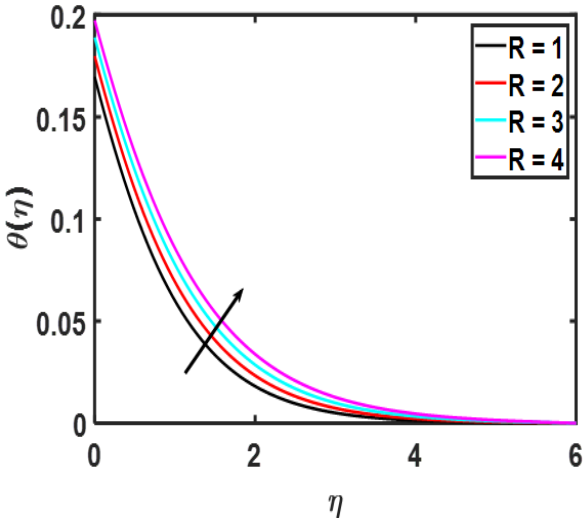


Figure 11. Temperature distribution of R.

Figure 6 shows the influence of thermal Grashof number ($0.5 \leq Gr \leq 2$) on the velocity distribution. Enhancement in Gr is identified to growing fluid velocity boundary layer thickness. The Gr acted like a buoyancy force which arrive the tri-nanocomposite nanoparticles and hereby increases the velocity.

Figure 7 exemplified the temperature of fluid with the effect of heat source/sink parameter ($-4 \leq \Delta_f \leq -1$). When the heat source/sink parameter value increases in a hybrid nanofluid flow, the temperature profile increases.

Figures 8 and 9 demonstrate implication of porosity parameter ($0.5 \leq K \leq 2.0$) on the velocity and temperature distributions of the fluid. Increased normal stress reduces boundary layer thickness, causing force to spread to nearby particles and accelerate their movement. Increasing porosity leads to a lower temperature profile due to reduced fluid velocity and improved fluid transfer. These characteristics enhance heat transfer.

Figure 10 illustrates inclined angle ($0 \leq \alpha \leq p/2$) on velocity profile. The trend observed indicates a decline in $f'(\eta)$ as experiences an increment. Increasing the values of α enhances Lorentz force and magnetic flux, resulting in friction between surface fluids and decreased velocity.

Figure 11 demonstrate the significance of thermal radiation ($1 \leq R \leq 4$) on the temperature distribution. The temperature profile ascended as the value of R increases. The present analysis shows that the absorption of thermal radiation within the layer alters the thickness of the thermal boundary layer and its temperature. Hence, the thermal radiation enhances the thermal and radiative analysis within the boundary layer.

Table 3. Comparison results for for various values of Pr

Pr	Manjunatha *et al.*[18]	Priyadarshini *et al.*[23]	Present Work
2.0	0.9113	0.9114	0.9113
7.0	1.8954	1.8954	1.8953
20	3.3539	3.3539	3.3538

Table 3 shows comparison of for various values of Pr, when $\phi_1 = \phi_2 = \phi_3 = R = M = \alpha = Gr = K = Ec = 0$. The investigation shows that there is excellent agreement between the current computations and the literature that has been published. Table 4 explores how the mixed effects of Magnetic field (M), Grashof (Gr), Radiation (R), Heat Source/ Sink (Δ_f), Eckert (Ec) effects the skin friction coefficient (C_{fx}) and Nusselt Number (Nu_x). The results reveal that rates of heat transfer increase with a higher Gr, R, Pr, Δ_f, and Ec declines for M. As the values of M, Gr, R,Δ_f and Ec increases skin friction coefficient decreases.

Table 4. The impact of Skin friction coefficient ($C_f \, \mathrm{Re}_x^{\frac{1}{2}}$), and Nusselt number ($Nu_x \, \mathrm{Re}_x^{-\frac{1}{2}}$) for various parameter

M	Gr	R	Δ_f	Ec	C_{fx}	Nu_x
0.5 1.0 1.5					1.7008 0.8743 0.0034	0.5172 0.5172 0.5172
	1.0 2.0 3.0				0.7843 1.4925 2.1106	0.6270 0.6270 0.6270
		2.0 3.0 4.0			0.8743 0.9417 1.0370	0.4949 0.5069 0.5272
			3.0 6.0 9.0		0.8743 0.9811 1.1946	0.2031 0.5366 0.7033
				1.0 2.0 3.0	0.8735 1.2050 1.5365	0.0141 0.5002 0.5285

4.0 Conclusion

The thermal and radiation analysis of three-phase nanoparticles suspended in has been solved numerically. The dynamics of the suspended nanoparticles in water were considered past a inclined stretching sheet with magnetic field, viscous dissipative, and thermal radiation.

The key outcomes in this analysis are:

1. The thermal profile is ascending, and momentum profile diminishes with an ascending value of the magnetism parameter. This is due to the creation of Lorentz force, which slows the movement of an electrically conducting fluid.
2. Ascending of Ec shows an increment in the profile of velocity along temperature and enhancement in Gr is identified to growing fluid velocity.
3. Increasing the heat source/sink parameter hiking the temperature distribution.
4. Enhanment in porosity leads to a lower temperature profile due to reduced fluid velocity and improved fluid transfer. These characteristics enhance heat transfer.
5. Increasing the values of α enhances decreased velocity. Conversely temperature profile ascended as the value of R increases.

5.0 References

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Nomenclature

f	Base fluid	K	Thermal conductivity $\left(Wm^{-1}K^{-1}\right)$
nf	Nanofluid	T_w	Gradient of temperature at the wall (K)
T	Temperature	hnf	Hybrid nanofluid
μ	Dynamic viscosity $\left(kgm^{-1}s^{-1}\right)$	thnf	Ternary hybrid nanofluid
C_{fx}	Coefficient of skin friction	φ	Inclination angle
c_p	Specific heat $\left(JKg^{-1}K^{-1}\right)$	T_∞	Stream temperature

c_p	Specific heat $\left(JKg^{-1}K^{-1}\right)$	T_∞	Stream temperature
ϕ_1	Volume fraction of Cu nanoparticle	Q_c	Heat generation coefficient
u and v	x and y-direction velocity $\left(ms^{-1}\right)$	q_r	Heat flux
ϕ_2	Volume fraction of TiO_2 nanoparticle	B_0	Magnetic strength
ρ	Fluid density	β_t	Volumetric capacity for temperature
ϕ_3	Volume fraction of SiO_2 nanoparticle	g	Acceleration base on gravity
σ	Electrical conductivity $\left(sm^{-1}\right)$		