GEOMETRICAL CALCULATIONS IN ŚULBASŪTRAS

MANJU.V.KUMAR.
ASSISTANT PROFESSOR, DEPARTMENT OF SANSKRIT,
SREE SANKARA COLLEGE, KALADY

The Vedas are considered as the earliest literary record and the most sanctified books of India. They are the eternal truth revealed by god to the ancient seers and these are the fountain head and illimitable depot of much of the ancient knowledge. The advancement in mathematics and astronomy is the notable aspect of vedic period. Śulbasūtra the part of vedic literature, stand first among these ancient mathematical texts. They are very important sources of geometrical and arithmetical knowledge. There are differences in the opinions among western and Indian scholars on the period of these books. They were written in the same language style adopted in Rg and Yajurvedas. Any way śulbasūtras have been included with literature written during 1000-500 B.C. The root word ŚULB or ŚULV means' to measure', and śulba means measuring rope or cord. However it is the word rajju and not śulba that occurs in the sutras themselves. 'रज्जुसमासं वक्ष्यामः' ।।

These form part of the vast literature, designated as kalpasūtras. It was perhaps primarily in connection with the construction of the sacrificial altars of proper size and shape. These books deal with the ritual in the śroutasūtra and this is a manual of geometrical calculations.

Geometry is said to be that branch of mathematical science which treats the properties of space and it is an inevitable part of mathematics. The science of geometry grew up and was codified in the śulbasūtras. They give modern geometrical concepts developed in ancient India. The geometry of the śulbasūtras is primarily constructive, though they occasionally notice and formulate some of the geometrical truths involved. The altars and fire places had different shapes, all geometrical. They contain the earliest extant verbal expression of the pytthagorean theorem. The diagonal rope (akayarajju) of an oblong (rectangle) produces both which the flank

(parshwamani) and the horizontal (thiryagmani) produce separately. Since the statement is a sutra, it is necessarily compressed and what the ropes elaborated on, but the content clearly implies the square areas constructed on their lengths. The Śulbasūtra has developed with the demands of the construction of the sacrificial vedis and therefore it must be very old. They contain lists of Pythagorean triples, which are particular cases of Diophantine equations. They also contain statements about squaring the circle and circling the square. There are four main śulbasūtras- the Baudhāyana, Āpastamba, Kātyāyana and Mānavaśulbasūtras.

REFLECTIONS IN BOUDHĀYANAŚULBASŪTRA

Boudhāyanaśulbaūtra is said to be the oldest of all śulbasūtras and its date is fixed as to be 8th century A.D. Boudhāyana composed this sulbasutra. It is divided into three chapters. It comprises of 525 sutras or verses. These works are of great importance due to the scientific and mathematical approach followed in the construction of fire altars, with the help of theorems and real measurements. It contains examples of simple Pythagorean triples as well as a statement of the Pythagorean theorm for the sides of a square. According to tradition the garhapatya fire altar has the measure of area one vyayama. It is a square according to one tradition and a circle by another. This is an example to show the emergence of the geometrical background in sulbasutra. For prougachiti a fire altar in the form of isosceles triangle is to be constructed shows how theorems and equations related to triangles emerged in ritual books. The diagonal of a square produces double the area of the square a rule to be followed in the construction of chiti. It is a geometrical theorem.

दीर्घचतुरस्रयाक्ष्णयारज्जु पार्श्वमानतिर्यिन्मानि।
च यत्पृथक्भूते कुरुतस्तदुभयं करोति।

The rope which is stretched across the diagonal of a square produces an area double the size of the original square. It also contains the general statement of the Pythagorean theorem; 'the rope stretched along the length of the diagonal of a rectangle makes an area which the vertical and horizontal sides make together. Boudhāyanaśulbasūtra contains geometric solutions of a linear equation in a single unknown. Quadratic equations and several values of pi also occur in

this sūtra. Boudhāyana also recommends his method for constructing a rectangle for the construction of a trapezium also.

REFLECTIONS IN ĀPASTAMBAŚULBASŪTRA

Āpastambaśulbasūtra is also supposed to be written between 800-500 B.C. this śulbasūtra is broadly divided into six patalas or sections. It consists of 223 sūtras for the construction of sacrificial altars using geometrical rules and mathematical ideas. Some methods and geometrical structures given in the Boudhāyanasūtras are also repeated herewith minor variations in language style. Different agni altars with varying areas are to be constructed in a Yajña ritual. Āpastamba improved and expanted on the rules given by Boudhāyana.

'एकवधिः पूरथमोऽग्नरि्द्ववधिः द्वतीयस्तृवधिस्तृतीयः ।
त एवमेवाद्यन्त्यैकशतवद्यिात् '।।

The first agni is one fold in area, the second two fold and third three fold and so on. Even for the same type of yajña, when repeated year after year by same yajamāna, the size of the altars is to be increased. This increase in size has a mathematical relation. Hence this also emphasizes geometry at various instances.

With the help of bamboo sticks, our ancient people made all the measurements with puruṣa unit. The measurements of the altars and the number of bricks required to construct the same, bricks are to be designed and made based on the geometrical knowledge. No broken or smaller bricks can be used for final adjustment in construction purposes. Hence the brickmaker is expected to have a perfected geometrical background of high standard. This obviously stands out as an example on the complicated bricks with the background of geometry. Sūtra says- The diagonal of the square produces double the area and it is root 2 of the side of the square. It is important to note that the method for the determination of root 2 given in Boudhāyanaśulbasūtra is explained under this Kātyāyanaśulbasūtra also.

ROOT 2= 1 + 1/3 + 1/3x4 - 1/3x4x34, so root 2=1.414

By third, increase the measurement and this third by its own fourth less the thirty fourth part of that fourth. This is the approximate value of root 2.

REFLECTIONS IN KĀTYAYANA AND MĀNAVASULBAŚUTRAS

Kātyāyanaśulbasūtra belongs to sage Kātyāyana. The work consists of 102 sūtras. This is divided in to two parts. The 1st part is composed in the style of the sūtras or aphorisms, while the second part is composed in verses. It also deals with the subject of geometry of fire altars. The altar is of the shape of a trough, chariot, rhombus and a kind of pot in the shape of wheel. In all these structures the sides, size, shape of the bricks and their numbers are specified. Hence the construction procedure has to be based on the geometrical information. Compared with the works of Baudhāyana and Āpastamba, the śulba of Kātyāyana exhibits the whole body of geometrical knowledge required for the vedic altar-builder in a more systematic form.

Mānavaśulbasūtra is said to be of later origin. It also discusses many rituals and geometry of altar and almost all mathematical calculations given in above three can be seen in this śulbasūtra also. However in this book the mathematical contents are deep, clear and more specific. Many fundamental knowledge which are not described in other śulbasūtras have found their place in this śulbasūtra. The method for finding the volume of a structure is given in this sūtra. Multiply the length with the breadth separately and that again by the height. This always gives the result in cubic measure ie,

'आयम बाहुम् नक्षिपिय वसितारस्तु तथापृथक्।
सो$ध्यर्धं गुणयेद्राशि स सर्वगुणितो घनः।

SOME IMPORTANT GEOMETRIC CONSTRUCTIONS IN ŚULBASŪTRAS

The following are some of the important geometrical constructions in śulba literature. They are-

1. To draw a straight line at right angles to a given straight line and from a given point to it.
2. To construct a square having a given side.
3. To construct a right angle of given sides.
4. To construct an isosceles trapezium of a given, altitude, face and base.
5. To construct a parallelogram having given sides at given inclinations.

And also it deals with the subject of combination of areas, transformation of areas etc.

Our ancient Vedic literature is very rich and they consist of eternal knowledge. Being unable to realize this wealth or our heritage, man proceeds to material prosperity without careful introspection into the facts imparted to us. These Śulbasūtras exhibit a thorough familiarity with the properties of the right triangle, or rather the properties of the sides and diagonals of figures with right angular corners. They provide many ideas in the growth of geometrical and arithmetical knowledge and its application to the construction of complex brick platforms. So it is obvious that the geometrical background was excellent in Śulbasūtras and they acted as the strong foundation for building up the modern mathematics.

NOTES AND REFERENCES

1. 'पुरोगचति चन्वितति' (Baudhayanasulba 4.100)
2. 'चतुरश्रस्याक्ष्णयारज्जुस्तदवष्टिावत भूमि करोति समस्य द्वकिराणि' (Apastambasulba 1.10.11)
3. Bibhutibhushan Datta, Ancient Hindu Geometry The science of the Sulba, Cosmo publications, New Delhi,1993.
4. Dr.T.A.Saraswathy Amma,Geometry in ancient and medieval India, Motilal Banarsidass publishers pvt ltd, Delhi, 1979.
5. BharatikrishnaTirthaji, Vedic Mathematics, ed: Dr.V.S.Agarwala, Motilal Banarsidass publishers pvt ltd. Delhi, 1992.
6. Swami Satya Prakash Saraswathy, Geometry in ancient India, Govindram hasanand, Nai sarak, Delhi.
7. David Rapport Lachterman, The ethics of geometry, Routledge, new york,1989.
8. Dr. Raghunath Purushotham Kulkkarni, Char Sulbasutra (Hindi), Maharshi Sandeepani rashtriya veda vidya prathishtan, delhi, 2003.
9. Dr. Sudhyumna acharya, ganithasastr ke vikas ki bharatheeya-parambara, Motilal Banarsidass publishers, Delhi, 2006.
10. Nishanth Shukla, Encyclopaedia of Vedic Mathematics, Cyber Tech Publications, New Delhi, 2011.