A Summed up Class of Ratio-cum-Product Synthetic Estimator for Small Domain

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Abstract— The necessity of domain estimation particularly small area estimation is wide now days because of the idea of intricacy include in it. This paper discusses on a summed up class of proportion cum-item manufactured estimator for small domain utilizing assistant data under fundamental examining structure. The distinctive types of manufactured estimator are considered including its quadratic structure. The predisposition and mean squared error (MSE) of the proposed summed up proportion cum-item manufactured estimator are likewise registered. Further, it shows the utilization of estimators for evaluating crop real esatate for Inspector Land Revenue Circle (ILRC) of Jodhpur tehsil, Rajasthan, India as little spaces and analyze the overall execution of proposed estimators as far as absolute relative bias (ARB) and simulated relative standard error (SRSE) through a simulation study.

Keywords- Synthetic Estimator, Small Domains, Ratio-Cum-Product Synthetic Estimators, Timely Reporting Scheme (TRS), Inspector Land Revenue Circle (ILRC).

I. INTRODUCTION

Customary estimators are basically inadequate to give dependable gauges because of the absence of satisfactory example for little spaces. Thus, specialists felt need to build up some other sort of estimators to give insights on little region or subsections and divisions utilizing the effectively gathered information in expansive scale test reviews. Gonzalez and Wakesberg (1973)and Gonzalez (1973) examined the estimators which are called manufactured estimator if a solid direct estimator for a huge territory covering a few little regions is utilized to determine a backhanded estimator for the little zone under the suspicions that the little zone have indistinguishable attributes from the substantial zone. Schaible et al. (1977) depicted that if little space test sizes are generally little, the manufactured estimator performs superior to the straightforward direct estimators though for the huge example direct estimators perform superior to engineered estimator. Helper factors region used to build the accuracy of the estimator. A summed up class of engineered estimators utilizing assistant data under basic irregular examining has been created by Tikkiwal and Ghiya (2000). A point by point writing is now accessible in Sarndal et al. (1992) and Rao (2003) among others.They thought about the general execution of basic proportion and item engineered estimators with the relative comparing direct estimators through a recreation think about. Tikkiwal and Pandey (2007)andPandey and Tikkiwal (2010) further worked over various inspecting plan to check to the effectiveness of proposed estimator The conventional structure based and elective estimators are accessible in the writing of the little zone estimation which additionally named as immediate and aberrant estimators individually Tikkiwal and Ghiya (2000) and Rai and Pandey (2013). Bhal and Sangeeta (2015) proposed some quadratic manufactured estimators under twofold inspecting plan and thought about their effectiveness. The above examination depends on the conventional truth of linearity between the auxilary and concentrate variable. To see the proposed estimator and their productivity under various conditions In the present paper we talk about some summed up class of engineered estimators when the idea of information is nonlinear and contrast them and relating quadratic manufactured estimators. The distinctive types of engineered estimator are considered including its quadratic structure. The predisposition and mean squared mistake (MSE) of the proposed summed up proportion cum-item manufactured estimator are likewise registered.

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Further, it shows the utilization of estimators for evaluating crop real esatate for Inspector Land Revenue Circle (ILRC) of Jodhpur tehsil, Rajasthan, India as little space and look at the general execution of proposed estimators as far as absolute relative bias (ARB) and simulated relative standard error (SRSE) through a recreation think about.

II. NOTATIONS AND TERMINOLOGY

Give us a chance to consider a limited populace U = {1, 2, . . . , N} is separated into A non covering small domains Ua of size Na(a = 1, 2, . . . , A) for which gauges are required. We indicate the trademark under investigation by y. We further expect that the assistant data is accessible and indicate this by x. An arbitrary example s of size n is chosen through straightforward irregular examining without substitution plot from populace U to such an extent that Na units in the example s originates from small domain Ua(a = 1, 2, . . . , A). Therefore,

$$\sum_{a=1}^{A} N_a = N \quad \text{and} \sum_{a=1}^{A} n_a = n.$$

We indicate the different populace and test methods for trademark Z = X, Y by

$\bar{Z}$ = mean of the populace dependent on N perceptions, $\bar{Z}$ a = populace mean of space a dependent on Na perceptions,

$\bar{z}$ = mean of the example s dependent on n perceptions,

$\bar{z}$a = test mean of area a dependent on na perceptions,

furthermore, zai , a = 1, 2, . . . , An and I = 1, 2, . . . , Na indicate the I th perception of the little space a for the trademark Z = X, Y .

Likewise the different mean squares and coefficient of varieties of the populace U for trademark Z are indicated by

$$S_z^2 = \frac{1}{N-1}\sum_{1}^{N}(z_i - \bar{Z})^2, \; C_z = \frac{S_z}{\bar{Z}}. \quad (1)$$

denoted by.

$$C_{xy} = \frac{S_{xy}}{\bar{X}\bar{Y}} \quad (2)$$

where

$$S_{xy} = \frac{1}{N-1}\sum_{1}^{N}(y_i - \bar{Y})(x_i - \bar{X}) \quad (3)$$

The relating different mean squares and coefficient of variation of little spaces Ua are meant by

$$S_{z_a}^2 = \frac{1}{N_a - 1}\sum_{1}^{N_a}(z_{a_i} - \bar{Z}_a)^2 \quad (4)$$

$$C_{z_a} = \frac{S_z a}{\bar{Z}_a} \; and \; C_{x_a y_a} = \frac{S_{x_a y_a}}{\bar{X}_a \bar{Y}_a} \quad (5)$$

$$S_{x_a y_a} = \frac{1}{N_a - 1}\sum_{1}(y_{a_i} - \bar{Y}_a)(x_{a_i} - \bar{X}_a) \quad (6)$$

III GENERALIZED CLASS OF SYNTHETIC ESTIMATORS

Tikkiwal and Ghiya (2000) proposed a summed up class of manufactured estimator of populace mean $\bar{Y}$ a dependent on the helper data x under straightforward arbitrary examining plan without substitution as pursues:

$$\bar{y}_{syn,a} = \bar{y}\left(\frac{\bar{x}}{\bar{X}_a}\right)^{\beta}, \quad (7)$$

Under the synthetic assumption

$$\bar{Y}_a(\bar{X}_a)^{\beta} = \bar{Y}(\bar{X})^{\beta}. \quad (8)$$

where β is a reasonably picked steady. The plan inclination and mean square blunder (MSE) of the estimator are gotten as

$$Bias(\bar{y}_{syn,a}) = \bar{Y}_a\left(\frac{N-n}{Nn}\right)\left[\frac{C_x^2}{2}(\beta^2 - \beta) + C_{xy}\beta\right] \quad (9)$$

and

$$MSE(\bar{y}_{syn,a}) = \bar{Y}^2\left(\frac{\bar{X}}{\bar{X}_a}\right)^{2\beta}\left[1 + \frac{N-n}{Nn}\{(2\beta^2 - \beta)C_x^2 + C_y^2 + 4\beta C_{xy}\}\right] - 2\bar{Y}_a\bar{Y}\left(\frac{\bar{X}}{\bar{X}_a}\right)^{\beta}\left[1 + \frac{N-n}{Nn}\left(\frac{\beta(\beta-1)}{2}C_x^2 + \beta C_{xy}\right)\right] + \bar{Y}_a^2. \quad (10)$$

The reasonable estimations of β gives different types of synthetic estimator.

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3.1 Simple Synthetic Estimator:

On substituting β = 0, we acquire basic manufactured estimator which is communicated as

$$\bar{y}_{ssyn,a} = \bar{y} \tag{11}$$

and the supposition (8) lessens to Y¯ a = Y¯ . The straightforward synthetic estimator is an unbiased estimator of Y¯ with MSE given as

$$MSE(\bar{y}_{ssyn,a}) = \bar{Y}^2(1 + C_y^2) - 2\bar{Y}_a\bar{Y} + \bar{Y}_a^2. \tag{12}$$

3.2 Ratio Synthetic Estimator:

if we put β = −1 in (7), at that point the summed up engineered estimator lessens to proportion manufactured estimator of the structure

$$\bar{y}_{rsyn,a} = \bar{y}\left(\frac{\bar{X}_a}{\bar{x}}\right), \tag{13}$$

under synthetic supposition Y/¯a, X¯/a = Y¯/X¯ . Here inclination and MSE of the estimator can be acquired as

$$Bias(\bar{y}_{rsyn,a}) = -\bar{Y}_a\left(\frac{N-n}{Nn}\right)C_{xy} \tag{14}$$

as

$$MSE(\bar{y}_{rsyn,a}) = \bar{Y}\left(\frac{\bar{X}_a}{\bar{X}}\right)^2\left[1 + \frac{N-n}{Nn}\left\{3C_x^2 + C_y^2 - 4C_{xy}\right\}\right] - 2\bar{Y}_a\bar{Y}\left(\frac{\bar{X}_a}{\bar{X}}\right)\left[1 + \frac{N-n}{Nn}(C_x^2 - C_{xy})\right] + \bar{Y}_a^2. \tag{15}$$

3.3 Product Synthetic Estimator:

For β = 1, the summed up synthetic estimator in (7) gives item engineered estimator composed as

$$\bar{y}_{psyn,a} = \bar{y}\left(\frac{\bar{x}}{\bar{X}_a}\right), \tag{16}$$

With synthetic assumption Y¯ aX¯ a = Y¯ X¯.

$$Bias(\bar{y}_{psyn,a}) = \bar{Y}_a\left(\frac{N-n}{Nn}\right)C_{xy} \tag{17}$$

as

$$MSE(\bar{y}_{psyn,a}) = \bar{Y}^2\left(\frac{\bar{X}}{\bar{X}_a}\right)^2\left[1 + \frac{N-n}{Nn}\left(C_x^2 + C_y^2 + 4C_{xy}\right)\right] - 2\bar{Y}_a\bar{Y}\left(\frac{\bar{X}}{\bar{X}_a}\right)\left[1 + \frac{N-n}{Nn}C_{xy}\right] + \bar{Y}_a^2. \tag{18}$$

Bhal and Sangeeta (2015) depicted summed up proportion cum item manufactured estimator and quadratic engineered estimator and guaranteed that they are progressively productive then regular estimators under twofold testing plan. we build up the summed up manufactured estimators and quadratic engineered estimator for little space

3.4 Ratio Quadratic Synthetic Estimator:

O If we put β = −2, in (7) we have an estimator called proportion quadratic synthetic estimator given as

$$\bar{y}_{rqsyn,a} = \bar{y}\left(\frac{\bar{X}_a}{\bar{x}}\right)^2, \tag{19}$$

ith the synthetic assumption

$$\frac{\bar{Y}_a}{\bar{Y}} = \left(\frac{\bar{X}_a}{\bar{X}}\right)^2.$$

$$Bias(\bar{y}_{rqsyn,a}) = \bar{Y}\left(\frac{N-n}{Nn}\right)(C_x^2 - 2C_{xy}) \tag{20}$$

$$MSE(\bar{y}_{rqsyn,a}) = \bar{Y}^2\left(\frac{\bar{X}_a}{\bar{X}}\right)^4\left[1 + \left(\frac{N-n}{Nn}\right)(6C_x^2 + C_y^2 - 8C_{xy})\right] - 2\bar{Y}_a\bar{Y}\left(\frac{\bar{X}_a}{\bar{X}}\right)^2\left[1 + \left(\frac{N-n}{Nn}\right)(6C_x^2 - 2C_{xy})\right] + \bar{Y}_a^2 \tag{21}$$

3.5 Product Quadratic Synthetic Estimator:

On the off chance that we put β = 2, in (7) the got estimator is named as item quadratic manufactured estimator given

$$\bar{y}_{pqsyn,a} = \bar{y}\left(\frac{\bar{x}}{\bar{X}_a}\right)^2, \tag{22}$$

With assumption $\bar{Y}_a\bar{X}_a^2 = \bar{Y}\bar{X}^2$.

$$Bias(\bar{y}_{pqsyn,a}) = \bar{Y}_a\left(\frac{N-n}{Nn}\right)(C_x^2 + 2C_{xy}) \tag{23}$$

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$$MSE(\bar{y}_{pqsyn,a}) = \bar{Y}^2\left(\frac{\bar{X}}{\bar{X}_a}\right)^4\left[1+\frac{N-n}{Nn}\left(2C_x^2+C_y^2+8C_{xy}\right)\right] - 2\bar{Y}_a\bar{Y}\left[1+\frac{N-n}{Nn}\left(C_x^2+2C_{xy}\right)\right]+\bar{Y}_a^2 \quad (24)$$

Further, the above proposed estimators are created for the situations where y variable is directly (positive or negative) related with x variable. In any case, it has been seen that in some genuine circumstances the assistant factors have a non-straight connection with the investigation variable. In such circumstances, the above characterized estimators could possibly give attractive outcomes. Here, we propose a summed up class of proportion cum-item engineered estimator as referenced beneath Bhal and Sangeeta (2015)

I. GENERALIZED RATIO CUM PRODUCT SYNTHETIC ESTIMATORS

We propose an estimator of summed up proportion cum-item engineered estimator as give

$$\bar{y}_{rpsyn,a} = \bar{y}\left[\omega\left(\frac{\bar{x}}{\bar{X}_a}\right)^{\beta+1}+(1-\omega)\left(\frac{\bar{x}}{\bar{X}_a}\right)^{\beta-1}\right] \quad (25)$$

For reasonably picked consistent of β which is a curved blend of engineered proportion and item manufactured estimators with certain loads

$$MSE(\bar{y}_r psyn,a) = \omega^2 MSE[\bar{y}\left(\frac{\bar{x}}{\bar{X}_a}\right)^{\beta+1}]+(1-\omega^2)MSE[\bar{y}\left(\frac{\bar{x}}{\bar{X}_a}\right)^{\beta-1}]+2\omega(1-\omega)cov\left[\left(\frac{\bar{x}}{\bar{X}_a}\right)^{\beta+1},\left(\frac{\bar{x}}{\bar{X}_a}\right)\right] \quad (26)$$

Neglecting the covariance term we get

$$MSE(\bar{y}_r psyn,a) = \omega^2 MSE[\bar{y}\left(\frac{\bar{x}}{\bar{X}_a}\right)^{\beta+1}]+(1-\omega^2)MSE[\bar{y}\left(\frac{\bar{x}}{\bar{X}_a}\right)^{\beta-1}] \quad (27)$$

To locate the ideal estimation of ω we separate the condition as for ω and apply the technique for limiting. Specifically situation when loads are similarly partitioned The summed up proportion cum-item engineered estimator is acquired as a raised mix of proportion and item manufactured estimators given as

$$\bar{y}_{rpsyn,a} = \frac{\bar{y}}{2}\left[\left(\frac{\bar{x}}{\bar{X}_a}\right)^{\beta+1}+\left(\frac{\bar{x}}{\bar{X}_a}\right)^{\beta-1}\right] \quad (28)$$

If β = −1, 0 and 1, the estimator is written as

$$\bar{y}_{-1,a} = \frac{\bar{y}}{2}\left[1+\left(\frac{\bar{X}_a}{\bar{x}}\right)^2\right] \quad (29)$$

$$\bar{y}_{0,a} = \frac{\bar{y}}{2}\left[\left(\frac{\bar{x}}{\bar{X}_a}\right)+\left(\frac{\bar{X}_a}{\bar{x}}\right)\right] \quad (30)$$

$$\bar{y}_{1,a} = \frac{\bar{y}}{2}\left[\left(\frac{\bar{x}}{\bar{X}_a}\right)^2+1\right] \quad (31)$$

V.COMPARISON OF ESTIMATORS-A SIMULATION STUDY

For some regulatory purposes the majority of the conditions of India is isolated into various locale. Each area is additionally isolated into various tehsils, and tehsils into investigator land income circles (ILRC). In spite of the fact that the ILRCs have further subdivisions, this examination considers ILRCs as small domain from the timely reporting scheme (TRS) perspective. TRS is utilized to improve courses of events and nature of yield grounds measurements and ther-by preparations of the rule crops amid every horticulture season. This plan has been in vogue since mid seventies in numerous conditions of India. So as to think about the execution of the estimators, we have taken the issue of harvest grounds estimation of 7 ILRCs of Jodhpur tehsil of Rajasthan. The harvest under thought is Bajra (Indian corn or millet) for the farming season 1993-94. The Bajra crop land for farming season 1992-93 is taken as helper trademark x. In this reproduction think about, we have taken 100 examples of size n = 12, 25, 38, 50, 63, almost 5 to 25 percent of test towns utilizing SRSWOR plot. The execution of the estimators is assessed regarding absolute relative bias (ARB) and reenacted relative.

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Table 1: ILRCs of Jodhpur Tehsil for Agricultural Season 1992-94

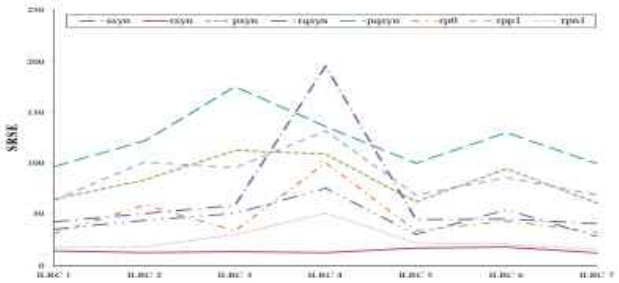
S.No.	ILRC of Jodhpur	No. of villages
1	Jodhpur	29
2	Keru	44
3	Dhundhara	32
4	Bisalpur	30
5	Luni	33
6	Dhava	40
7	Jajiwal Kalan	44
Total		**252**

Standard error (SRSE) or essentially coefficient of variety. The ARB and SRSE of every estimator is determined by the accompanying equation

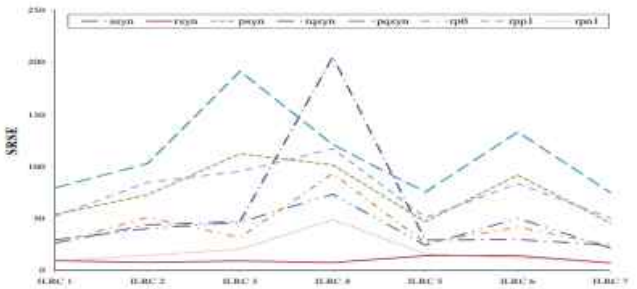
$$ARB(\bar{y}_a) = \frac{\left|\frac{1}{100}\sum_{i=1}^{100}\bar{y}_a^{(i)} - \bar{Y}_a\right|}{\bar{Y}_a} \times 100 \qquad (32)$$

$$SRSE(\bar{y}_a) = \frac{\sqrt{\frac{1}{100}\sum_{i=1}^{100}\left(\bar{y}_a^{(i)} - \bar{Y}_a\right)^2}}{\frac{1}{100}\sum_{i=1}^{100}\bar{y}_a^{(i)}} \qquad (33)$$

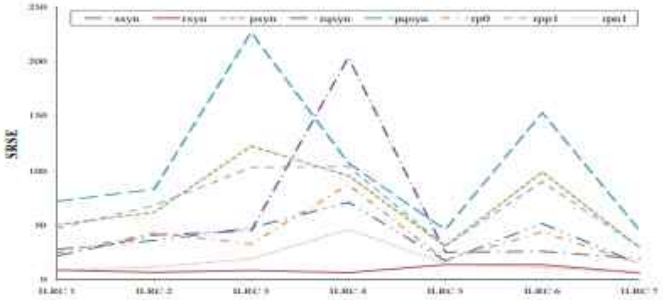
Where ¯y (I) an is an expected estimation of Y¯a for I th test. The reproduction results are accounted for in the tables. where the estimators basic engineered, proportion manufactured, item engineered, proportion quadratic manufactured, item quadratic manufactured and proportion cum-item engineered estimators at various estimations of (β = 0, −1, 1) signified by e1, e2, e3, e4, e5, e6, e7ande8. Among the proportion cum-item engineered sort estimator for β = 0, −1, 1 alongside basic manufactured, proportion manufactured, item manufactured, proportion quadratic and item quadratic manufactured estimators the proportion engineered estimator and proportion cum-item manufactured estimator for β = −1, which is a mix of basic engineered and item quadratic engineered estimator are having low estimations of ARB and SRSE in a large portion of the ILRC's. Different estimators don't perform well when contrasted with these two for various ILRC's.The ARB estimations of proportion engineered estimator and proportion cum-item manufactured estimator for β = −1 changes (1.1017, 29.9095),(1.1593, 57.1305) individually while there SRSE fluctuates (12.0277, 17.7053)?? furthermore, (15.3828, 50.9841) separately for test estimate 12. while for test measure 63 the ARB and SRSE benefits of relating estimators differs (0.5905, 14.8440),(1.4276, 21.3635) and (3.9695, 13.4243),(4.0611, 44.7573) individually.



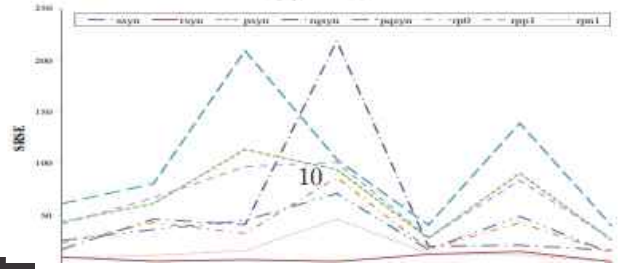
(a) $n = 12$



(b) $n = 25$



(c) $n = 38$



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CONCLUSION

This paper tends to the issue of estimating the populace implies Y of the investigation variable y utilizing data on an assistant variable x. A class of ratio cum-double to proportion estimators has been proposed and its properties are examined under substantial example guess. Distinctive estimators of the populace mean Y have been recognized as an individual from the proposed class of proportion cum-double to proportion estimators. Districts of execution have been gotten in which recommended class of estimators perform superior to other existing estimators. It is a bound together methodology. Properties of a few estimators having a place with the proposed class of estimators can be considered effectively.

REFERENCES

[1] Bhal, S. and Sangeeta (2015). Genealized synthetic estimator using double sampling scheme and auxiliary information. Mathematical Journal of Interdisciplinary Sciences, 4(1):15–21.

[2] Gonzalez, M. (1973). Use and evaluation of synthetic estimates. Proceedings of social statistical section of americanstatistical association, pages 33–36.

[3] Gonzalez, M. E. and Wakesberg, J. (1973). Estimation of the error of synthetic estimates. Paper presented at first meeting of the international association of survey statisticians, Vienna, Austria, pages 18–25

[4] Pandey, K. and Tikkiwal, G. (2010). Generalized class of synthetic estimators for small areas under systemetic sampling scheme. statistics in transition, 11(1):75–89.

[5] Rai, P. K. and Pandey, K. K. (Spring, 2013). Synthetic estimators using auxiliary information in small domains. Statistics in Transition-new series, 14(1):31–44.

[6] Rao, J. N. K. (2003). Small Area Estimation. Wiley Interscience. Sarndal, C. E.,

[7] Swensson, B., and Wretman, J. (1992). Model Assisted Survey Sampling. Springer-Verlag. Schaible, W., Brock, D., Casady, R., and Schmack, G. (1977). An empirical comparison of the simple inflation, synthetic and composite estimators for small area statistics. Proceedings of the Americal Statistical Association, Social Statisitcs Section, pages 1017–1021.

[8] Tikkiwal, G. and Pandey, K. (2007). On synthetic and composit estimators for small area estimation under lahri midjuno sampling scheme. statistics in transition, 8(1):111–123.

[9] Tikkiwal, G. C. and Ghiya, A. (2000). A generalized class of synthetic estimators with application to crop acreage estimation for small domains. Biometrical Journal, 42(7):865– 876

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