In Brief

# On some results in quotient Boolean algebra and Boolean logic

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**Abstract**
The aim of this paper is to derive some results in quotient Boolean algebra and as a consequence also in Boolean logic. The work carried out in the present paper is in fact an extension of Behara, Biswas and Datta {cf.[1]}.

**Keywords:** Quotient Boolean Algebra, Boolean Logic, Principle of Duality, Ideal, Filter, Conjunctive Normal Form (CNF), Disjunctive Normal Form (DNF).

**Introduction:**
The development of Boolean algebra is strongly motivated by the basic laws described by the English Mathematician and Logician George Boole (1815-1864) in a small book 'The Mathematical Analysis of Logic'. Our works in this paper on Quotient Boolean Algebra and Boolean Logic are mainly the improvement of the paper by Behara, Biswas and Datta {cf.[1]}. We do not explain the standard theories and notations as well as definitions of Boolean Algebra, Duality Principle, Ideal, Filter and Boolean Logic as those are available in [1], [2], [3], [4] and [5]. Studing the theories of quotient Boolean algebra by its ideal and filter as developed in [1], we wish to prove some fundamental theorems related to above in the following manner:

**Some Fundamental Theorems**:
Let B be a Boolean algebra and $I$, $F$ be respectively its ideal and filter.

**Theorem 1:** In a Boolean Algebra $< B/I, +, . >$ the following hold:

(i) the elements $0 + I$ and $1 + I$ are unique.

(ii) Each $a + I \in B/I$ has a unique complement $a' + I \in B/I$

(iii) for each $a + I \in \frac{B}{I}$, $(a' + I)' = a + I$

(iv) $0' + I = 1 + I$ and $1' + I = 0 + I$

(v) (Idempotent law): $(a + I) + (a + I) = a + I$ and $(a + I).(a + I) = (a + I)$ for every $a + I \in B/I$

(vi) $(a + I) + (1 + I) = 1 + I$ and $(a + I).(0 + I) = 0 + I$ $\forall \alpha + I \in B/I$

(vii) (Absorption law) : $(a + I).((a + I) + (b + I)) = a + I$ and $(a + I) + ((a + I).(b + I)) = a + I$ $\forall (a + I), (b + I) \in B/I$

**Proof:**

(i) If possible, let $0_1 + I$ and $0_2 + I$ be two zero elements of $B/I$.

Then by definition, $(a + I) + (0_1 + I) = (a + I)$ and $(a + I) + (0_2 + I) = (a + I)$ $\forall \alpha \in B$

Hence $(0_2 + I) + (0_1 + I) = (0_2 + I)$ and $(0_1 + I) + (0_2 + I) = (0_1 + I)$

But $(0_2 + I) + (0_1 + I) = (0_1 + I) + (0_2 + I)$.

Therefore, $0_1 + I = 0_2 + I$,

i.e., zero element of $B/I$ is unique.

By the duality principle, unit element of $B/I$ is unique.

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(ii) Let $a_1' + I$ and $a_2' + I$ be two complements of $a + I$ in $B/I$.

Then $(a + I).(a_1' + I) = 1 + I$, $(a + I).(a_1' + I) = 0 + I$ and $(a + I).(a_2' + I) = 1 + I$, $(a + I).(a_2' + I) = 0 + I$

Now $a_1' + I$

$= (a_1' + I).(1 + I)$

$= (a_1' + I).((a + I) + (a_2' + I))$

$= (a_1' + I).(a + I) + (a_1' + I).(a_2' + I)$

$= (0 + I) + (a_1' + I).(a_2' + I)$

$= (a_1' + I).(a_2' + I)$

Similarly we can show $a_2' + I = (a_2' + I).(a_1' + I)$

Hence, $a_1' + I = a_2' + I$.

(iii) For each $a + I \in B/I$, $\exists$ a unique element $a' + I \in B/I$ s.t.

$(a + I) + (a' + I) = 1 + I$ *and* $(a + I) + (a' + I) = 0 + I$

$(a' + I) + (a + I) = 1 + I$ *and* $(a' + I) + (a + I) = 0 + I$

This implies that a is a complement of $a_1' + I$, *i.e.*, $a + I = (a' + I)$.

(iv). Since $(a + I) + (0 + I) = a + I$ *and* $(a + I).(1 + I) = a + I \;\forall\; (a + I) \in B/I$, replacing a+I by $1 + I$ and $0 + I$ respectively we get that

$(1 + I) + (0 + I) = 1 + I$ and

$(0 + I).(1 + I) = (1 + I).(0 + I) = 0 + I$.

Therefore by $(P_5)$, $0 + I$ is the complement of $1 + I$, i.e., $1' + I = 0 + I$.

The 2nd part is the dual of 1st part.

(v). $(a+I)+(a+I)$

$=((a+I)+(a+I)).(1+I)$

$=((a+I)+(a+I)).((a+I)+(a'+I))$

$=(a+I).((a+I).(a'+I))$

$=(a+I)+(0+I)$

$=a+I$

The 2nd part is the dual of 1st part.

(vi). $(a+I)+(1+I)$

$=((a+I)+(1+I)).(1+I)$

$=((a+I)+(1+I)).((a+I)+(a'+I))$

$=(a+I)+((1+I).(a'+I))$

$=(a+I)+((a'+I).(1+I))$

$=(a+I)+(a'+I)$

$=1+I$

The 2nd part is the dual of 1st part.

(vii). $(a+I).((a+I)+(b+I))$

$=(a+0+I).((a+I)+(b+I))$

$=(a+I)+((0+I).(b+I))$

$=(a+I)+(0+I)$

$=a+I$

The 2nd part is the dual of 1st part.

**Theorem 2**: In a Boolean Algebra $<B/I,+,.>$ the following hold:

For all $(a+I),(b+I),(c+I) \in B/I$

(i) If $(b+I)+(a+I)=(c+I)+(a+I)$ and $(b+I)+(a'+I)=(c+I)+(a'+I)$ then $(b+I)=(c+I)$

(ii)

$(a+I)+((b+I)+(c+I))=((a+I)+(b+I))+(c+I)$

$(a+I).((b+I).(c+I))=((a+I).(b+I)).(c+I)$

(iii) $((a+I)+(b+I))'=(a'+I).(b'+I)$

$((a+I).(b+I))'=(a'+I)+(b'+I)$

(iv) $((a+I)+(b+I))=((a'+I).(b'+I))'$ and $(a+I).(b+I)=((a'+I)+(b'+I))'$

(v) $(a+I)+(b'+I)=1+I$ iff $(a+I)+(b+I)=(a+I)$

Also $(a+I).(b'+I)=0+I$ iff $(a+I).(b+I)=a+I$

(vi) $(a+I)+((a'+I).(b+I))=(a+I)+(b+I)$ and $(a+I).((a'+I)+(b+I))=(a+I).(b+I)$

**Proof:**

(i) We assume $(b+I)+(a+I)=(c+I)+(a+I)$ and $(b+I)+(a'+I)=(c+I)+(a'+I)$

Then $(b+I)$

$=(b+I)+(0+I)$

$=(b+I)+((a+I).(a'+I))$

$=((b+I)+(a+I)).((b+I)+(a'+I))$

$=((c+I)+(a+I)).((c+I)+(a'+I))$

$=(c+I)+((a+I).(a'+I))$

$=(c+I)+(0+I)$

$=c+I$

The 2nd part is the dual of 1st part.

(ii) $(a+I)+\left((b+I)+(c+I)\right)$

$=(1+I).\left[(a+I)+\left((b+I)+(c+I)\right)\right]$

$=((a+I)+(a'+I)).\left[(a+I)+\left((b+I)+(c+I)\right)\right]$

$=(a+I)+(a'+I).\left((b+I)+(c+I)\right)$

$=a+I+((a'+I).(b+I))+((a'+I).(c+I))$

Also $\left((a+I)+(b+I)\right)+(c+I)$

$=(1+I).\left[\left((a+I)+(b+I)\right)+(c+I)\right]$

$=((a+I)+(a'+I)).\left[\left((a+I)+(b+I)\right)+(C+I)\right]$

$=(a+I).\left[(a+I)+\left((b+I)+(c+I)\right)\right]+(a'+I)$

$.\left[(a+I)+\left((b+I)+(c+I)\right)\right]$

$=(a+I).\left((a+I)+(b+I)\right)+(a+I).(c+I)+(a'+I)$

$.\left((a+I)+(b+I)\right)+(a'+I)(c+I)$

$=(a+I)+(a+I).(c+I)+(a'+I).(a+I)+(a'+I)$

$.(b+I)+(a'+I).(c+I)$

$=(a+I)\boxplus(\square+I)+(a'+I)\ (b+I)+(a'+I).(c+I)$

$=(a+I)\boxplus(a'+I).(b+I)+(a'+I).(c+I)$

Hence

$(a+I)+\left((b+I)+(c+I)\right)=\left((a+I)+(b+I)\right)+(c+I)$

(iii) $\left((a+I)+(b+I)\right)+\left((a'+I).(b'+I)\right)$

$=[\left((a+I)+(b+I)\right)+(a'+I)]$

$.[\left((a+I)+(b+I)\right)+(b'+I)]$

$=[(a'+I)+\left((a+I)+(b+I)\right)]$

$.\left[(b'+I)+((a+I)+(b+I))\right]$

$=\left[\left((a'+I)+(a+I)\right)+(b+I)\right]$

$.[(a+I)+\left((b+I)+(b'+I)\right)]$

$=\left((1+I)+(b+I)\right).\left((1+I)+(a+I)\right)$

$=(1+I).(1+I)$

$=1+I$

Again,

$\left((a+I)+(b+I)\right)+\left((a'+I).(b'+I)\right)$

$=\left[\left((a+I)+(b+I)\right).(a'+I)\right].(b'+I)$

$=\left[(a'+I).\left((a+I)+(b+I)\right)\right].(b'+I)$

$=\left[(a'+I).(a+I)+(a'+I).(b+I)\right].(b'+I)$

$=\left((0+I)+(a'+I).(b+I)\right).(b'+I)$

$=((a'+I).(b+I)).(b'+I)$

$=(a'+I).\left((b+I)(b'+I)\right)$

$=(a'+I).(0+I)$

$=0+I$

Hence $((a+I)+(b+I))'=(a'+I)(b'+I)$

The 2nd part is the dual of 1st part.

(iv) We have by De-Morgans Law

$$\left((a+I)+(b+I)\right)'=(a'+I).(b'+I)$$

Hence $(((a+I)+(b+I))')'=((a'+I).(b'+I))'$
which implies that

$$((a+I)+(b+I))=((a'+I).(b'+I))'$$

The 2nd part is the dual of 1st part.

(v) Let $(a+I)+(b'+I)=1+I$

Now $(a+I)+(b+I)$

$=(a+I)+(b+I).(1+I)$

$=((a+I)+(b+I)).\left((a+I)+(b'+I)\right)$

$=(a+I)+\left((b+I).(b'+I)\right)$

$=(a+I)+(0+I)$

$=(a+I)$

Conversely, let $(a+I)+(b+I)=a+I$

Now, $(a+I)+(b'+I)$

$=(1+I).((a+I)+(b'+I))$

$=\left((a+I)+(a'+I)\right).\left((a+I)+(b'+I)\right)$

$=(a+I)+\left((a'+I).(b'+I)\right)$

$=(a+I)+\left((a+I)+(b+I)\right)'$

$=(a+I)+(a'+I)$

$=1+I$

The 2nd part is the dual of 1st part.

(vi) $(a+I)+\left((a'+I).(b+I)\right)$

$=\left((a+I)+(a'+I)\right).\left((a+I)+(b+I)\right)$

$=(1+I).\left((a+I)+(b+I)\right)$

$=\left((a+I)+(b+I)\right)$

The 2nd part is the dual of 1st part.

**Theorem 3:** In a Boolean Algebra <$B/F$,+,.> the following hold:

(i) the elements $0+F$ and $1+F$ are unique.

(ii) Each $a+F\in B/F$ has a unique complement $a+F\in B/F$

(iii) for each $a+F\in \frac{B}{F}$, $(a'+F)'=a+F$

(iv) $0'+F=1+F$ and $1'+F=0+F$

(v) (Idempotent law): $(a+F)+(a+F)=a+F$ and $(a+F).(a+F)=(a+F)$ for every $a+F\in\frac{B}{F}$

(vi) $(a+F)+(1+F)=1+F$ and $(a+F).(0+F)=0+F\ \forall\alpha+F\in B/F$

(vii) (Absorption law) : $(a+F).((a+F)+(b+F))=a+F$ and $(a+F)+((a+F).(b+F))=a+F\ \forall\ (a+F),(b+F)\in B/F$

**Proof:**

(i) If possible, let $0_1+F$ and $0_2+F$ be two zero elements of $B/I$.

Then by definition, $(a+F)+(0_1+F)=(a+F)$ and $(a+F)+(0_2+F)=(a+F)\ \forall\alpha\in B$

Hence $(0_2+F)+(0_1+F)=(0_2+F)$ and $(0_1+F)+(0_2+F)=(0_1+F)$

But $(0_2+F)+(0_1+F)=(0_1+F)+(0_2+F)$.

Therefore, $0_1+F=0_2+F$,

i.e., zero element of $B/F$ is unique.

By the duality principle, unit element of $B/F$ is unique.

(ii) Let $a_1'+F$ and $a_2'+F$ be two complements of $a+F$ in $B/F$.

Then $(a+F).(a_1'+F)=1+F$, $(a+F).(a_1'+F)=0+F$

and $(a+F).(a_2'+F)=1+F$, $(a+F).(a_2'+F)=0+F$

Now $a_1'+F$

$=(a_1'+F).(1+F)$

$=(a_1'+F).((a+F)+(a_2'+F))$

$=(a_1'+F).(a+F)+(a_1'+F).(a_2'+F)$

$=(0+F)+(a_1'+F).(a_2'+F)$

$=(a_1'+F).(a_2'+F)$

Similarly we can show $a_2'+F=(a_2'+F).(a_1'+F)$

Hence, $a_1'+F=a_2'+F$.

(iii) For each $a+F\in B/F$, $\exists$ a unique element $a'+F\in B/F$ s.t.

$(a+F)+(a'+F)=1+F$ *and* $(a+F)+(a'+F)=0+F$

$(a'+F)+(a+F)=1+F$ *and* $(a'+F)+(a+F)=0+F$

This implies that a is a complement of $a_1'+F$, *i.e.,* $a+F=(a'+F)'$.

(iv). Since $(a+F)+(0+F)=a+F$ *and* $(a+F).(1+F)=a+F\ \forall\ (a+F)\in B/F$, replacing a+I by $1+F$ and $0+F$ respectively we get that

$(1+F)+(0+F)=1+F$ and

$(0+F).(1+F)=(1+F).(0+F)=0+F$.

Therefore by $(P_5)$, $0+F$ is the complement of $1+F$, i.e., $1'+F=0+F$.

The 2nd part is the dual of 1st part.

(v) $(a+F)+(a+F)$

$=((a+F)+(a+F)).(1+F)$

$=((a+F)+(a+F)).((a+F)+(a'+F))$

$=(a+F).((a+F).(a'+F))$

$=(a+F)+(0+F)$

$=a+F$

The 2nd part is the dual of 1st part.

(vi) $(a+F)+(1+F)$

$=((a+F)+(1+F)).(1+F)$

$=((a+F)+(1+F)).((a+F)+(a'+F))$

$=(a+F)+((1+F).(a'+F))$

$=(a+F)+((a'+F).(1+F))$

$=(a+F)+(a'+F)$

$=1+F$

The 2nd part is the dual of 1st part.

(vii) $(a+F).((a+F)+(b+F))$

$=(a+0+F).((a+F)+(b+F))$

$=(a+F)+((0+F).(b+F))$

$=(a+F)+(0+F)$

$=a+F$

The 2nd part is the dual of 1st part.

**Theorem 4**: In a Boolean Algebra <$B/F$,+,.> the following hold:

For all $(a+F),(b+F),(c+F)\in B/F$

(i) If $(b+F)+(a+F)=(c+F)+(a+F)$ and $(b+F)+(a'+F)=(c+F)+(a'+F)$ then $(b+F)=(c+F)$

(ii)

$(a+F)+((b+F)+(c+F))=((a+F)+(b+F))+(c+F)$

$(a+F).((b+F).(c+F))=((a+F).(b+F)).(c+F)$

(iii) $((a+F)+(b+F))'=(a'+F).(b'+F)$

$((a+F).(b+F))'=(a'+F)+(b'+F)$

(iv) $((a+F)+(b+F))=((a'+F)(b'+F))'$ and $(a+F).(b+F)=((a'+F)+(b'+F))'$

(v) $(a+F)+(b'+F)=1+F$ iff $(a+F)+(b+F)=(a+F)$

Also $(a+F).(b'+F)=0+F$ iff
$(a+F).(b+F)=a+F$

(vi) $(a+F)+((a'+F).(b+F))=(a+F)+(b+F)$
and $(a+F).((a'+F)+(b+F))=(a+F).(b+F)$

**Proof:**

(i) We assume $(b+F)+(a+F)=(c+F)+(a+F)$
and $(b+F)+(a'+F)=(c+F)+(a'+F)$

Then $(b+F)$

$$=(b+F)+(0+F)$$
$$=(b+F)+((a+F).(a'+F))$$
$$=((b+F)+(a+F)).((b+F)+(a'+F))$$
$$=((c+F)+(a+F)).((c+F)+(a'+F))$$
$$=(c+F)+((a+F).(a'+F))$$
$$=(c+F)+(0+F)$$
$$=c+F$$

The 2nd part is the dual of 1st part.

(ii) $(a+F)+((b+F)+(c+F))$

$$=(1+F).\left[(a+F)+((b+F)+(c+F))\right]$$
$$=((a+F)+(a'+F)).\left[(a+F)+((b+F)+(c+F))\right]$$
$$=(a+F)+(a'+F).((b+F)+(c+F))$$
$$=a+F+((a'+F).(b+F))+((a'+F).(c+F)$$

Also $((a+F)+(b+F))+(c+F)$

$$=(1+F).\left[((a+F)+(b+F))+(c+F)\right]$$
$$=((a+F)+(a'+F)).\left[((a+F)+(b+F))+(C+F)\right]$$
$$=(a+F).\left[(a+F)+((b+F)+(c+F))\right]+(a'+F)$$
$$.\left[(a+F)+((b+F)+(c+F))\right]$$
$$=(a+F).((a+F)+(b+F))+(a+F).(c+F)+$$
$$(a'+F).((a+F)+(b+F))+(a'+F)(c+F)$$
$$=(a+F)+(a+F).(c+F)+(a'+F).(a+F)$$
$$+(a'+F).(b+F)+(a'+F).(c+F)$$
$$=(a+F)+(0+F)+(a'+F).(b+F)+(a'+F).(c+F)$$
$$=(a+F)+(a'+F).(b+F)+(a'+F).(c+F)$$

Hence
$(a+F)+((b+F)+(c+F))=((a+F)+(b+F))+(c+F)$

(iii) $((a+F)+(b+F))+((a'+F).(b'+F))$

$$=[((a+F)+(b+F))+(a'+F)]$$
$$.[((a+F)+(b+F))+(b'+F)]$$
$$=[(a'+F)+((a+F)+(b+F))]$$
$$.\left[(b'+F)+((a+F)+(b+F))\right]$$
$$=\left[((a'+F)+(a+F))+(b+F)\right]$$
$$.[(a+F)+((b+F)+(b'+F))]$$
$$=((1+F)+(b+F)).((1+F)+(a+F))$$
$$=(1+F).(1+F)$$
$$=1+F$$

Again,
$((a+F)+(b+F))+((a'+F).(b'+F))$

$$=\left[((a+F)+(b+F)).(a'+F)\right].(b+F)$$
$$=\left[(a'+F).((a+F)+(b+F))\right].(b'+F)$$
$$=\left[(a'+F).(a+F)+(a'+F).(b+F)\right].(b'+F)$$
$$=((0+F)+(a'+F).(b+F)).(b'+F)$$
$$=((a'+F).(b+F)).(b'+F)$$
$$=(a'+F).((b+F)(b'+F))$$
$$=(a'+F).(0+F)$$
$$=0+F$$

Hence $((a+F)+(b+F))'=(a'+F)(b'+F)$

The 2nd part is the dual of 1st part.

(iv) We have by De-Morgans Law
$((a+F)+(b+F))'=(a'+F).(b'+F)$

Hence $(((a+F)+(b+F))')'=((a'+F).(b'+F))'$
which implies that

$$((a+F)+(b+F))=((a'+F).(b'+F))'$$

The 2nd part is the dual of 1st part.

(v) Let $(a+F)+(b'+F)=1+F$

Now $(a+F)+(b+F)$

$$=(a+F)+(b+F).(1+F)$$
$$=((a+F)+(b+F)).((a+F)+(b'+F))$$
$$=(a+F)+((b+F).(b'+F))$$
$$=(a+F)+(0+F)$$
$$=(a+F)$$

Conversely, let $(a+F)+(b+F)=a+F$

Now, $(a+F)+(b'+F)$

$$=(1+F).((a+F)+(b'+F))$$
$$=((a+F)+(a'+F)).((a+F)+(b'+F))$$
$$=(a+F)+((a'+F).(b'+F))$$

$$=(a+F)+((a+F)+(b+F))'$$
$$=(a+F)+(a'+F)$$
$$=1+F$$

The 2nd part is the dual of 1st part.

(vi) $(a+F)+((a'+F).(b+F))$
$$=((a+F)+(a'+F)).((a+F)+(b+F))$$
$$=(1+F).((a+F)+(b+F))$$
$$=((a+F)+(b+F))$$

The 2nd part is the dual of 1st part.

Now we will derive DNF and CNF of some Boolean expressions in the respective quotient Boolean algebras namely $B/I$ and $B/F$.

To express the following Boolean expression in DNF over $B/I$

1. $(x+y+z+I)(xy+x'z+I)'$
$$=(x+y+z+I)\left((xy)'.(x'z)'+I\right)$$
$$=(x+y+z+I)((x'+y').(x+z')+I)$$
$$=(x+y+z+I)(x'x+x'z'+y'x+y'z'+I)$$
$$=(x+y+z+I)(0+x'z'+y'x+y'z'+I)$$
$$=(x+y+z+I)(x'z'+y'x+y'z'+I)$$
$$=(xx'z'+xy'x+xy'z'+yx'z'+yy'x+yy'z'+zx'z'+zy'x+zy'z'+I)$$
$$=0+xy'+xy'z'+yx'z'+0+0+0+zy'x+0+I$$
$$=xy'+xy'z'+yx'z'+zy'x+I$$
$$=xy'(z+z')+xy'z'+yx'z'+zy'x+I$$
$$=xy'z+xy'z'+xy'z'+yx'z'+zy'x=xy'z+xy'z'+x'yz'+I.$$

To express the following Boolean expression in DNF over $B/F$

2. $\left(x'y+(xz)'+F\right)(x+yz+F)'$
$$=(x'y+x'+z'+F).\left(x'.(yz)'+F\right)$$
$$=(x'y+x'+z'+F).(x'.(y'+z')+F)$$
$$=(x'+z'+F).x'.(yz)'+F$$
$$=(x'+z').x'.(y'+z')+F$$
$$=(z'+x').(z'+y').x'+F$$
$$=(z'+x'y').x'+F$$
$$=z'x'+x'y'x'+F$$
$$=z'x'+x'y'+F$$
$$=z'x'(y+y')+x'y'(z+z')+F$$
$$=z'x'y+z'x'y'+x'y'z+x'y'z'+F$$
$$=x'yz'+x'y'z'+x'y'z+F.$$

To express the following Boolean expression in CNF over $B/I$

3. $(x+y+z+I)(xy+x'z+I)'$
$$=(x+y+z+I)\left((xy)'.(x'z)'+I\right)$$
$$=(x+y+z+I)((x'+y').(x+z')+I)$$
$$=(x+y+z+I)((x'+y'+z.z'+I).(x+z'+y.y'+I)$$
$$=(x+y+z+I)(x'+y'+z+I)(x'+y'+z'+I)(x+z'+y+I)(x+z'+y'+I).$$

To express the following Boolean expression in CNF over $B/F$

4. $\left[(x+y').(xy'z)'+F\right]'$
$$=(x+y')'+(xy'z)+F$$
$$=(x'y)+x.y'z+F$$
$$=(x'y+x)(x'y+y'z)+F$$
$$=(x+x')(x+y)(x'y+y')(x'y+z)+F$$
$$=(x+y)(x'+y')(y+y')(x'+z)(y+z)+F$$
$$=(x+y+zz')(x'+y'+zz')(x'+z+yy')(y+z+xx')+F$$
$$=(x+y+z)(x+y+z')(x'+y'+z)(x'+y'+z')(x'+z+y)(x'+z+y')(y+z+x)(y+z+x')+F$$
$$=(x+y+z)(x+y+z')(x'+y'+z)(x'+y'+z')(x'+z+y)+F$$

**References:**


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