

Study of Residual Stresses in Welding

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For quite a long period, various researches have been carried out to study the residual stresses developed due to welding. The influence of residual stresses on the mechanical behaviour of steels, the different methods for measuring the stress and the effects of welding on residual stress generation have been studied by different research workers. Some of the relevant works¹ are described in brief below.

The amount and distribution of residual stress and distortion in a welded joint is affected by many factors such as :

- (1) Variations in welding techniques,
 - (2) Degree of constraint,
- and (3) Welding sequence.

Many studies² on the effects of these factors have already been made.

INFLUENCE OF THE RESIDUAL STRESS ON MECHANICAL BEHAVIOUR OF STEELS

Mild Steel in static tension

Presence of residual stress causes local deviation of stress-strain diagram from Hooke's law and local plastic flow occurs under external load which is well below the yield point of the material. In the case of residual stress, plastic deformation undergoes no further change for unloading from or reloading to the same applied load². Further plastic flow is only possible if the external load is increased. Thus plastic flow and relief of residual stress are closely related. Relief of residual stress occurs because plastic flow sets in. Conversely, plastic flow starts as a result of the combined effect of applied load

and residual stress. However, deviation from Hooke's law is not restricted to the zone of high residual tension ; it also occurs in the region of residual compression. Obviously no condition for premature plastic flow can exist in this region. Therefore, the deviation from Hooke's law must be regarded here as a result of stress relief caused by plastic flow elsewhere, in this case in the region of high residual tension.

The foregoing is true only when the applied load is below the yield point of the material. When the applied load exceeds the yield point, the deformation of the specimen is governed solely by the plastic properties of the base metal as would be the case of the stress-free specimen.

Mild Steel in direct and reverse bending

The general feature of stress-strain diagram as found in tensile test, is also exhibited in direct and reverse bending. As a result of residual stress, the stress-strain diagram deviates locally from Hooke's law under an applied load well below the yield point of the material. Plastic flow sets in and progresses in direct bending only when the load is increased. However, if the specimen is reversed, an additional flow will start without any increase of load, but further reversals are without influence. Obviously, the plastic flow caused by the first reversal is due to the relief of stress in parts which were only slightly affected in direct bending.

Effect of residual stress on fracture

So long as sufficient ductility on the weldment is present, the effect of residual stress on fracture is practically nil. However, high residual stress generated near the weld may act as a trigger to initiate a brittle fracture if there are sharp notches such as weld flaws and may

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cause the complete fracture of structure even when the magnitude of external stress is not so high—a fact which has been confirmed through experiments.

Crack growth⁴ is retarded by residual compressive stress and increased by residual tensile stresses. Fatigue crack growth rate is appreciably higher in the stress relieved condition than in the as-welded condition if the crack propagation is in the compressive residual stress field and vice versa when crack propagation is in the tensile residual stress field.

Effects of residual stress in buckling

Researches on the above item have been confined to long welded columns⁴. Conclusively, it can be said that under the existence of high residual stresses, the ultimate strength of a column in plastic range is reduced considerably and the initial deflection should be carefully taken into account because of its remarkable influence on the strength of the column.

METHODS OF RELIEVING RESIDUAL STRESS

Residual stresses produced during welding of rigidly fixed assemblies or pieces of large thickness can occur locally, which may be nearing the elastic limit of the parent metal⁴.

Before the utilisation of such an assembly and if the same is subjected to external loading, the superimposition of the two types of stresses can become dangerous. It may thus be necessary to relieve the residual stresses by suitable methods.

Well known methods to relieve residual stresses are :

- (1) Stress Relief Annealing,
- (2) Low Temperature Stress Relief,
- (3) Peening,
- (4) Ultrasonic Stress Relief.

Such relieving treatments other than peening are expensive. Furthermore, had there been any crack developed earlier due to the existence of residual stresses, the same cannot be made good.

From this standpoint, it would be immediately felt that prevention of the generation of residual stresses would definitely be a better and a radical method than the post welding stress relieving treatments. In order to arrive at the goal, some means must be found out to

determine the residual stress in magnitude and direction in connection with welding process.

DETERMINATION OF RESIDUAL STRESS

The estimation of residual stresses in arc welding is complicated from analytical considerations⁵. However, the problem may be simplified to a good degree of approximation with the help of dimensional analysis. The model thus obtained, in terms of the important controlling parameters has already been verified physically by researches.

The determination of residual stresses in arc welding analytically is a difficult mathematical problem but an approach to the problem can be made with the help of dimensional analysis.

Practically, it is not possible to find the effect of any one of the welding parameters keeping the others constant. To overcome the difficulty, a set of dimensionless combination of residual stresses^{5,6} and other important welding parameters may be formed.

In doing so, the following functions are obtained :

$$\frac{RD}{w} = f\left(\frac{H}{wD}\right)$$

and $\frac{RB}{w} = f\left(\frac{H}{wB}\right)$

where R = Residual stress due to welding, kgf/cm².

D or B = Diameter or Width of specimen, cm.

w = Weight of weld metal per unit length of weld, kgf/cm.

and H = Heat input per unit length of weld, kgf.

MEASUREMENT OF RESIDUAL STRESS

Residual stresses can be measured by different methods which are classified into following three major fields :

- (1) Non-destructive method,
- (2) Semi-destructive method and
- (3) Destructive method.

NON-DESTRUCTIVE METHODS

X-Ray method

The technique of measuring residual stress by means of X-rays is based on the fact that X-rays are diffracted from the atoms in metals. From the examination of

the diffraction phenomena in an actually stressed material, the component of strain in any desired direction can be readily computed. Thus it is not necessary to make measurements in the unstressed state as and when using other methods to actually calculate the stress. This circumstance makes the X-ray diffraction method particularly designed for measuring the residual stress. Another advantage of the X-ray technique is that diffraction of X-rays requires only a very small volume of material. Like other methods, this method also measures stresses only at the surfaces.

The fundamental theory of stress measurement by means of X-rays is based on the fact that the interplanar spacing of the atomic planes within a specimen is changed when subjected to stress.

The basic expression when using the X-ray method can be derived using principles of theory of elasticity⁷.

$$\sigma_{\phi} = \frac{E}{1+\mu} \cdot \frac{1}{d_0} \cdot \frac{\delta d_{\phi, \psi}}{\delta \sin^2 \psi}$$

where,

- ϕ = direction of measurement on the specimen surface
- ψ = angle between diffraction planes and surface
- E = Young's modulus
- μ = Poisson's ratio
- d_0 = interplanar spacing in unstressed condition
- $d_{\phi, \psi}$ = interplanar spacing in direction determined by ϕ and ψ

X-ray strain measurements have frequently been shown to be useful in very different fields of applied and basic research. Uptill now, however, the equipment necessary for accurate measurements has limited its use to small specimens. Furthermore, in some materials, such as with plastically deformed steel, interpretation of the results may be difficult.

Indentation method

Several means of using hardness measurements for the determination of residual stress have been proposed, based on the principle that the hardness of metal parts depends on stresses acting on those parts. Investigations conducted on steel indicated that the relationship between stress and change of hardness is practically linear as long as those stresses are within the linear

range. Furthermore, it is also seen that the change of hardness is greater for tensile than compressive stresses. Using the same principle⁸, biaxial residual stresses can be measured utilising the Knoop indenter. The change in hardness ΔH is defined as,

$$\Delta H = \frac{H_s - H_0}{H_0}$$

where,

- H_0 = hardness of unstressed state
- H_s = hardness of stressed state

The biaxial residual stresses are determined from the following equations suggested by Oppel.

$$\sigma_1 = \frac{E}{2} [A (\Delta H_1 + \Delta H_2) + B (\Delta H_1 - \Delta H_2)]$$

$$\text{and } \sigma_2 = \frac{E}{2} [A (\Delta H_1 + \Delta H_2) - B (\Delta H_1 - \Delta H_2)]$$

where A and B are empirically derived constants and the subscripts 1 and 2 indicate the principal stress directions.

The hardness-test method is nondestructive and has a special application in the range of nonlinear material behaviour. It is, however, limited to materials for which initial hardness is known and its accuracy is dependent on numerous factors.

SEMIDESTRUCTIVE METHODS

Gunnert's method

In this method, stress relieving is achieved by trepanning a groove round the gauging area by means of a core drilling⁹. The core drilling is guided by drilling a small hole in the center of the measuring surface and by using a spring guide.

The method has the advantage of evaluating local residual stresses close to the yield point such as residual stresses due to welding.

The trepanning method accomplishes relaxation by removing a plug of metal containing gauges by drilling a series of overlapping holes. If the directions of the principal stresses are not known, strain gauges are arranged in rosette form. Residual stresses are calculated based on the initial and final strain-gauge readings and using principles of strength of materials. The method can be reliable when the stresses are fairly uniform over the area to be measured.

Subdivision method

This method, commonly used for measuring welding stresses, consists of marking gauge points, a known distance apart, on lines parallel and perpendicular or radially or tangentially, with respect to the completed weld. The distance between the gauge points are measured by means of measuring instruments like extensometer. The metal containing the gauge points is removed from the welded part by machining. All forces acting on this strip of metal due to its surroundings having thus been destroyed, the strip becomes essentially free from stresses. The distance between gauge points is again measured. If the distance has increased, then the strip was originally under compression; if the distance has decreased, there must have been no tensile stress acting along the line between the gauge points. The extensometer readings are converted to stress by means of a calibration secured on a specimen under known loads.

DESTRUCTIVE METHOD

Sachs' method

In this method, residual stresses are determined by removing concentric layers from a cylindrical rod or tube and measuring the resulting elongation or contraction. With sufficient number of steps, the longitudinal stress distribution is determined.

The method is, however, an approximation since only longitudinal stresses are considered. The presence of transverse and radial stresses that would be present in the general case are ignored. These limitations were recognised by Mesnager¹⁰ wherein he proposed the removal of the material from the center of the cylindrical rod or tube and measuring the longitudinal and circumferential strains in the remaining portion. Sachs¹¹ greatly simplified the calculation and to-day the method is popularly known by his name.

Sachs' equations for the determination of the longitudinal, tangential and radial residual stresses in an axially symmetric cylinder are given by,

$$\sigma_e = \frac{E}{1-\mu^2} \left[(A_e - A) \cdot \frac{d\lambda}{dA} - \lambda \right]$$

$$\sigma_t = \frac{E}{1-\mu^2} \left[(A_e - A) \frac{d\theta}{dA} - \frac{(A_o - A)\theta}{2A} \right]$$

$$\text{and } \sigma_r = \frac{E}{1-\mu^2} \left[\frac{(A_o - A)\theta}{2A} \right]$$

where,

E = Young's modulus

μ = Poisson's ratio

A_o = Original cross section of cylinder

A = bored area

λ = $\epsilon_e + \mu\epsilon_t$

θ = $\epsilon_t + \mu\epsilon_e$

ϵ_e, ϵ_t = strains in longitudinal and circumferential directions.

MODIFICATION OF SACHS' 'BORING OUT' METHOD

A. G. Cepolina and D. A. Carronica¹² modified Sachs' method for determining residual stresses. This technique was used for measuring the residual stresses present in gas tungsten arc welds made in Hastelloy N, an alloy that is being employed in the Molten Salt Reactor Experiment.

The specimen was 1/2 in. thick plate, 12 in. in diameter. Circular welds 6 in. in diameter were simultaneously deposited on both flat faces. The equations applicable for the calculation of residual stresses in this specimen geometry are follows:

$$(\sigma_r)_{\bar{R}} = \frac{E\epsilon_T}{2} \left(\frac{R_o^2}{R^2} - 1 \right)$$

$$\text{and } \sigma_T = \sigma_R + \frac{R d\sigma_R}{dR}$$

where, $(\sigma_r)_{\bar{R}}$ is the residual radial stress at any specific location (\bar{R})

σ_T = tangential radial stress

E = Young's modulus for the disc material

ϵ_T = tangential strain

R_o = outside radius.

The maximum value of tangential residual stress was measured to be 50,000 psi and was not particularly affected by either the shielding gas or heat input. Stress relieving at 160°F for 4.5 hr. proved to be the optimum heat treatment and reduced the tangential residual stress to about 5,000 psi. Lowering of the maximum residual stress to about 10,000 psi. was achieved at 1400°F after 6 hr.; however, lower temperatures even for times as long as 100 hr. only reduced the maximum residual stress by about 25%.

RESIDUAL STRESS IN OXY-ACETYLENE WELDS

Without restraint

Investigations of the residual stress in oxy-acetylene welded plates free to move during welding, have been made by Bellenrath¹³, Mies¹⁴, Bierett and Gruning¹⁵

and Siebel and Pfender¹⁶. Bollenrath used the Mathar method. In his experimentation, maximum residual tensile stress of 76,000 psi parallel to the weld was found a little beyond the weld, and 36,000 psi perpendicular to the weld.

Mies measured residual stress in free gas welded plates. The plates to be welded were machined from full annealed, 0.02% carbon steel originally 0.79 inch thick and 0.16 inch was machined from each side and surfaces were ground to avoid initial surface stresses. The plates were 24 inches (welded edge) \times 28 inches and were oxy-acetylene welded with 0.16 inch rod in 54 minutes (with 4 rest periods of about 3 minutes each). The weld was 60°V, not tacked, 0.12 inch shoulder, with an allowance of 40 minutes for angular distortion. The stress of 28,500 psi was everywhere exceeded at some time during welding and cooling. The residual stress in the weld itself was not measured. Equilibrium calculations indicated that stresses perpendicular to the weld exceeded 43,000 psi (tension) at the edges. Compressive stresses parallel to the weld were 14,000 to 17,000 psi.

Siebel and Pfender measured residual stresses in a continuous 90° double V oxyacetylene weld in 24 inches long mild steel plates 8 inches wide, 1.18 inch thick. The distribution of residual stress was unaccountably complex, the maximum values being 28,500 psi compressive perpendicular to the weld and 14,000 psi compressive parallel to the weld. The starting end of the weld had a residual tensile stress of 28,500 psi. The other end had a compressive stress of equal magnitude.

Bierett and Gruning carried out experiment with specimen oxy-acetylene welded with neutral flame and observed that the maximum residual stress in the weld and parallel thereto was 17,000 psi tensile at the centre.

With restraint

Buhler and Lohmann¹⁷ carried out experiments with disc of soft steel and low alloy structural steel, oxy-acetylene welded made under conditions of more or less restrained shrinkage. The discs were of 9.8 or 19.6 inches outside diameters having circular patches 5.9 inches diameter as shown in Fig. 1. The stresses were found out using Sachs' method.

In mild steel plate of 9.8 inches outside diameter, maximum tangential tensile stress on weld was 4,300 psi with right hand welding and that for 19.6 inches outside diameter 20,000 psi while the radial tensile stresses were 1,400 and 20,000 psi respectively.

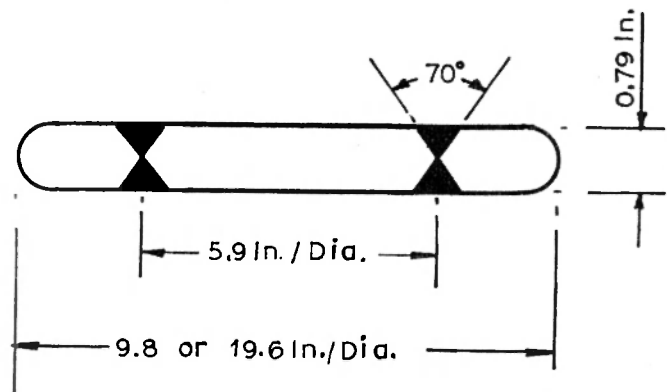


Fig. 1. Disc specimen used by Buhler and Lohmann

The residual stresses on oxy-acetylene welded butt joint in the form of circular patch, have also been determined by Bierett and Gruning. The specimen consisted of a patch 8 inches in diameter, left hand welded from one side in the centre of mild steel plate 80 inches square. Average residual stresses, both radial and tangential, so found out are shown in Fig. 2.

Von Rosler¹⁸ measured residual stress perpendicular to the weld in single-V oxy-acetylene butt joint. Residual stresses parallel to the weld were measured by observing the change in width of the plate by means of a dial gauge as the weld was gradually cut out by slitting saw. For rigidly wedged specimen which was right hand welded, a tensile stress parallel to weld of 19,200 psi and tensile stress of 36,400 psi perpendicular to the weld, were noted.

RESIDUAL STRESS IN ARC WELDS

Without restraint

A comprehensive investigation, using Mathar method in butt welds in free plates, has been made by Bollenrath¹⁹. The plates were of structural steel having a dimension of 12 inches \times 24 inches of different thickness. 60° V-welding was done on 24 inches edge. The plates were annealed by heating the same to 850°C for 8 hours and subsequent cooling in furnace for 24 hours prior to welding. The maximum stresses, parallel and perpendicular to weld, as found were 1,35,000 and 33,000 psi in case of plates having a thickness of 0.59 inch welded with covered electrodes in two-layers with 0.16 inch electrodes using 120 amperes and three layers with 0.20 inch electrodes using 140 amperes. In order to explain the extraordinarily high elastic stresses in the weld, Bollenrath assumed that multiaxial stress condition affected a rise in the yield point of as much as 90%. Since it was difficult to believe that biaxial stress alone

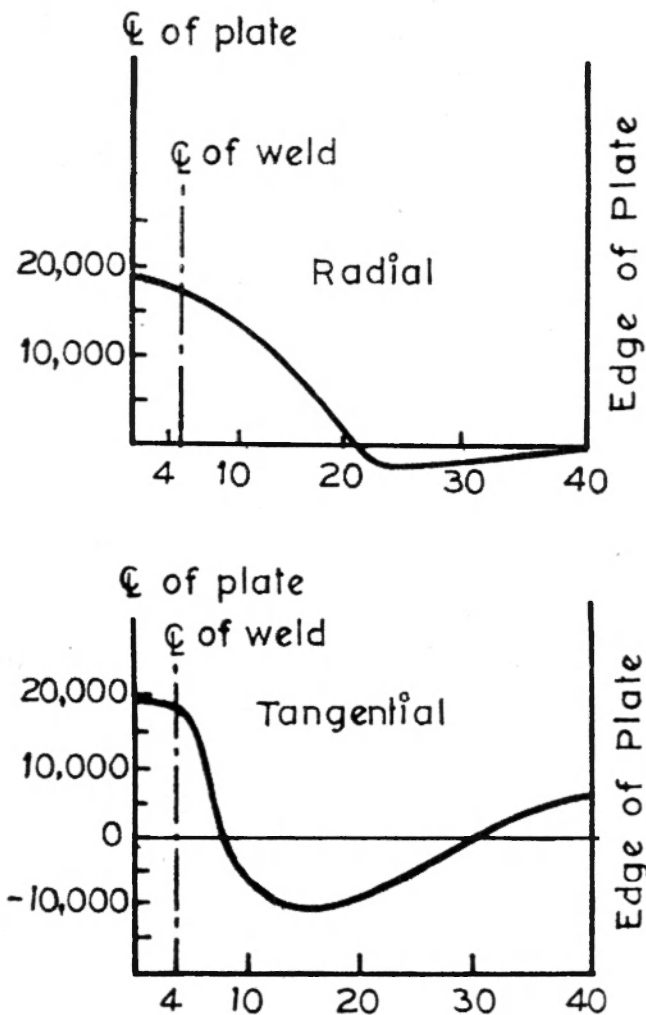


Fig. 2. Average residual stresses along a radius of patch welded disc

could be responsible for such a rise, Bollenrath inferred that the rise was occasioned by triaxial stress, realising that the results of the Mathar method were valid only if inelastic action did not take place, he carried out further experimentation where the stresses were calculated by using modified Mathar method to take care of plastic action. The maximum residual stress parallel to the weld was found to be 28,500 psi tensile and that perpendicular to the weld 43,000 psi tensile in case of structural steel plate specimen of 0.59 inch thickness, arc-welded with 0.16 inch covered electrode in three layers, step-back welding using 160 amperes.

Arc welded butt joints were also studied by Siebel and Pfender, Bierett, Theisinger²⁰, Akeson²¹, Mies, Reinhard and Matting²². All of these investigators used the subdivision method. Siebel and Pfender used 90° X-welds in 28 inches long mild steel plates, 8 inches wide, 1.18 inches thick, having a yield point of 36,000

to 37,000 psi, tensile strength of 59,000 to 60,000 psi. Bare electrodes of 0.16 inch diameter using 230 amperes, were deposited in a single layer in each side, the layers being continuous or stepped back in five steps. Deformations parallel and perpendicular to weld as a result of subdivision of the entire plate into strips $1\frac{1}{8}$ inches wide were measured by means of a wedge extensometer accurate to 0.00002 inch in 0.79 inch length. Deformations were reduced to stress by means of Young's modulus and Poisson's ratio. The residual stresses in both welds were maximum in the vicinity of the weld.

Very elaborate measurements on a single free arc welded specimen, were made by Mies; the plate was 24 inches (welded edge) \times 28 inches and was machined and ground to 0.47 inch thickness from an original thickness of 0.71 inch and was annealed by heating the same to 600°C to 650°C for 2 hours with subsequent furnace cooling. The weld was 60°V, 0.12 inch shoulder with an allowance for 10 minutes distortion. The weld was made in three layers, 12 minutes for each layer, total welding time being 40 minutes, using 0.16 inch bare electrodes, 17 volts, 145 to 150 amperes. Specimen was tacked over a distance of 0.39 inch at ends and middle. The maximum residual stresses appeared in the weld averaging 36,000 psi tensile, but not exceeding 45,000 psi tensile locally, in the direction parallel to the weld. Stresses in the weld perpendicular to the same were tensile in the middle, maximum being 15,200 psi.

Bierett also determined the residual stress in annealed mild steel plates 40 inches long, 12 inches wide, 0.47 inch thick, free to move during continuous welding on the 12 inches edge with bare mild steel electrode with 120 amperes D.C. for first layer, 160 amperes for second and third layers. A root gap of 0.12 inch and 60°V preparation were used. The residual stress perpendicular to the weld along the centre line of the same was noted to be 14,000 psi at the middle. The residual stress in the weld parallel to the same was, of course, zero at the edges rising uniformly to 30,000 psi in the middle of the weld.

Residual stresses varying from 3,000 to 20,000 psi were measured by Reinhard²³ in a 90°V weld with 0.08 inch root spacing, 60 inches long mild steel plate 0.55 inch thick, 24 inches wide, free to move during welding.

Theisinger studied 90°V weld in 36 inches long, 12 inches wide, $\frac{7}{16}$ inch thick as-rolled structural steel tacked at ends and middle. The weld was made in eight layers manually, using $\frac{3}{16}$ inch coated electrode, 200 amperes, 45 volts, each layer being deposited in the opposite direction to the preceding one. Each layer was

brushed and the completed weld was cooled in air. The maximum residual stress parallel to the weld was 42,000 psi tensile on the weld itself. The stresses perpendicular to the weld are not indicated.

Akeson found out the residual stresses in double V and unbevelled butt weld, 20 inches long in mild steel plates, 6 inches wide, 0.39 inch thick. The double V weld was made in three layers on each side with 0.16 inch coated electrodes. The same type of electrode, but 0.20 inch diameter, was used in making the unbevelled weld, one layer on each side. Residual stresses perpendicular to the weld was 37,000 psi at the mid-point of the double V weld and 38,500 psi at the mid-point of the unbevelled weld, both being tensile.

Paton²⁴ used the subdivision method in determining the effect of plate thickness on residual stress parallel to 80°V arc welds in mild steel plates.

In addition to the investigation on the residual stresses by means of Mathar and Subdivision method, several attempts have been made to estimate residual stresses in unconstrained arcwelded butt joint by measuring the change between two gauge points on plate before and after welding. If all the metal within the gauge length under consideration is known to have remained below the yield point during welding, the change in the gauge length then provides a value of strain that may be used directly for calculation of residual stress. Since it is known that maximum residual stresses in welds occur in just those regions near the weld, which undergo inelastic action, the above direct method can not be used to determine that residual stresses.

Nevertheless, Nikolaev²⁵ and Sarazin²⁶ had employed direct method to measure residual stresses in arc welded butt joints. The maximum tensile stress usually appeared on the gauge length nearest to the weld.

With Restraint

Mies studied an annealed specimen welded with 0.16 inch bare electrode at an arc voltage of 20 using 160 amperes. The results thus obtained, demonstrated that the principal stresses were parallel and perpendicular to the welds. Maximum stress of 25,300 psi tensile occurred in the weld. Comparing the results of the arc weld made with and without restraint. Mies found that the stress distribution in the weld and in its vicinity was not very different. In both cases, the maximum tensile stress occurred in the weld and was parallel thereto; this stress was 23,000 psi. The average tensile stress perpendicular to the weld on the weld itself was measured to

be 18,400 psi. He estimated his overall accuracy of measurement to be $\pm 1,520$ psi.

THERMAL STRESS AND RESIDUAL STRESS

Several investigators tried to predict residual stresses from the consideration of thermal stresses. Watanabe and Satoh²⁷ made such an investigation on circular specimen of 200 mm diameter and 6 mm thickness, which were bead welded over the central portion using electrode of 4 mm diameter, for ten seconds. Residual stresses were measured using Sachs' method. Stresses were calculated on consideration of thermal cycles. The stresses so found out were a bit smaller than the experimental value.

Koch and Freiherr²⁸ carried out an experiment to define residual stresses caused through welding by calculating the field of thermal stresses. The pre-requisites for a valid application of this method of calculation have been stated and a comparison made with the results obtained by experimental means.

CONCLUSION

The effect of residual stress on a component is one of the most troublesome problems in the practical field. The problem is more acute in case of welded structures where so many service failures occur due to the pressure of residual stresses. So it has always been the endeavour of engineers and scientists to control the generation of residual stresses upto the zero level but if not possible, then at least within a safer limit. Much research work has been done and is still being done in this field. Though complete elimination of residual stresses is not feasible yet, it is possible to control within a safe limit by stress-relieving and other methods. But the main problem is how much of the residual stress is to be relieved. It requires the knowledge of the magnitude and direction of the residual stresses present and its nature whether tensile or compressive. For this, a reliable method is needed. A number of methods have been developed for the measurements of residual stresses. But most of them are destructive by nature. Regarding non-destructive method, X-ray technique is no doubt a useful approach but the technique along with the instrument is very costly and it is not adaptable to the shop-floor applications. The other non-destructive methods are still in their nascent stages.

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