RESIDUAL STRESS ANALYSIS IN WELDMENTS - THEORETICAL APPROACH

by

S. Murugan, P.V. Kumar, Baldev Raj & M.S.C. Bose*

Metallurgy and Materials Group
Indira Gandhi Centre for Atomic Research, Kalpakkam, India

*Department of Mechanical Engineering
Indian Institute of Technology, Powai, Bombay, India

INTRODUCTION

Residual stresses are self equilibrating stresses existing in materials or components under uniform temperature conditions. When two pieces of plates/pipes are joined together by welding, localised residual stresses coupled with shrinkage are generated in the vicinity of the weld. The presence of these residual stresses can be detrimental to the strength of the joint. Tensile residual stresses are generally detrimental, increasing the susceptibility of a weld to fatigue damage, stress corrosion and fracture. During welding, temperature conditions range from melting point of the material to room temperature. Mechanical and thermal properties of the material are temperature dependent and this change with temperature during the welding process. The material stress-strain behaviour is elastic-plastic and temperature dependent. Thermal stresses are produced in the material during the process of heating and cooling. When the material is cooled to room temperature, the locked up stresses present in the material are retained residual tresses.

A comprehensive paper on the determination of residual stresses in weldments by experimental methods has been published [1]. The present paper is complement to that paper and deals mainly with the theoretical methods of determining the residual stresses in weldments.

Definition of Residual Stress

Since the residual stress are self equilibrating stresses, the resultant moment produced by them must be zero. The residual stresses can generally be classified into residual stresses of 1st. 2nd and 3rd kind. these are shown in **Fig. 1** [2].

Residual stresses of the 1st kind are nearly homogeneous across large areas say, several grains of

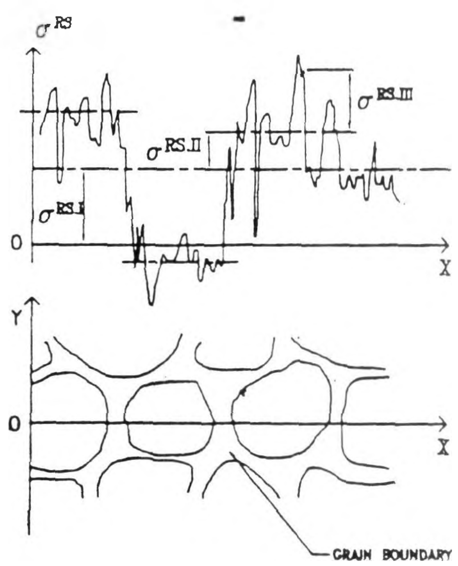


FIG 1 : TOTAL RESIDUAL STRESS DISTRIBUTION ALONG SEVERAL GRAINS OF A POLYCRYSTAL (SCHEMATICALLY) AND THEIR SEPERATION IN 1st, 2nd AND 3rd KIND RESIDUAL STRESSES [2]

a material and are in equilibrium over the bulk of the material. Residual stress of 2nd kind are nearly homogeneous across microscopic areas, say on grain or parts of a grain of a material and are equilibrated across a sufficient number of grains.

Residual stress of 3rd kind are homogeneous across submicroscopic areas of a material, say by some atomic distances within a grain and are equilibrated across small parts of a grain. Usually a superposition of residual stresses of the 1st, 2nd & 3rd kind determines total residual streess acting at a particular point of a material or component.

The classification of residual stresses discussed above corresponds with the commonly used subdivision of residual stresses into macro residual stresses and micro residual stresses. Macro residual stresses are of 1st kind residual stresses. Generally these are of more practical importance. Micro residual stresses can be regarded as combination of residual stresses of 2nd & 3rd kind.

DIFFERENT SOURCES OF WELDING STRESSES

Shrinkage Residual Stresses

An important source of residual stresses which is present in fusion welded joints is the shrinkage of hot zone, which is impeded by colder zones which shrink by a very little amount.

In a single pass fusion welded plate, the longitudinal and transverse residual stress distributions along the weld line (Y-axis) and along a line perpendicular to the weld line (X-axis) are shown in **Fig. 2** [3]. the longitudinal residual stresses. that is the stress component parallel to the weld line, are tensile stresses within the weld and in the heat affected zone. These are maintained in equilibrium by compressive longitudinal stresses in zones farther away from the weld (X-axis). If the weld line is sufficiently long (Y-axis), the tensile longitudinal stresses in the centre of the weld have a constant value a certain distance. Towards the ends of the weld line, they fall to zero. This longitudinal stress distribution along the weld is responsible for the distribution of transverse strains represented schematically on the far left of Fig. 2. These are equal to zero at the ends of the weld line and increase towards the middle. This means that the transverse strains are hindered along the length of the weld line and thus even if no external forces or restraints impede the transverse shrinkage, residual stresses must arise transverse to the weld line. The distribution of these transverse residual stresses is also represented schematically in Fig. 2. Their magnitude is consistantly smaller than that of the longitudinal stresses. The maximum value of shrinkage stresses in the centre of the seam depends on many factors. These are given in the **Table 1** [3]. The direction of arrows indicates whether the shrinkage stresses increase as the influencing factor increases or decreases.

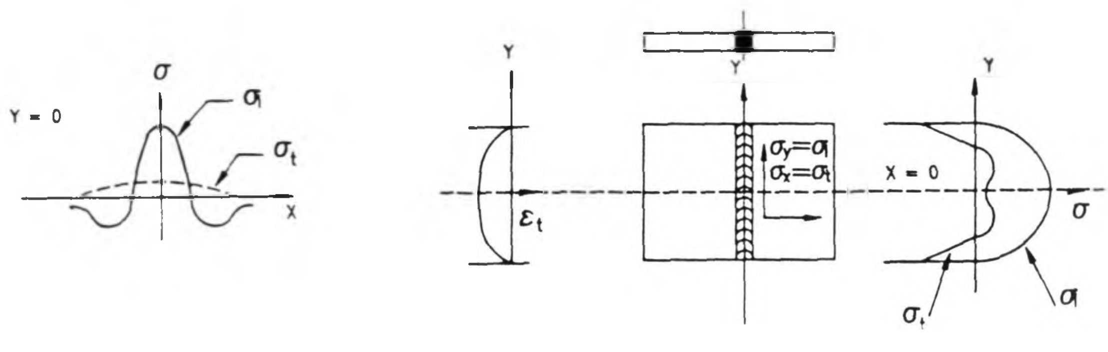


FIG 2 : DISTRIBUTION OF LONGITUDINAL (σ_l) AND TRANSVERSE (σ_t) RESDUAL STRESSES AND OF TRANSVERSE STRAINS (ε_t) ALONG THE Y AND X-AXIS IN A SINGLE-PASS WELDED SHEET (A TYPICAL CASE) [3]

Transformation Residual Stresses

Transformations associated with a change in volume can give rise to residual stresses if parts of the weld and of the heat affected zone which reached a high temperature pass through a transition during the course of cooling while less heated regions do not pass through. The transformation of austenite to ferrite, bainite or martensite in known to involve an increase in the volume. Therefore, in case of welding of steel, compressive stresses would arise in the transformed region if phase transformation were the only stress generating process and if it takes place simultaneously in the entire region which has reached a sufficiently high temperature. **Table 2** [3] shows how different factors would affect the magnitude of these transformation induced residual stresses.

In reality, however, the interaction of the shrinkage process and the transformation process has to be taken into account. As a consequence of the combined effect of both processes, tensile or compressive stresses can arise in the weld seam.

RESIDUAL STRESSES IN SINGLE PASS WELDS

Interaction of various stress generating sources yield complex residual stresses even in single pass welds. Simple models that additively superpose different kinds of residual stresses yield the first qualitative explanations

TABLE 1
Effect of Various Factors on Residual Stresses due to Shrinkage [3]

Factors influencing shrinkage residual stresses
The magnitude of the tensile residual stress at the weld centre line increase if the following factors increase (▲) or decrease (▼):

Material Parameters
Coefficient of thermal expansion ▲
Young's Modulus ▲
Yields Strength ▲

Geometry
Length of the seam (influence on σ_l and σ_t is different);
Restraint of the weld seam ▲
Shape and build up of weld seam;
Sheet thickness ▲

Welding Process
Width of seam ▼; width of HAZ ▼;
Number of layers and shape of bead
Heat inputs ▼; interpass temperature ▼;
Preheat temperature ▼; welding sequence

TABLE 2
Effect of Various Factors on Residual Stresses due to Phase Transformation [3]

Factors influencing transformation induced stresses

The magnitudes of the compressive transformation stresses increase if the folowing factors increase (▲) or decrease (▼):

Material Parameters
Transformation temperature ▼
Young's Modulus ▲

Geometry
Restraint of the weld seam ▲
Sheet thickness ▲

Welding Process
Width of the zone with $T > T_t$ ▼;
Heat inputs ▼;
Cooling rate ▲

of experimentally observed residual stress distributions which deviate from the basic pattern shown in Fig. 2 in possessing additional maxima and minima. But for a complete and accurate description of the production of the welding stresses, this simple addition of different kinds of stress is inadequate. The interaction of the various stress producing processes during the entire cooling process must be taken into account. If no external forces or restraints impede the transverse shrinkage, transverse residual stresses in single pass welds are solely a consequence of the inhomogeneous distribution of the longitudinal residual stresses.

Fig. 3 shows the schematic representation of changes in temperatures and stresses for a single pass, butt weld joint when welding arc is moving along X-axis [4]. O is the origin representing the present position of the arc. Section AA is ahead of welding arc. Section BB is crossing the origin where the metal is at the molten condition, Section CC is at some distance behind the welding arc, and Section DD is far behind the welding arc. Fig. 3b shows the corresponding temperature distribution change in each section and Fig. 3c shows the distribution of residual stresses along the cross sections. The shaded portion in Fig. 3a represents the areas where plastic deformations have taken place due to welding process.

Since the section AA is far ahead, the temperature changes due to welding are zero and correspondingly there is no residual stress formation. Along Section BB, the temperature distribution is very steep (Fig. 3b-2). The corresponding residual stress distribution is shown in Fig. 3c-2. Origin O contains molten metal and cannot support residual stresses. However, due to the presence of high temperature on both sides of the origin, the adjacent material experiences compressive residual stresses. To balance the compressive stresses in the region near the weld, tensile stress form in the regions away from the weld. Along Section CC, the temperature distribution is shown in Fig. 3b-3. The molten metal has cooled and the resulting shrinkage has been restrained by the adjacent material. Therefore tensile residual stress is present at the weld centre. At Section DD, the temperature has cooled down to room temperature. Therefore, high tensile residual stresses are produced in areas near the weld, while compressive residual stresses are produced in areas away from the weld. The distribution of residual stresses that remain after welding is completed, is shown in Fig. 3c-4.

The above explanation pertain to stresses acting parallel to the welding direction. However, stresses acting in the direction perpendicular to weld axis, and those stresses acting across the thickness are also to be considered for complete analysis.

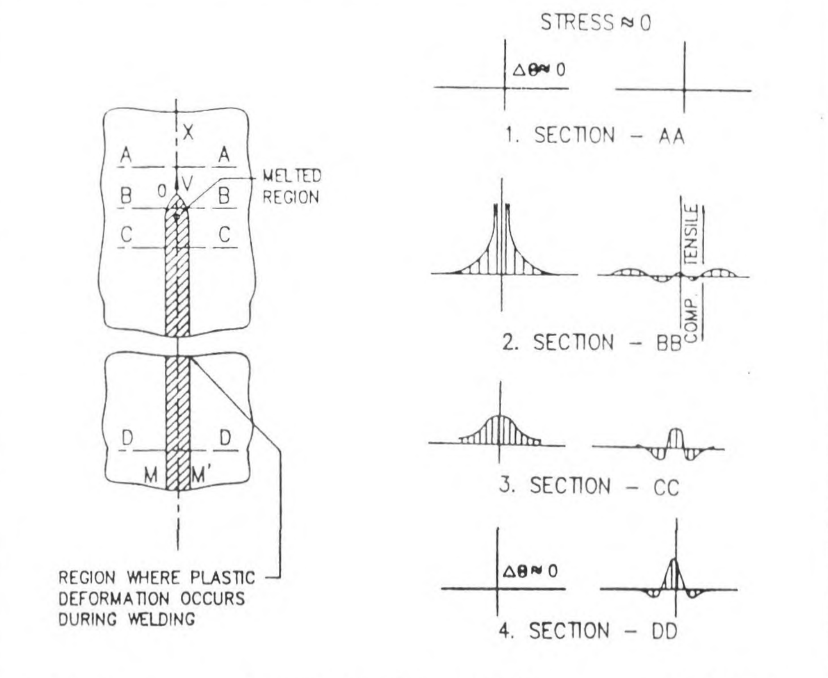
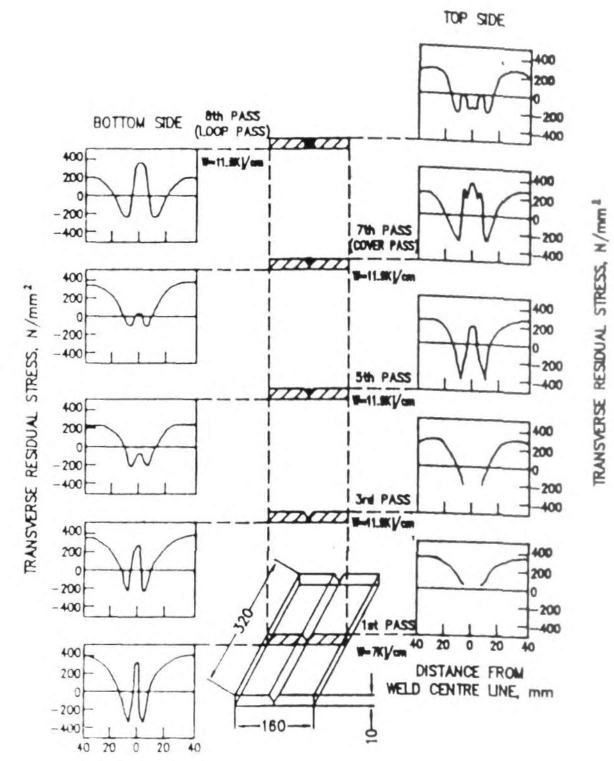


FIG 3 : SCHEMATIC REPRESENTATION OF CHANGES IN TEMPERATURE AND STRESS DURING WELDING [4]

RESIDUAL STRESS DISTRIBUTION IN MULTIPASS WELDS

Knowledge of the residual stresses in multiple pass welds is important in view of diverse applications of thick welded plates. On account of the diversity of the welding conditions, seam geometries and welding procedures and their effect on the nature of the internal stresses, the experimental or computed results of stress determinations in multiple pass welds can turn out to be very different (3,5). It is appropriate to consider at first the characteristic features of the formation of shrinkage stresses and transformation stresses in multiple pass welds.

After the first pass of a multipass weld, transverse shrinkage as well as transverse volume expansion due to phase transformation (if applicable) are directly and indirectly and increasingly hindered with such successive pass. As a result, the magnitudes of the transverse residual stresses can become as high as the magnitudes of the longitudinal stresses, for instance, as high as the yield point of material if shrinkage stresses have a dominating effect. Besides the distribution of residual stress transverse to the weld, may differ considerably from the residual stress distribution in single pass welds. In multipass welds, the new weldmetal fuses a small region surrounding it and relieves the residual stress which is already present. After redistribution of the residual stress, the characteristics of the succeeding phenomena are considered as a single pass weld although some part of the joint may be subjected to loading and unloading in the plastic range. **Fig. 4** illustrates a typical residual stress distribution in multipass welds.



DISTANCE FROM WELD CENTRE LINE, mm

FORMATION OF TRANSVERSE RESIDUAL STRESS IN A TIG-WELDING PLATE (StE 690), W=HEAT INPUT

FIG 4 : DISTRIBUTION OF TRANSVERSE RESDUAL STRESSES ON THEBOTTOM SIDE AND ON THE TOP OF A MULTI-PASS WELDED SHEET AFTER WELDING OF DIFFERENT PASSES, FINE-GRAINED STRUCTURAL STEEL StE 690, SHEET THICKNESS 10mm [3]

RESIDUAL STRESS DISTRIBUTION IN PIPES

In the case of welded pipes, the residual stress distribution is largely affected by the material, pipe diameter, the pipe thickness, heat input and the size of the weld bead compared to the pipe thickness [6]. Many of the pipes that are used in the pressure vessel and piping industry are schedule 80 pipes. For this size of pipe, the conventional girth welds generally lead to high tensile residual stresses on the inside surface of the pipe. The weld induced inner surface residual stresses can be at the level of the yield stress of the material. Thicker pipes (schedule 160 pipes) generally have less tensile, or compressive, residual stress on the inner surface of the pipes than pipes that of schedule 80 or lower schedules. The high tensile stresses in the inner surface of pipes have been associated with cracking in welded pipe and are of concern to the industries. Intergranular stress corrosion cracking is influenced by the presence of high tensile stresses. To reduce tensile stresses and to create compressive residual stresses, various methods have been developed including heat-sink welding, last pass heat-sink welding, back-lay welding and induction heating [6].

THEORITICAL ANALYSIS

The theoretical determination of residual stresses in weldments is a complex problem. Many variables such as efficiency of the welding arc, metallurgical, mechanical and physical properties of the material etc. play an important role. The mathematical formulation of both heat transfer and stress analysis aspects are highly non-linear and time dependent thus adding complexity to the theoretical models.

The determination of the thermomechanical effects of welding is generally carried out in the following steps [7] :

1. Analysis of heat flow
2. Analysis of transient thermal stresses during welding
3. Determination of incompatible strain after the weldment cools to the initial temperature
4. Determination of residual stresses and distortion due to incompatible strains.

Finite element method is widely used to find out the residual stress distribution in the welded material.

Thermal analysis

Analytical Method

Rosenthal's [8-11] classical solution of solving the heat transfer equation for moving heat sources is applied to find the temperature distribution in the welded plates/pipes. The general three dimensional partial differential equation of heat conduction in solids is given by equation (1)

$$\frac{\delta}{\delta x}\left(K_x\frac{\delta T}{\delta x}\right)+\frac{\delta}{\delta y}\left(K_y\frac{\delta T}{\delta y}\right)+\frac{\delta}{\delta z}\left(K_z\frac{\delta T}{\delta z}\right)+\dot{Q}_g=\rho c\frac{\delta T}{\delta t} \quad \ldots\ldots\ldots (1)$$

Where T = temperature (K)

t = time (sec)

Qg = rate of heat generated per unit volume (W/m^3)

C = specific heat (J/KgK)

p = density (Kg/m^3)

x,y,z = Cartesian coordinates

The above equation is solved for a moving heat source and the temperature distributing due to this moving heat source in an infinite plate is given by the following equation (2) :

$$T=T_o+\frac{Q}{4\pi KR}\exp\left(-\left|\frac{V\xi}{2\alpha}+\frac{VR}{2\alpha}\right|\right) \quad \ldots\ldots (2)$$

where T = temperature (K)

To = ambient temperature (K)

Q = rate of heat input (W)

K = thermal conductivity (W/mK)

V = speed of heat source (m/sec)

ε = X-distace from heat source (m)

R = distance from the heat source (m)

α = thermal diffusivity (m^{2}sec)

Quasi-stationary state is assumed in the above relationship. Time does not appear explicitly as a variable in the above equation because although the temperature distribution is time dependent with respect to a stationary point in the solid, it is unchanging with respect to the location of the heat source since steady state conditions are assumed. Time appears implicitly in equation as $\varepsilon = Xo - Vt$ where Xo is the distance between the stationary point and the heat source when $t = 0$.

The computational model approximates the temperature rise due to the moving heat source in a finite thickness plate by superposing a series of heat sources. As long as the conductivity and heat capacity are assumed to be temperature independent, the consideration of linear and superposition principle is valid. This enables the temperature distribution in an infinite plate of a finite thickness to be found by superpositioning a series of heat sources which simulate perfect insulation of plate surfaces. One heat source is located in the plate or pipe representing the actual heat source. Other heat sources are placed outside of the plates or pipes to eliminate the heat transfer through the surfaces. The required number of auxilary sources depends on the heat input and thickness of the material.

Finite Element Method

Finite element method for the analysis of transient two-dimensional heat conduction problems to determine resulting transient stresses and distortions in an elastoplastic strain hardening materials have been developed. The programs are structured on the assumption that the thermoplastic coupling can be neglected and thermal model is solved independently of the mechanical model in each time step. Argyris, et al. investigated the influence of mechanical coupling terms on the temperature field and they found that the influence is very small [12, 13].

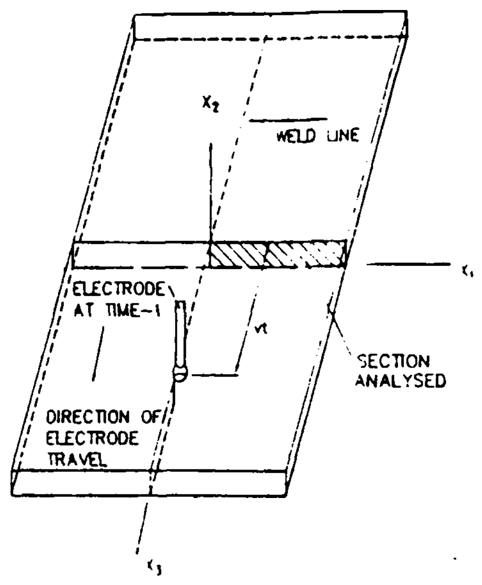


FIG 5 : WELDMENT CONFIGURATION [15]

The thermophysical and mechanical properties of the material, including yield strength are taken to be dependent upon temperature. The accompanying nonlinearity in the analysis is handled by assuming a piece-wise linear relationship between the material properties and the temperature variations within a small temperature interval.

Hibbitt and Marcal [14] used finite element method for developing a numerical thermomechanical model for welding and loading of a fabricated structure. The finite element model for the thermal nanlysis was developed by uncoupling of fundamental energy balance by introducing uncoupling assumptions. Numerical techniques for planar analysis in section normal to the weld direction have been developed under quaşi-stationary thermal conditions. The weld thermomechanical model developed by Friedman [15] also utilised finite element method to model both thermal and mechanical behaviour.

Calculation of transient temperature distribution is based on the attainment of quasi stationary conditions, which are developed when the welding heat source is moving at a constant speed in a regular path and the end effects resulting from either initiation or termination of heat source are neglected. The temperature distribution is then stationary with respect to a moving coordinate system whose origin coincides with the point of application of the heat source. Considering the planar weld illustrated in **Fig. 5**, the temperature at any point of weldment is expressed functionally as.

$$T(x1,x2,x3,t) = T(x1,x2,x3\text{-}Vt,0) \quad .....3$$

where 'V' is the welding speed. Thus given the transient temperature distribution at any one section of the weldment defined by 3=0, the temperature at any other section is determined by apropriate shift in the time scale as given in equation (4) :

$$T(x1,x2,x3,t) = T(x1,x2,0,t\text{-}x3/V) \quad .....4$$

The problem is therefore reduced to finding the two dimensional unsteady temperature field at a section normal to the line. A planar analysis may be used for this purpose when weld speed, relative to a characteristic heat diffusion rate for the material, is sufficiently high so that the amount of heat conducted ahead of the weld torch is very small relative to the total heat input. In this case, the net heat flow across any infinitesimal thin slice of weldment normal to the weldline is assumed to be negligible relative to the heat being diffused within the slice itself. Two dimensional thermal analysis at a section x3=0, normal to the direction of weld is thus treated in Friedman's work [15].

The finite element formulation for transient thermal problems may be developed by variational methods, Galerkin's principle or suitable application of the principle of virtual work for heat conduction problems [15-16]. A set of simultaneous equations can be obtained in matrix form. The nonlinearities encountered in welding thermal analysis resulting from temperature dependent material properties are handled by specifying the temperature dependent material coefficients for a given time increment, which will be those evaluated at the known temperatures at the beginning of the increment. The coefficients are updated for every time step and thus are made to be piecewise constant function of temperature. Therefore, except for those region undergoing phase change, the analysis procedure is linear and noniterative in each time step.

The finite element discretisation, in effect, reduces the transient heat conduction equation, which is a partial heat conduction equation to a set of simultaneous ordinary differential equations which contain time derivatives of the nodal temperatures. Numerical solutions are employed to solve these equations.

Due to fusion, the material phase changes and it is accompanied by a latent heat effect. This is modelled by use of a modified specific heat since alloys, which are mostly used for welding, melt over a finite temperature range. Alternatively, latent heat can be included in each time step of thermal analysis by taking energy balance.

Models for Welding Heat Sources

The most critical input data required for welding thermal analysis are the parameters necessary to describe heat input to the weldment from arc. The magnitude as well as distribution of heat input will influence the dimensions of weld metal & heat affected zones, the cooling rates, the peak temperature distribution, and the temperature gradient necessary to calculate stresses and distortions. The welding parameters required to formulate the heat flux boundary conditions are the magnitude of heat input from the arc Q, the distribution of the heat input, characterised by a length parameter, "r", and the weld speed v. Taking arc efficiency as "n", Q is found as Q = nEI, where E and I are the arc voltage and current respectively.

Pavelic, et al. [17] have suggested that the heat from welding arc, at any given time, can be assumed to be deposited on the surface of the weldment as a radially symmetric normal distribution function. Taking "r" as the distance from the centre of the heat source, which is coincident with the axis of the electrode, the heat flux "q" is given by equation (5).

$$q(r) = qe^{-Cr^2} \quad .....5$$

where q and C are constants determined by the magnitude and distribution of the heat input.

The flux is brightest at the centre and in most cases a Gaussian energy distribution can be as-

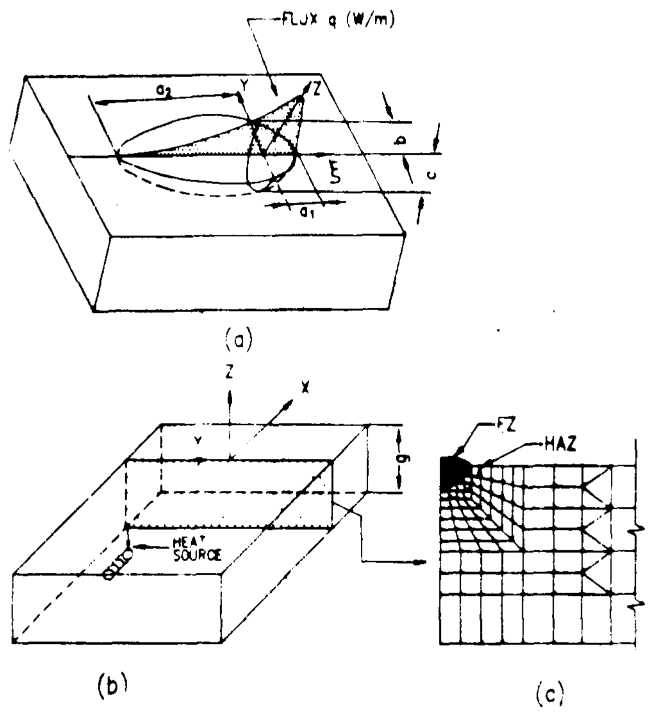


FIG 6 : FEM WELD ANALYSIS
a. DOUBLE ELLIPSOID SOURCE MODEL
b. REFERENCE PLANE CONCEPT AND
C. MESH USED FOR THE ANALYSIS [18]

sumed. Presently models do not explicitly incorporate stirring in the weld pool or digging of arc to depress weld pool surface. The capability of computing accurate temperature fields is essential in computing accurate residual stresses and strain fields in realistic welds.

Goldak, et al. [18,19] have developed a different model to define the energy distribution in the arc (Fig. 6). In these models, arbitary functions are used to define the distribution of flux on the surface of the weld and the power density throughout the volume of the weld. For arc welds, a double elliptical disc with a Gaussian distribution of flux on the surface of the weld, together with one double ellipsoid function with a Gaussian distribution of power density to model the direct impingement of the arc, and a second double ellipsoid with a gaussian distribution to model the energy distributed by stirring the molten metal has been used. This model of welding arc has reportedly given the most accurate temperature field computed to date.

Stress Analysis

In both heat transfer and stress analysis, the basic equations are derived in the incremental forms on the assumption that any changes during a small increment of time are linear. Accumulation of the solution to the basic equation for each time step furnishes the entire history of stress and strain. Thus determination of residual stress analysis problem is thermal elasticplastic analysis problem [5,8,15,20]. The welded zone is divided into a number of elements. During each welding pass, thermal deformations are calculated from the temperature distribution determined by thermal model. The residual deformation for each increment is added to the nodal point locations to determine an updated configuration of the model before the next load increment. By this way, a large deformation elastic-plastic problem is represented as a series of incrementally linear problems.

Thermal strain increment $\{de^T\}$ is expressed by instantaneous linear expansion coefficient $\{\alpha\}$ and temperature increment [dT] as

$$\{de^T\} = \{\alpha\}\, dT \qquad .....6$$

The above instantaneous lineat expansion coefficient $\{\alpha\}$ is a coefficient which indicates the magnitude of expansion or shrinkage due to temperature changes at every instant. $\{\alpha\}$ is used to express expansion or shrinkage due to temperature changes, and due to transformation, when the material is in the temperature range of trnasformation. In the elastic range, total strain increment {de} is represented as the summation of thermal strain increment $\{de^T\}$ and elastic strain increment $\{de^e\}$ which are produced to satisfy the conditions of compatibility, that is

$$\{de\} = \{de^T\} + \{de^e\} \qquad ....7$$

Elastic strains are related to stress $\{\delta\}$ as

$$\{\delta\} = [D]\ \{e^e\} \qquad .....8$$

where [D] is elasticity matrix.

During subsequent temperature change, all the above three parameters change but they should still satisfy the equation (8). In plastic range, applying incremental plasticity theory, the plastic strain rates are replaced by plastic strain increments. Total plastic strain is accumulated in a finite number of increments.

When a set of stresses produced at a point reaches certain magnitudes, the material yields and this condition can be represented by a yield criterion. Generally the shape, size and position of yield surface change with progress of plastic deformation of the material. This means, yield criterion changes when being subjected to plastic work and the law governing the changes in yield criterion is called work hardening rule or strain hardening rule. Isotropic or kinematic strain hardening rules are generally used. Isotropic work hardening rule assumes that the size of yield surface changes with increase of plastic work but position and shape do not change. This implies that the initial yield surface expands uniformly during the subsequent plastic flow. In kinematic work hardening rule, it is assumed that the yield surface does not change its initial size and shape but moves in the stress space like a rigid body. Any of these work hardening rules or combination of work hardening rules are generally used in stress analysis programs. In plastic range, total strain increment {de} is expressed by summation of components as.

$$\{de\} = \{de^e\} + \{de^p\} + \{de^T\} \qquad ....9$$

Basic Equations for Thermal Elastic Plastic Analysis by Finite Element Method

The basic concept of finite element method, if expressed simply, is to regard a component as an assembly of simple structural elements interconnected at a finite number of nodal points, where the equilibrium and compatibility conditions are satisfied. Accordingly, the component under consideration is divided into a finite number of elements at the beginning of the analysis, such as triangular finite elements for plane stress or strain problems.

The basic equations in the finite element method, in the continuum, will be derived as follows [20]

(i) The displacement {s} of an arbitary point in an element will be defined as a function of nodal displacements {w}

$$\{s\} = [N]\ \{w\} \qquad ......10$$

where [N] is the shape function

(ii) Components of strain in an arbitrary point can be written as

$$\{e\} = [B]\ \{w\} \qquad ......1$$

where [B] is strain - displacement matrix

In the case of infinitesimal displacement problem, the matrix [B], can be regarded as a constant matrix. In the incremental form, the above equation is expressed as

$$\{de\} = [B]\ \{dw\} \qquad ......12$$

(iii) When temperature of the element changes during an increment of time, dt, the stress component is expressed as

$$\{d\sigma\} = [D]\ \{de\} - \{dC\} \qquad .....13$$

Where [D] = elasticity matrix

{dC} = stress component due to creep

(iv) From the basic equations which are given above, the incremental relationship between nodal forces and nodal displacements, that is, stiffness equation will be derived by applying the principal of virtual displacement.

Nodal forces {F} of an element are defined, as forces which are statically in equilibrium with the stresses acting on the boundary of the element.

Each of the forces {Fi} will contain the same number of components the corresponding nodal displacements {w}. It can be shown that

$$\{F\} = \int [B]^T\ \{\sigma\}\ d(vol) \qquad ....14$$

In the incremental form this is

expressed as

$\{dF\} = \int [B]^t \{d\sigma\}\ d(vol)$ 15

where [B] is regarded as a constant matrix.

Substitution of equations (13) and (12) in (15) provides stiffness equation for time increment "dt", that is

$\{dF\} = [K]\ \{dw\} - \{dL\}$ 16

where

$[K] = \int [B]^T [D] [B]\ d(vol)$: stiffness martix of the element

$\{dL\} = \int [B]^T \{dC\}\ d(vol)$: equivalent nodal force due to creep

Generally creep component is taken only in very detailed analysis.

Stiffness equation for the whole structure is obtained as the summation of stiffness equation (16) for all elements at each node.

$\Sigma\{dF\} = \Sigma[K]\{dw\} - \Sigma\{dL\}$ 17

Once the above equation (17) is solved for nodal displacement increment {dw}, satisfying the specified boundary conditions, total strain increment {de} and stress increment {dσ} of each element can be evaluated from equations (12) and (13).

CASE STUDIES

No theoretical work is completed unless it is substantiated by experimental results. Experimental techniques generally employed in determining residual stresses in weldments are dissection method, X-ray diffraction method and neutron diffraction method.

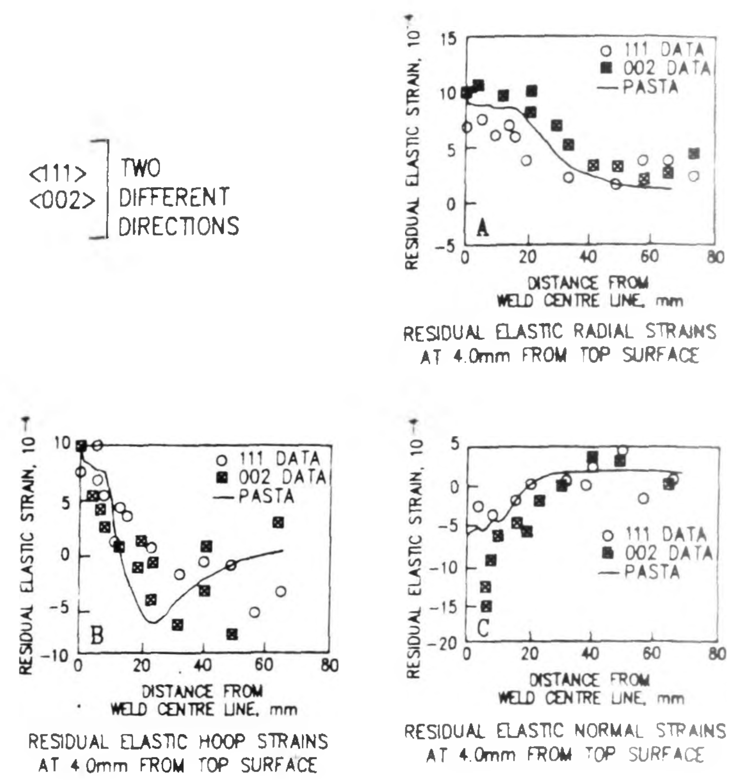


FIG 7 : COMPARISON BETWEEN THE PASTA2D PREDICTIONS OF RESIDUAL ELASTIC STRAIN AND THE NEUTRON DIFFRACTION MEASUREMENTS AT THE MIDPLANE OF THE SPECIMEN (4.0mm FROM TOP SURFACE). [21]

a. RADIAL COMPONENT OF STRAIN
b. HOOP COMPONENT OF STRAIN
C. NORMAL COMPONENT OF STRAIN

The residual elastic strain distribution developed in a stationery gas tungsten arc weld on AISI type 304L austenitic stainless steel was determined using experimental methods and finite element simulation techniques by Mahin, et al. [21]. Numerical analysis of the weld was performed using a fully coupled thermo-mechanical finite element code, called PASTA - 2D. Experimental method of determining residual stress in and around the as-solidified weld pool was carried out by neutron diffraction techniques. It has been reported that agreement was excellent in both the heat affected zone and within the as - solidified weld pool.

Fig. 7 shows the comparison between computer prediction and neutron diffraction measurements at the midplane of AISI type 304L austenitic stainless steel specimens.

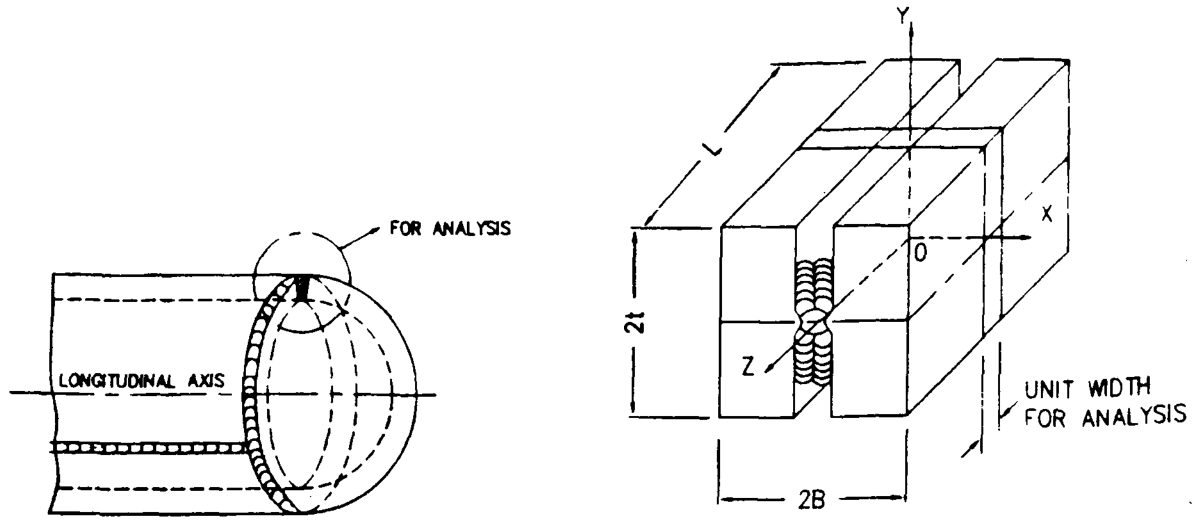
Ueda, et al. [20] have estimated welding residual stresses prodiced in a cylinder headjoint (U-Groove) of pressure vessel steel made of very thick 2 1/4 Cr-1 Mo steel plates. For analysis and experiment, double U-groove joint model **Fig. 8** of 300 mm plate thickness was taken. Each pass of welding was applied to the model, alternatively on each side of the groove. In the experiment, submerged arc welding was applied and number of passes were 167. **Fig. 9** shows transverse welding ttansient and residual stresses on the top surface and in the middle cross section of the 300 mm thick plate by both theoretical analysis and experiment and it can be seen that both are in good agreement. Transient stresses produced when the groove was welded halfway and cooled to the interpass temperature of 473 K are also whown in Fig. 9.

Rybicki. et al.[8] have developed an axisymmetric finite element computer model for predicting transient termperatue distrubution, residual stresses, and residual deflections in butt welded pipes. comparison of predicted and measured temperarures, for a two-pass welded pipe of AISI type 304 austeninic stainless steel showed agreement within 9 and 17 percent of the measured values, for passes one and two respectively. The computer modes is based on finite element representation, recognising indivfidual passes, temperature dependent elastic-plastic constitutive behaviour, elastic unloading for material in the non-linear stress strain range, and changes in geometry due to deformation of each weld pass. It has been reported that results for a two-pass butt welded pipe of AISI type 304 austenitic stainless steel showed good correlation between residual stress and residual deflections obtained from the computational model and data obtained from experiments. **Fig. 10 & 11** show comparison of computed and experimentally obtained residual stresses on inner and outer surface for two-pass butt weld.

Similarly, Rybicki et al. [5] have also developed a computational model to calculate the magnitude and distribution of residual stresses for multipass butt welded pipes. Residual stresses obtained with the model, for a seven-pss weld and a thirty-pass weld, have been compared with residual stresses obtained from laboratory measurements. Good aggreement has been reported between computed and measured values of residul stresses for both welds. **Figs. 12 & 13** show the comparison of calculated and experimentally determined sesidual stresses for inner surface of seven-pass weld and thirty-pass weld respectively.

Fujita, et al. [13,22] used a combined analytical and numerical approach to calculate residual stresses, due to a circumferential weld at the joint between a cylinderical drum and a hemispherical head plate, In this work, comparitively thin shells were considered and the deformations were assumed to be axisymmetric. This means, that the welding is assumed to be performed instantaneously around the circumference. The temperature was assumed to be uniform across the shell thickness and a one-dimensional solution for the temperature field in the direction transverse to the weld was used. In the thermal elastic- plastic solution, the change in radial displacement was assumed to have the same shape as the displacement due to a circumferential line load on the cylinder and the hemisphere. The usual relationships between strains and displacements for axisymmetric shells were used. Calculated and measured values of axial and hoop stresses on the inner and outer surface are shown in **Fig.14.** Calculated and measured values of the displacements are also shown in Fig. 14. A good overall agreement is found between the calculated and measured residual axial and hoop stresses.

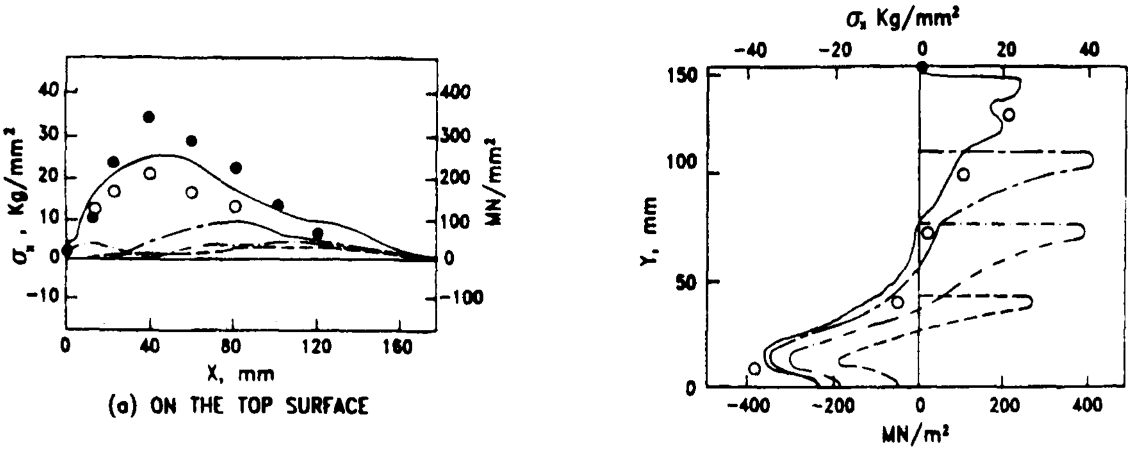
Computer codes are useful in the theoretical analysis of determining residual stresses. Exisiting codes are predominantly utilised



a) CYLINDER-HEAD WELDED JOINT

b) IDEALIZED RESEARCH MODEL

FIG 8 : CYLINDER HEAD WELDED JOINT OF A PRESSURE VESSEL AND ITS RESEARCH MODEL [20]



(a) ON THE TOP SURFACE

(b) AT THE MIDDLE CROSS SECTION

EXPERIMENT (FOR BOTH GRAPHS)
● RESIDUAL STRESSES (3-DIMENSION)
○ RESIDUAL STRESSES (2-DIMENSION)

ANALYSIS (FOR BOTH GRAPHS)
– – – AFTER 5th LAYER WELDING (T=200°C)
- –·- AFTER 9th LAYER WELDING (T=200°C)
–·–· AFTER 13th LAYER WELDING (T=200°C)
——— RESIDUAL STRESSES

FIG 9 : TRANSVERSE WELDING TRANSIENT AND RESIDUAL STRESSES [20]

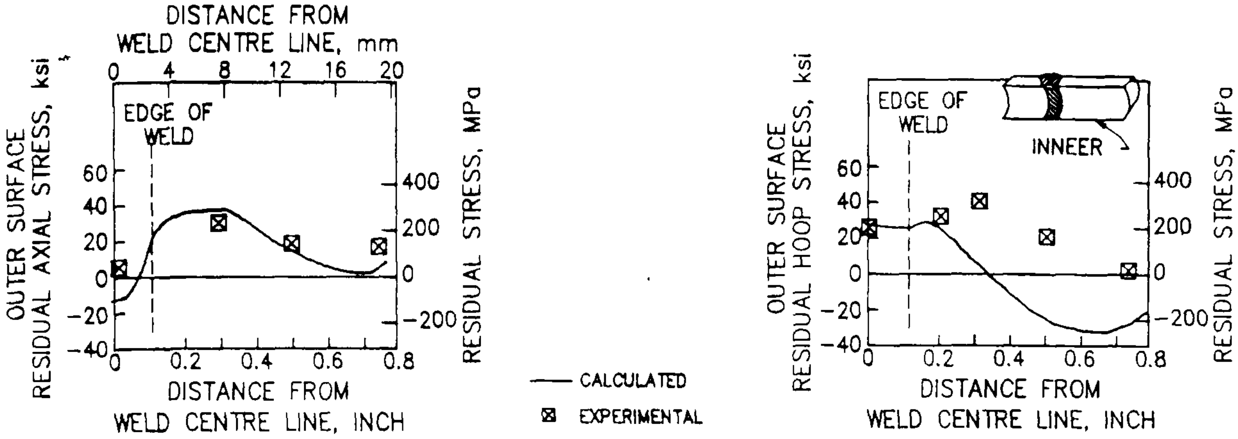


FIG 10 : COMPARISON OF COMPUTED AND EXPERIMENTALLY OBTAINED RESIDUAL STRESS AT THE INNER SURFACE FOR TWO—PASS GIRTH—BUTT WELD [8]

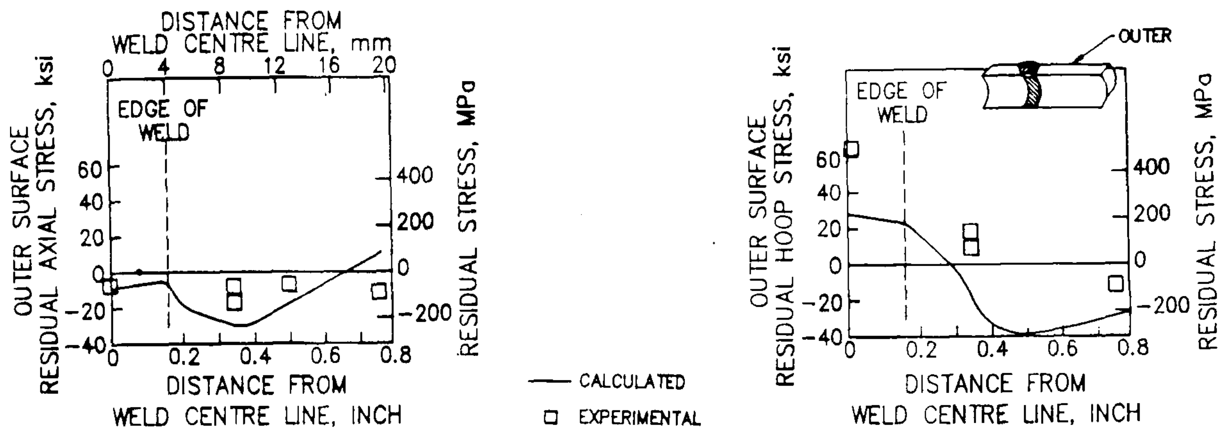


FIG 11 : COMPARISON OF COMPUTED AND EXPERIMENTALLY OBTAINED RESIDUAL STRESS AT THE OUTER SURFACE FOR TWO—PASS GIRTH—BUTT WELD [8]

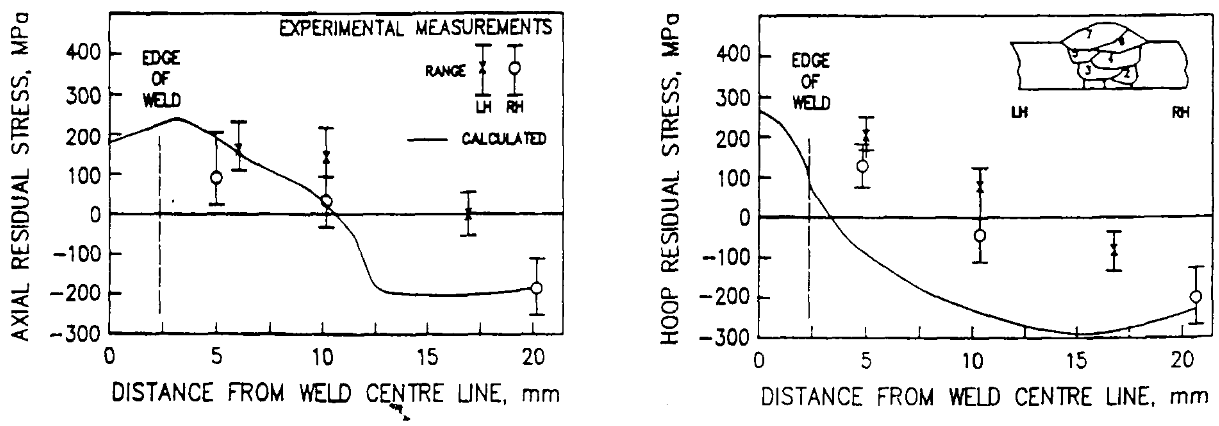


FIG 12 : COMPARISON OF COMPUTED AND EXPERIMENTALLY DETERMINED RESIDUAL STRESS FOR THE INNER SURFACE OF THE SEVEN—PASS WELD [5]

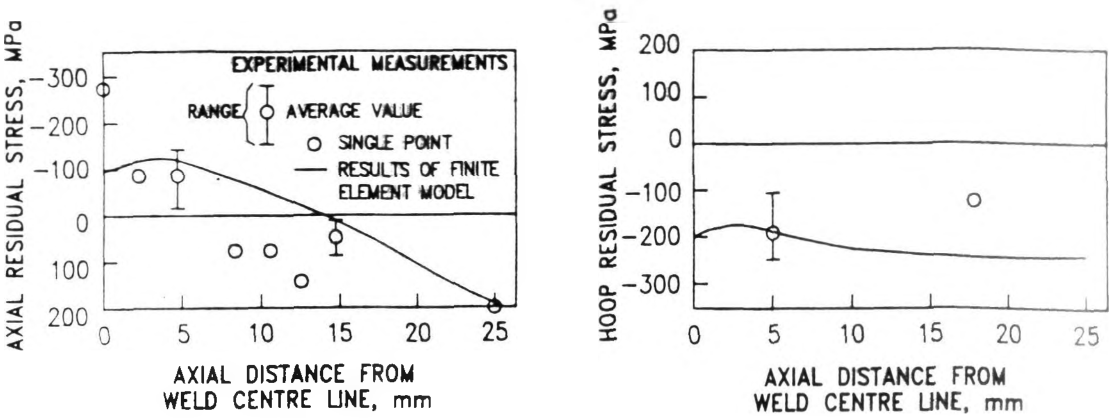
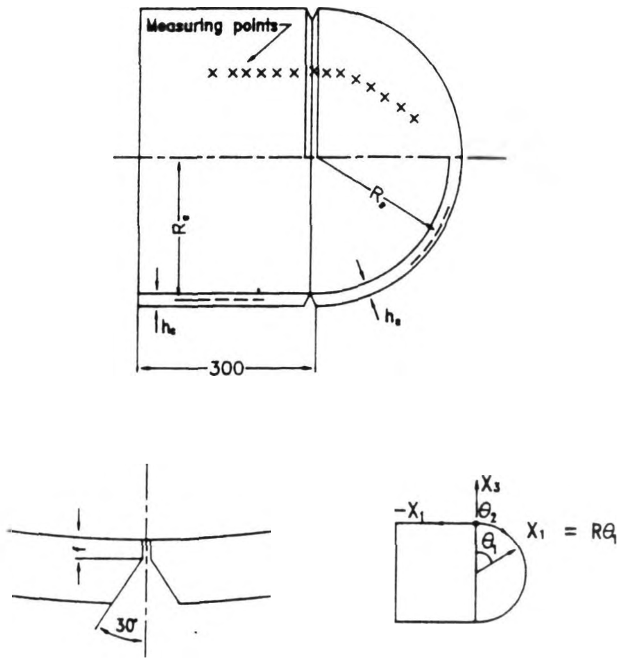


FIG 13 : COMPARISON OF CALCULATED AND EXPERIMENTALLY DETERMINED RESIDUAL STRESS FOR THE INNER SURFACE OF THE THIRTY—PASS WELD [5]



	CYLINDRICAL SHELL	HEMISPHERICAL SHELL
RADIUS R	299.6	304.5
THICKNESS h	9.5	8.4
ROOT FACE f	2.0	

FIG 14A

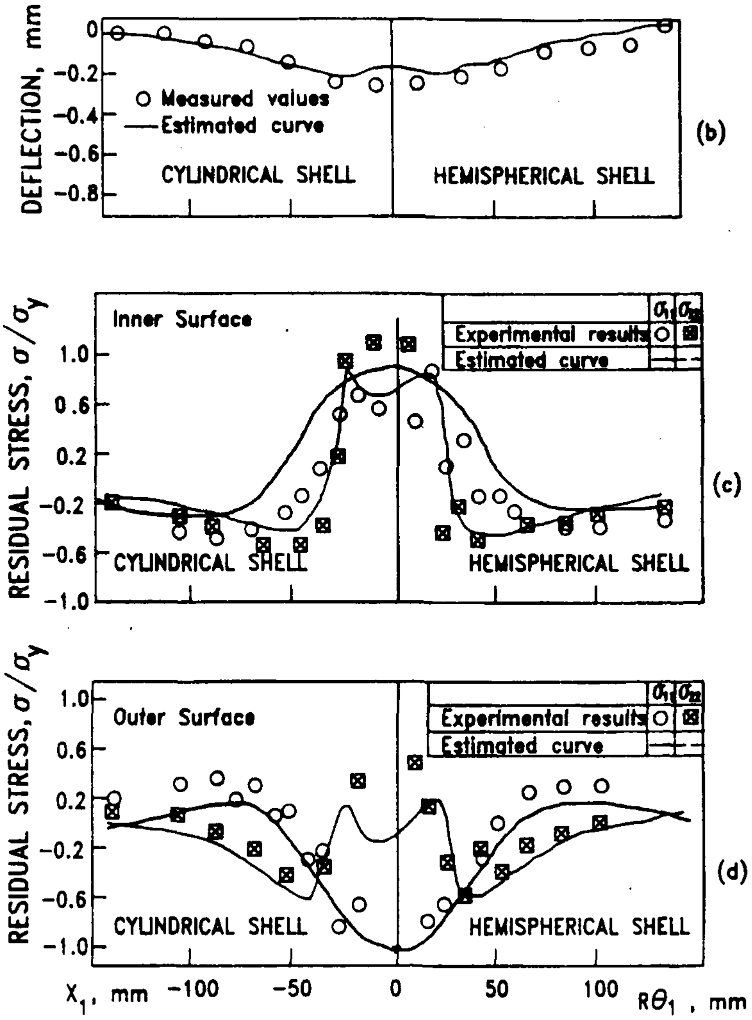


FIG 14 a, b, c : CALCULATED AND MEASURED RESIDUAL DISPLACEMENTS (b) and stresses (c&d) DUE TO A CIRCUMFERENTIAL WELD OF THE JOINT BETWEEN A CLINDRICAL DRUM AND HEMISPHERICAL HEAD PLATE (14a) [13,22]

to perform specific functions. Several assumptions are made during analysis to reduce the computing time. For instance, in multipass welding, several intermediate passes are consedered equivalent to a single pass and computation is carried out. Computer modelling of residual stresses in weldments is a progressing area and the work has been limitd to simple geometries until now.

CONCLUSION

Theoretical methods of determining residual stresses in weldments which are found in literature are discussed in this report. Finite elemnt method is widely used for finding out residual stresses in weldments. computational methods of determining residual stresses are complex, nevertheless these methods have been fairly established in plates and pipes and to some extent in structures. computer models for specific applications are to be developed from basic principles or existing codes should be suitably modified to meet the requirements and this poses an interesting and challenging task. Computational methods offer good opportunity for better understanding of the distribution of residual stresses in weldments and for prediction of the consequent behaviour of the welded joints.

ACKNOWLEDGEMENT

The authors are thankful to Dr. Placid Rodriguez, Director, Indira Gandhi Centre for Atomic Research, kalpakkam for constant encouragement and useful suggestions.

REFERENCES

1. P. Palanichamy, A. Josepb, D. K. Bhattacharya and Baldev Raj, "Residual Stresses and their Evaluation in welds", in Welding Engineering Hand Book ISBN : 81-85373-02-7, Radiant Publications Private Limited, Secunderabad, India, 1992

2. Eckard Macherauch, " Introduction to Residual Stress", Advances in Surface Treatments : Vol. 4 - International Guide Book on residual Streses, 1986.

3. Helmut wohlfahrt, "Residual Stress as a Consequence of Welding", ibid.

4. K. Masubuchi, "The Need for Analytical / Experimental Orchestrated Approaches to Solve Residual Stress Problems in Real Structures", in Nondestructive Methods for material Property Determination, Eds. C.O Ruud and R.K. Green Jr., (Plenum Press, London) 1984, pp 123-134.

5. E.F Rybicki, R.B Stonesifer, "Computation of Residual Stresses due to Multipass Welds in Piping Systems", Jl. of Pressure Vessel Technology, May 1979, pp 149-154.

6. E.F. Rybicki, "Computational Methods for Calculating Weld Residual Steess in Pipes", in Welding : Theory and Practice, Edited by D.L. Olson, R. Dixon and A.L. Liby, Elsevier Publishers, B.V., 1990.

7. K. Masubuchi, "residual Stresses and Distortion in weldments : A Review of the Present State-of-the-art" in Residual stress and Stress relaxation, Edited by Kula. E and Weiss. V, Plenum Press, N.Y, 1982.

8. E. F. Rybicki, D.W. Schmueser, R. B. Stonesifer, J.J. Groom, and H.W. Mishler. "A Finite-element Model for Residual Stress and Deflections in Girth-Butt Welded Pipes" Jl. of Pressur Vessel Technology, Aug 1978, pp 256-262.

9. U, Chandra. "Determination of Resisual stresses due to Girth-Butt Welds in Pipes", Jl. of Pressure Vessel Technology, vol 107, may 1985.

10. D. Rosenthal, "Theory of Moving sources of Heat and its Application to metal Treatments", Trans, ASME, Vol. 68, 1946, pp 849-865.

11. D. Rosenthal, Welding, Jl., Vol. 20, 1941, pp 220s-234s.

12. J.H. Argyris, J. Szimmat and K.J. William, "Computational Aspects of Welding Stress Analysis", Comp. Mothods. App. Mech. Engg., 33, pp 635-666, 1982.

13. Lennart Karlsson, "Thermal Stresses in Welding", in Thermal Stresses I, Edited by R.B. Hetnarski. Elsevier Publishers B.V. 1986

14. H. D. Hibbitt and P.V. Marcal. "A Numerical Thermo- mechanical Model for the Welding and Subsequent Loading of a Fabricated Structure", Computers & Structures Vol.3 1973, pp 1145-1174.

15. E. Friedman, "Thermomechanical Analysis of the Welding Process Using the Finite Element Method" Jl of Pressure Vessel Technology, Aug 1975, pp 206-213.

16. O. C. Zienkiewicz, "The finite Element Method", Mc Graw-Hill-Book Company, New York, 1977.

17. V. Pavelic, R. Tanbakuchi, O. A Uyehara and P.S. Meyers, Welding Jl., Vol. 48, 1969, pp 295s - 305s.

18. J. Goldak, M. Bibby, R. Moore, R House and B. Patel, "Computer Modeling of Heat flow in Welds", Metallurgical Transactions B, vol 17B, Sep. 1986, pp. 587-600

19. J. Goldak, B. Patel, M. Bibby and J. More, "Computational Weld Mechanics", AGARD presentation Structures and Materials, 61 st Panel Meeting, Oberammergau, Germany, Sep. 1985

20. Y. Ueda, K. Nakacho, "Theories of Analysis and measurement of Welding Residual Stresses and PWHT" Jl. of Mechanical Behaviour of Materials, Vol 2, Nos. 1-2, 1989

21. K.W. Mahin, W. Winters, T.M. Holden, R.R. Hosbons and S.R. MacEwen, "Prediction and Measurement of Residual Elastic Strain distributions in Gas Tungsten Arc Welds", Welding Jl., Sep. 1991, pp. 245S - 260S.

22. Y. Fujita, T. Nomoto and H. Hasegawa, "Welding Deformations and Residual Stresses due to Circumferential Welds at the Joint between Cylinderical Drum and Hemisperical Head Plate", International Institute of Welding, Document X-985-81, 1981