# Development of a Mathematical Model for Predicting Angular Distortion in Butt Welded Stainless Steel 409M Plates in GMAW Process

**Pradeep Khanna[1] and Sachin Maheshwari[2]**
[1]Associate Professor, MPA Engineering Division, NSUT, New Delhi
[2]Professor, MPA Engineering Division, NSUT, New Delhi
Email: 4.khanna@gmail.com

*DOI : 10.22486/iwj.v53i1.191280*

**ABSTRACT**

Angular distortion is almost inevitable in fusion welding processes as it involves localized heating and cooling of materials. Keeping the weld distortion to its minimum is the endeavor of every weld engineer as excessive distortion can not only spoil the physical appearance but also can cause mismatch of joint in fabricated structures. In the present investigative work, an attempt was made to predict the influence of input weld parameters like wire feed rate, voltage, speed of welding and the angle of groove on angular distortion, by developing a mathematical model. This was accomplished by developing a mathematical equation which included the direct, quadratic and interaction effects of input weld parameters and could be used to predict the effects of these parameters on the resulting angular distortion. The experimentation was carried out in a structured manner by using Central Composite Face Centered Design (CCFD) technique. All the selected welding parameters were taken at three levels to accommodate the curvature effect. The final model was developed by using regression analysis and its adequacy was tested by Analysis of variance (ANOVA) approach. Response surface methodology (RSM) was used to graphically plot direct and interaction effects of weld parameters on angular distortion. The validity of the developed model was checked by conducting test runs at different values of input parameters. The comparison of actual and predicted results indicated good conformance.

**Keywords:** Angular distortion; fusion welding; weld parameters; mathematical model; regression analysis.

## 1.0 INTRODUCTION

Fusion welding invariably is a process involving localized heating and cooling of materials, making distortion almost inevitable. During welding, the base metal experiences a compressive plastic strain during heating which is greater than tensile plastic strain during cooling, resulting in distortion. It has been recognized in ship building and heavy industries that the initial deflection due to distortion of the weldments reduces the buckling strength [1]. Distortion can significantly impair the reliability and final quality of the welds and need to be dealt with carefully during design and fabrication of welded structures [2-5]. Excessive distortion may result in the mismatch of the joints, thereby increasing the possibility of a defective weld and in extreme conditions, severe disasters due to buckling instability. This distortion generally results from the no-uniform shrinkage across the depth of the plates [6, 7]. It is affected by many factors like welding current, arc voltage, welding speed, welding process, groove geometry, type of joint, number of runs, etc. Angular distortion in welded structures is one of the most troublesome problems [8]. Angular distortion (α) in butt welds, as shown in **Fig. 1**, occurs when transverse shrinkage is not uniform in the thickness direction which is particularly the case when welding is done from one side with medium thickness plates.

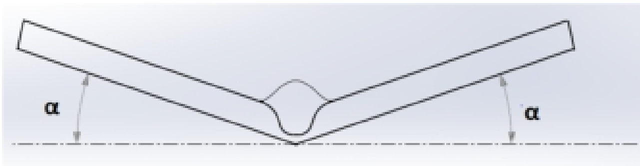


**Fig. 1 : Angular distortion**

If GMAW is to be used in conjunction with robotic welding it is essential that, to produce quality welds, it should have least distortion in the welded structures. That requires not only the prior knowledge of the effects of different welding parameters and physical parameters on distortion but also demands that such a knowledge should be available in the form of mathematical formulation which, when fed to the automated systems, the desired end results are achieved. Present work is an attempt to develop such mathematical model to predict distortion as influenced by the important weld parameters [9, 10].

In the present investigative work, a mathematical model was developed using (CCFD) technique. This model was expected to be able to estimate the direct, quadratic and interaction effects of input welding parameters on the angular distortion. All the selected welding parameters were taken at three levels to accommodate the curvature effects. ANOVA technique was used to check the adequacy of the mathematical model finally developed by regression analysis treatment [11]. The statistical technique CCFD guides to conduct experiments in a structured way, based on which mathematical equation(s) were developed. The direct and interaction effects of input weld parameters on angular distortion as obtained from the developed mathematical model have been plotted graphically for easy understanding and were analyzed by employing Response Surface Methodology (RSM) technique [12].

## 2.0 THE WELDING SET-UP

**Fig. 2** shows the complete welding set-up used for the present investigative work. It consisted of a weld power source of rated capacity 400 A, with flat V-I characteristics. A mechanized carriage unit to ensure reproducibility of results and to carry out welding at set speeds within a range of 0-50 cm/min. A control panel to facilitate easy and step-less control of welding speed via a variable frequency drive. Industrially pure Argon gas was used for shielding purpose. Stainless steel 308L wire with diameter 1.5 mm was used to make the welds.

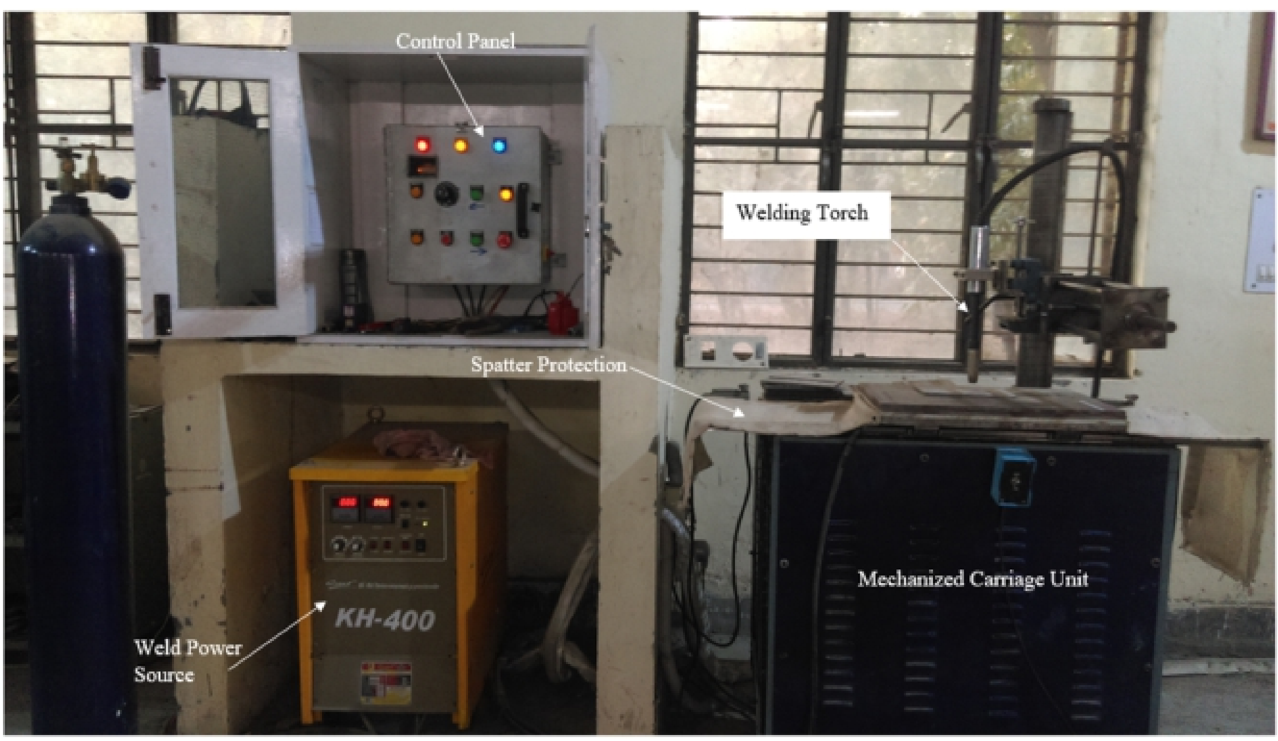


**Fig. 2 : The complete welding set-up**

## 3.0 PLAN OF INVESTIGATION

The effect of input welding parameters on the angular distortion was investigated by following the stepwise investigation plan given below;

1. Identification of the input welding parameters affecting the angular distortion.
2. Estimating the working limits of these parameters.
3. Development of the experiment design matrix.
4. Conducting the experiments as per the design matrix.
5. Development of mathematical model.
6. Testing the model for its adequacy.
7. Testing the regression coefficient's significance.
8. Analysis of results and discussions.
9. Validation of results
10. Conclusions

### 3.1 Identification and Selection of Welding Parameters

In the present investigative work, out of many possible input parameters, wire feed rate (W), welding speed (S), arc voltage (V) and the groove angle (θ) were chosen as the ones affecting the angular distortion significantly.

### 3.2 Establishment of Useful Limits of the Selected Parameters

To have a good quality weld without any visible defect, it is essential to have the welding parameters in a proper balance. To ensure this, the useful working limits of all the selected parameters must be determined. A large number of trial runs were made to find the limits or the operating ranges of the parameters. This was done by changing the values of one factor at a time while keeping the other factors at a constant level. The process was repeated for all the factors till working ranges of all factors were determined [13]. The established limits are presented in **Table 1**.

**Table 1: Selected welding parameters and their useful limits**

| Welding parameters | Unit | Symbol | Useful Limits | | |
|---|---|---|---|---|---|
| | | | (-1) | (0) | (+1) |
| Wire feed rate | m/min | W | 4 | 7 | 10 |
| Welding speed | cm/min | S | 25 | 35 | 45 |
| Arc voltage | volts | V | 24 | 27 | 30 |
| Groove angle | degrees | θ | 0 | 20 | 40 |

### 3.3 Development of the Design Matrix

A design matrix consisting of all possible combinations of input parameters levels was constructed as shown in **Table 2**. This design is based on CCFD approach with four factors taken at three levels and consisting of 30 sets of coded conditions derived from a simple formula [30 = 2k + 2k + 6], where k represents the number of welding parameters [14]. It therefore consisted of a full replication of 24 = 16 factors, 2x4=8-star points and six center points.

### 3.4 Conducting the Experiments

In total, 30 numbers of experimental runs were made. Since the length of plates does not have any effect on angular distortion and comparatively wider plate would facilitate measurement of even a small angular change, a specimen size of 150 mm x 100 mm x 6 mm was selected for the experiments. Keeping in mind the requirement of the trial and also to cater for any eventuality, 70 plates of the required size were cut with a plasma cutter. Referring to the design matrix, it can be seen that the three levels of groove angles were 0°, 20° and 40°. Therefore 24 plates with bevel angle 10° and 18 plates with bevel angle 20° were prepared on a vertical milling machine; the remaining 18 plates had square edges. During the machining of plates, a root face of 1.5 mm was maintained.

In order to maintain a root opening of 1.5 mm for minimum distortion and also to facilitate handling of specimen during subsequent stages of experimentation, two tack welds, one at each end was made, after placing a 1.5 mm thick gauge in-between the plates. The **Fig. 3** shows important parameters of edge preparation for the plates to be welded.

A nozzle to plate distance of 15 mm was maintained. The welding runs were made in a random sequence so that any systematic error in the system may be avoided. The response parameter, which is angular distortion in this case was measured and placed in the same table.

#### 3.4.1 Measurement of angular distortion

**Fig. 4** shows the arrangement used for the measurement of angular distortion of the welded plates.

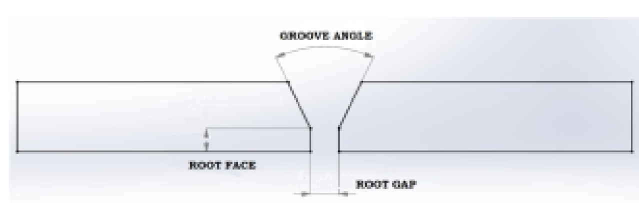


**Fig. 3 : Plate edge parameters**

**Table 2 : The design matrix**

| Standard Order | Run Order | W | Variables in coded form | | | Response |
|---|---|---|---|---|---|---|
| | | | V | S | θ | Angular Distortion |
| 5 | 1 | -1 | -1 | 1 | -1 | 3.36 |
| 15 | 2 | -1 | 1 | 1 | 1 | 3.46 |
| 29 | 3 | 0 | 0 | 0 | 0 | 4.03 |
| 18 | 4 | 1 | 0 | 0 | 0 | 4.39 |
| 14 | 5 | 1 | -1 | 1 | 1 | 3.92 |
| 16 | 6 | 1 | 1 | 1 | 1 | 4.86 |
| 3 | 7 | -1 | 1 | -1 | -1 | 3.50 |
| 25 | 8 | 0 | 0 | 0 | 0 | 4.06 |
| 28 | 9 | 0 | 0 | 0 | 0 | 4.09 |
| 20 | 10 | 0 | 1 | 0 | 0 | 3.88 |
| 1 | 11 | -1 | -1 | -1 | -1 | 3.34 |
| 13 | 12 | -1 | -1 | 1 | 1 | 3.28 |
| 23 | 13 | 0 | 0 | 0 | -1 | 3.73 |
| 27 | 14 | 0 | 0 | 0 | 0 | 4.02 |
| 12 | 15 | 1 | 1 | -1 | 1 | 6.52 |
| 26 | 16 | 0 | 0 | 0 | 0 | 3.64 |
| 4 | 17 | 1 | 1 | -1 | -1 | 6.02 |
| 22 | 18 | 0 | 0 | 1 | 0 | 3.34 |
| 11 | 19 | -1 | 1 | -1 | 1 | 3.98 |
| 21 | 20 | 0 | 0 | -1 | 0 | 4.16 |
| 17 | 21 | -1 | 0 | 0 | 0 | 2.81 |
| 2 | 22 | 1 | -1 | -1 | -1 | 5.11 |
| 9 | 23 | -1 | -1 | -1 | 1 | 3.38 |
| 6 | 24 | 1 | -1 | 1 | -1 | 3.99 |
| 19 | 25 | 0 | -1 | - | - | 3.33 |
| 30 | 26 | 0 | 0 | 0 | 0 | 3.64 |
| 24 | 27 | 0 | 0 | 0 | 1 | 3.94 |
| 7 | 28 | -1 | 1 | 1 | -1 | 3.09 |
| 10 | 29 | 1 | -1 | -1 | 1 | 5.15 |
| 8 | 30 | 1 | 1 | 1 | -1 | 4.48 |

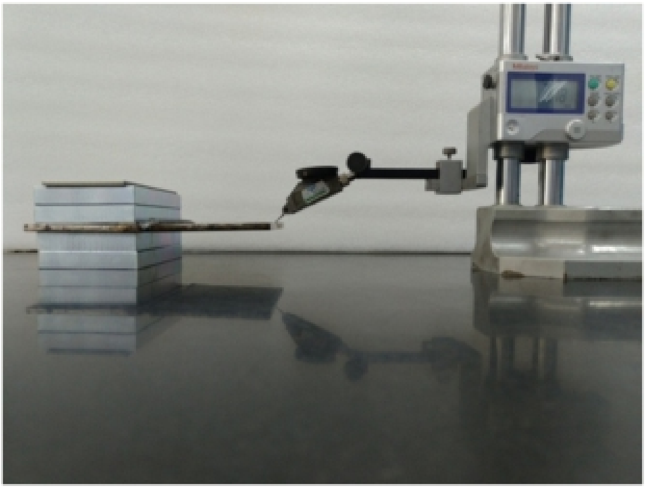

**Fig. 4 : Arrangement for the measurement of angular distortion**

To keep one half of the weldment in air, a distance piece and a counter weight were used. Height of the elevated edge ($h_1$) at one end and also the height of the horizontal half of the specimen ($h_2$) were measured with a digital vernier height gauge of least count 0.01 mm. A lever type dial indicator was attached to the anvil tip of the height gauge to ensure equal measuring pressure each time a reading was taken. The difference ($h_2 - h_1$) gave the height through which the upper edge was elevated. This height was then used to calculate the angle ($\sin\theta = \frac{h_2-h_1}{w}$), by which one of the plates was rotated against the other due to angular distortion.

### 3.5 Development of Mathematical Model

Equation (1) shows the manner in which the input parameters can be related to the response parameter [15-17].

$$Y=f\,(I, V, S, \theta) \tag{1}$$

where, Y represents angular distortion in present case.

A quadratic relationship, shown in equation (2), can best represent the present case

$$Y = \beta_0 + \Sigma^4_{(i=1)}\beta iXi+\Sigma^3_{(i=1)}\Sigma^4_{(j=i+1)}\beta ijXiXj + \Sigma^4_{(i=1)}\beta iiX^2 \tag{2}$$

Based on equation (2), general equation (3) can be arrived at:

$$Y = \beta_0 + \beta_1 W + \beta_2 V + \beta_3 S + \beta_4 \theta + \beta_{12} WV + \beta_{13} WS + \beta_{14}W\theta + \beta_{23} VS + \beta_{24}V\theta + \beta_{34}S\theta + \beta_{11} W^2 + \beta_{22} V^2 + \beta_{33} S^2 + \beta_{44} \theta^2 \tag{3}$$

### 3.6 Testing the Model for Adequacy

The model was tested for its adequacy by using ANOVA, where the calculated and standard values of the F and R-ratio for the developed model are compared separately. For the model to be adequate within a confidence level of 95%, the calculated value should be less than the standard value of F-ratio. Also, the calculated value should be higher than the standard value of R-ratio [15, 18, 19]. For the present case,

Standard F-ratio$_{(10,6,0.05)}$ = 4.06

The numbers in parenthesis correspond to df (degree of freedom) for lack of fit, df for experimental error and significance level respectively.

Standard R-ratio$_{(14,6,0.05)}$ = 3.96

The numbers in parenthesis correspond to df for lack of fit, df for both first and second order terms, and significance level respectively.

The F and R-ratio are calculated using the following relations [17, 20].

$$\text{F-ratio} = \frac{\text{Mean square of lack of fit}}{\text{Mean square of error}} \tag{4}$$

$$\text{R-ratio} = \frac{\text{Sum of mean square of first and second order terms}}{\text{Mean square of error}} \tag{5}$$

**Table 3 : ANOVA table for adequacy of the developed models**

| Sl. No. | Response Parameter | First Order terms | | Second order terms | | Lack of fit | | Error-terms | | F-ratio | R-ratio | R | Adequacy of the model |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | SS | df | SS | df | SS | df | SS | df | | | | |
| 1 | Angular distortion | 15.84 | 4 | 3.57 | 10 | 0.32 | 10 | 0.17 | 5 | 0.914 | 39.71 | 0.975 | Adequate |

### 3.7 Checking the Regression Coefficients for Significance

The quantitative effect of the input welding parameters on angular distortion is given by the values of regression coefficients. Insignificant coefficients which probably do not contribute much in predicting the response can be conveniently dropped, along the terms they are associated with, without losing accuracy of the model and thereby avoiding unnecessary calculation work. Student's t-test was used for this purpose [15, 19].

The t value can be obtained by using the relation given in equation (6) [19].

$$t = \frac{\beta_j}{S\beta_j} \tag{6}$$

For a regression coefficient to be significant, the standard value of t that can be obtained from a table (for a confidence level of 95%) must be greater than its calculated value obtained from the above relation.

The values for $S\beta_j$ can be calculated for linear, quadratic and interaction terms using the following equations respectively for four factors central composite design matrix employed in the present study [17, 19].

$S\beta j = 0.204$ S. E

$S\beta jj = 0.185$ S. E

$S\beta ij = 0.250$ S. E

Where S. E is the standard error calculated from the replication of centre point.

In the present case, the tabulated value of t for 6 df of error is 2.447.

In this way, the identified insignificant coefficients along with their associated terms were eliminated from the model [20, 21]. The final mathematical model constructed by dropping the insignificant terms is given in the form of equation (7).

$$\text{Angular Distortion} = 4.36 - 0.005W - 0.11V + 0.005S - 0.06\theta + 0.02W\,V - 0.009W\,S + 0.004W\,\theta - 0.003V\,S + 0.0018V\,\theta - 0.0014\,S\,\theta + 0.002W2 + 0.0026V2 + 0.0016\,S2 + 0.006\,\theta 2 \tag{7}$$

The scatter diagram generated by the design expert software is shown in **Fig. 5**. The closeness of the predicted and actual values on the diagram proves the validity of the developed model, graphically.

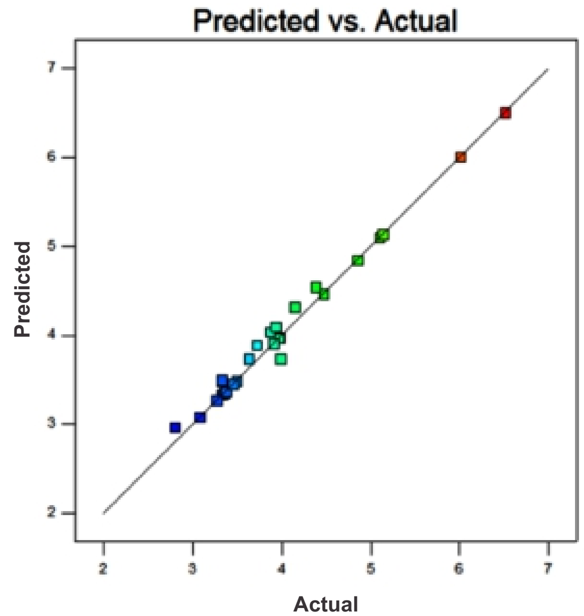


**Fig. 5 : Scatter diagram showing actual Vs predicted values of angular distortion**

### 3.8 Analysis of Results

The results obtained through design expert software in the form of graphs are given in **Fig. 6** through **15**. The graphs and 3D response surfaces so drawn depicted satisfactorily the effects of input parameters on angular distortion. The probable reasons for these effects were analysed and presented under the following headings;

#### 3.8.1 Direct influence of wire feed rate on angular distortion

Angular distortion increased directly and more pronouncingly with the increase in W as shown in **Fig. 6**, keeping V, S and θ constant. This is because with the increase in W, the welding current increased resulting in more heat input into the weld, causing increased angular distortion.

#### 3.8.2 Direct influence of arc voltage on angular distortion

It is evident from **Fig. 7** that angular distortion increased with the increase in V but not as strongly as with the increase in W. The minor increase in angular distortion might be associated with the fact that increased voltage cause more input of heat and more spreading of arc thereby increasing the temperature around the weld bead, causing more distortion of the base metal.

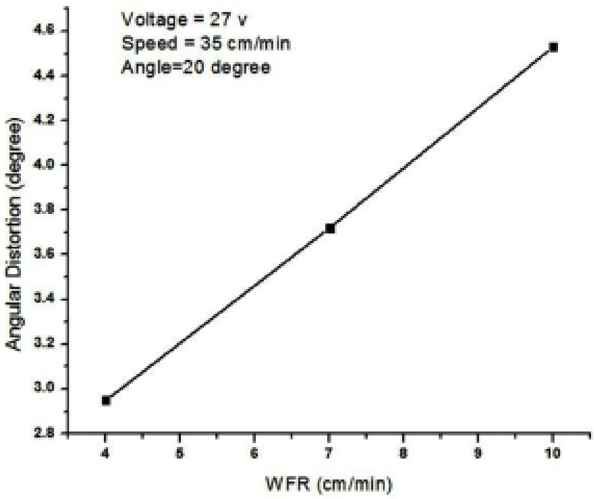


Fig. 6 : Effect of WFR on angular distortion

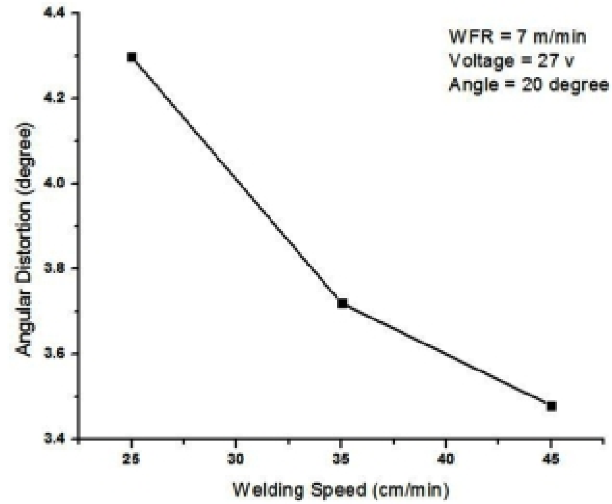


Fig. 8 : Effect of welding speed on angular distortion

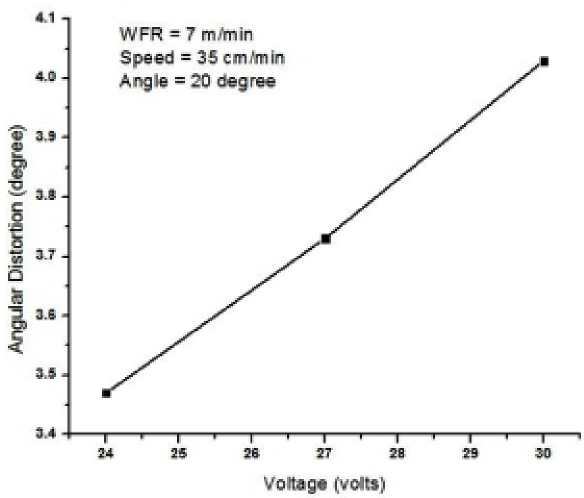


Fig. 7 : Effect of voltage on angular distortion

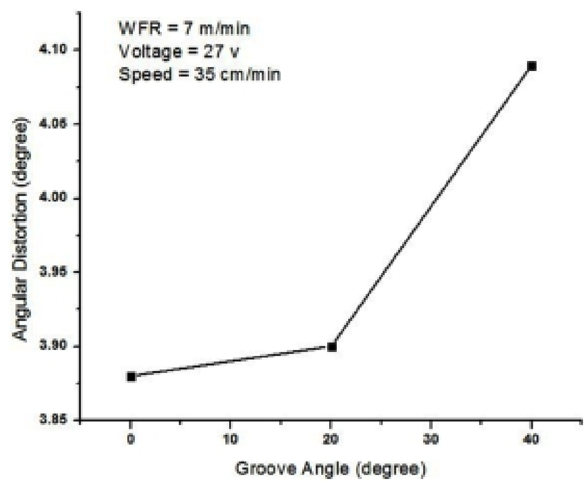


Fig. 9 : Effect of groove angle on angular distortion

### 3.8.3 Direct influence of welding speed on angular distortion

It is apparent from **Fig. 8** that the angular distortion reduced as the welding speed increased; this is because with the welding speed increased, the amount of heat going into the weld was reduced. Due to the reduction in heat input, the amount of metal melted was less and the heat did not get enough time to spread into the base metal resulting in less likelihood of angular distortion.

### 3.8.4 Direct influence of groove angle on angular distortion

It is evident from **Fig. 9** that angular distortion increased slightly with the increase in groove angle. The reason could be attributed to as the groove angle increased the amount of molten metal that could be accommodated in the groove also increased. This lead to more heating of the base plates up to the full depth causing more distortion, whereas, in square butt joint, the molten metal tends to sit on the top surface of the joint, resulting in shallow penetration of heat.

### 3.8.5 Interaction effects of WFR and arc voltage

A 3-D response surface plot in **Fig. 10** shows the combined effect of WFR and V on angular distortion. For all the values of voltage, angular distortion increased with increase in WFR. A maximum angular distortion of 5° was obtained at maximum values of WFR and voltage. The effect of WFR on angular distortion was more than that of V. When WFR is between 4 – 7 m/min, there is insignificant change in angular distortion with

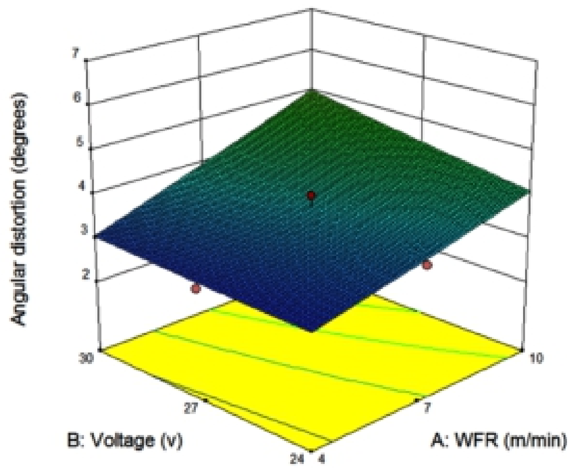


**Fig. 10 : Influence of WFR and voltage on angular distortion [Welding speed = 35 cm/min, Groove angle = 20°]**

increase in V. from 7 – 10 m/min WFR, the positive effect of V on angular distortion is visible.

### 3.8.6 Interaction effects of WFR and welding speed

It is clear from **Fig. 11** that WFR has increasing and wielding speed has decreasing effect on angular distortion. A maximum distortion of 5.3° was therefore obtained at maximum value of WFR (10 m/min) but minimum value of welding speed (25 cm/min) and a minimum distortion of 3° were obtained at maximum welding speed (45 cm/min) and minimum WFR (4 m/min). It is also evident that the rate at which distortion increased with WFR is higher at lower values of welding speed, than at its higher values.

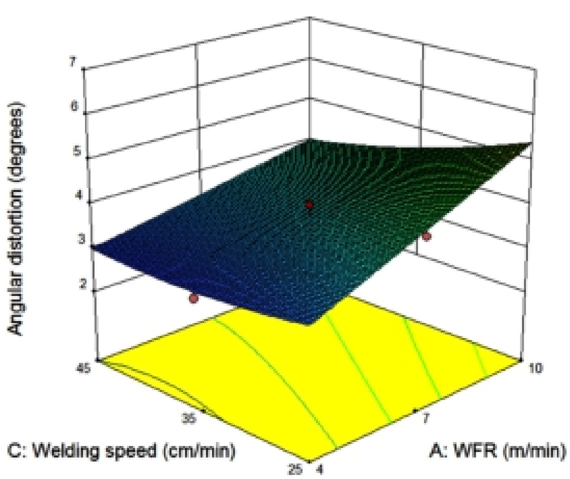


**Fig. 11 : Influence of WFR and welding speed on angular distortion [Voltage = 27 V, Groove angle = 20°]**

### 3.8.7 Interaction effects of WFR and groove angle

From **Fig. 12**, it is evident that for all value of θ, angular distortion increased with increase in WFR. Though θ also had a positive effect on distortion but its effect was less as compared to WFR within the selected range of groove angle. Therefore, there is a marginal increase in distortion with increased value of θ. Maximum value of distortion (4.9°) was obtained at maximum values of WFR and θ. Whereas minimum distortion (3°) was obtained at minimum values of WFR and θ.

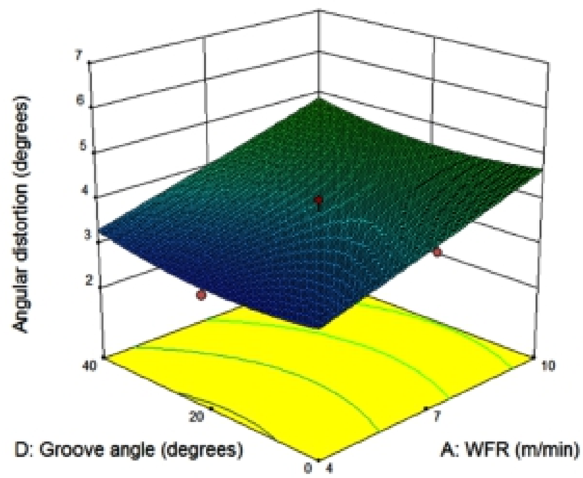


**Fig. 12 : Influence of WFR and groove angle on angular distortion [Voltage = 27V, Welding speed = 35 cm/min]**

### 3.8.8 Interaction effects of welding speed and voltage

It is evident from **Fig. 13** that voltage has increasing while speed has decreasing effect on angular distortion. Therefore, maximum distortion of 4.6° was obtained at maximum voltage of 30V and minimum speed of 25 cm/min. whereas minimum distortion (3.2°) was obtained at minimum voltage of 24V and maximum speed of 45cm/min.

### 3.8.9 Interaction effects of voltage and groove angle

**Fig. 14** shows that voltage and groove angle has mild positive effect on distortion. Maximum distortion of 4.6° was obtained at maximum values of voltage and groove angle, whereas, minimum value of distortion (3.6°) was obtained at minimum values of voltage and groove angle.

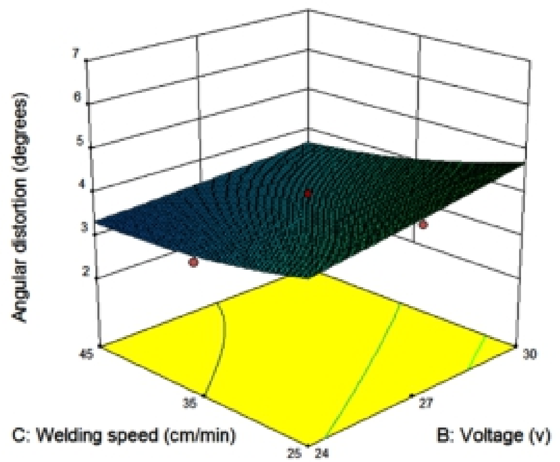


**Fig. 13 : Influence of welding speed and voltage on angular distortion [WFR = 7 m/min, Groove angle = 20°]**

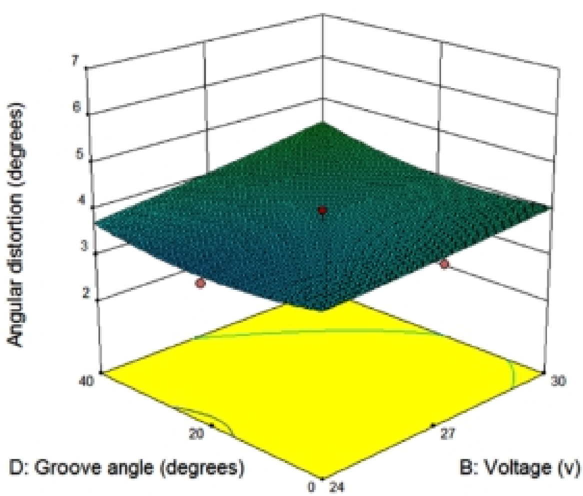


**Fig. 14 : Influence of voltage and groove angle on angular distortion [WFR = 7 m/min, Welding speed = 35 cm/min]**

### 3.8.10 Interaction effects of welding speed and groove angle

It is evident from **Fig. 15** that groove angle has increasing and speed has decreasing effect on angular distortion. A maximum distortion (4.7°) was obtained at maximum value of groove angle and minimum value of speed, whereas a minimum value of distortion (3.5°) was obtained at maximum speed and minimum groove angle.

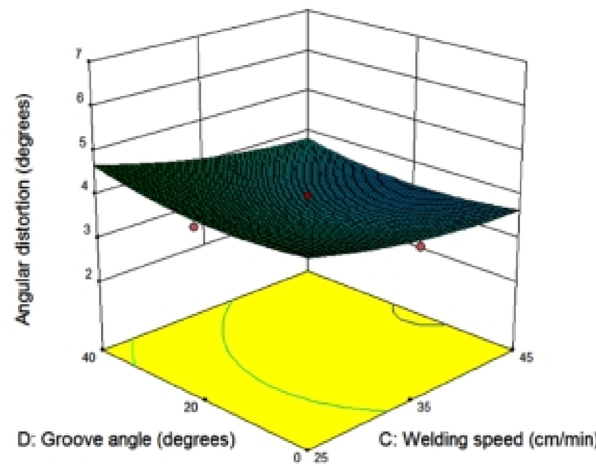


**Fig. 15 : Influence of groove angle and welding speed on angular distortion [WFR = 7 m/min, Voltage = 27V]**

## 3.9 Validation of the Developed Model

Experimental runs were carried out at different values and combinations of the input parameters. The actual values of angular distortion were measured as explained in section 3.4.1. These values were then compared with the results obtained by putting the same input parameters in equation (A) developed in section 3.8. All these observations are summarized in **Table 4**.

**Table 4** shows a percentage error between a range of ± 3.5%, which is quite within the practical permissible limits.

**Table 4 : Validation Table**

| Experiment No. | Input paraeters (actual values) | | | | Angular Distortion | | (%) Error |
|---|---|---|---|---|---|---|---|
| | W (m/min) | V (volts) | S (cm/min) | Θ (deg.) | Predicted | Measured | |
| 1 | 5 | 25 | 30 | 0 | 3.75 | 3.88 | - 3.35 |
| 2 | 6 | 25 | 30 | 20 | 6.32 | 6.12 | 3.26 |
| 3 | 8 | 28 | 40 | 0 | 4.40 | 4.28 | 2.8 |
| 4 | 10 | 30 | 25 | 10 | 6.13 | 6.00 | 2.16 |

## 4.0 CONCLUSIONS

On the basis of the investigative work undertaken, the concluding remarks are as under.

1. The CCFD approach has proved its suitability to the extent of satisfaction for mathematical modelling for predicting the direct, interactive and quadratic effects of input welding parameters on angular distortion.
2. WFR has the strongest positive effect on angular distortion.
3. Arc voltage has a less positive effect on angular distortion when compared with WFR.
4. Speed of welding has a negative effect on angular distortion.
5. Groove angle has a lesser positive effect on angular distortion when compared with voltage within the selected range.

## REFERENCES


[1] Seong WJ (2019); Prediction and Characteristics of Angular Distortion in Multi-Layer Butt Welding, Journal of Materials, 12, pp.1-19.

[2] Kozak J and Kowaslski J (2015); Problems of Determination of Welding Angular Distortions of T-Fillet Joints in Ship Hull Structures, Polish Maritime Research, 22, pp.79-85.

[3] Arifin A, Gunawan, Mataram A, Yani I, Pratiwi DK, Yanis M and Sani KA (2019); Optimization of Angular Distortion on Weld Joints Using Taguchi Approach, Jurnal Kejuruteraan, 31(1), pp.19-23.

[4] Duhan S, Mor A and Malik D (2015); A Review of Angular Distortion and Its Prevention Techniques, International Journal of Recent Research Aspects, pp.135-137.

[5] Kumar A (2011); Effect of Various Parameters on Angular Distortion in Welding, International Journal of Current Engineering and Technology, pp.132-136.

[6] Yukler A I, Kurtulmus M and Dogan E (2018); Angular Distortion in Butt Arc Welding, International Journal of Engineering Science and Application, 2(4), pp.137-144.

[7] Murugan Vel V and Gunaraj V (2005); Effect of Process Parameters on Angular Distortion of Gas Metal Arc Welded Structural Steel Plates, Welding Journal Supplement, 84, pp. 165s-171s.

[8] Arya SK (1985); Development of mathematical models for angular distortion in butt welds, M.Tech Thesis submitted to the Department of Mechanical Engineering, IIT, Delhi, India.

[9] Arya SK and Parmar RS (1986); Mathematical models for predicting angular distortion in $CO_2$ shielded flux cored arc welding, Proceedings of the International Conference on Joining of Metals (JOM–3), Denmark, pp. 240–245.

[10] Mochizuki M and Okano S (2018); Effect of Welding Process Conditions on Angular Distortion Induced by Bead-on-plate Welding. ISIJ International, 58(1), pp.153-158.

[11] Murugan N and Gunaraj V (2005); Prediction and control of weld bead geometry and shape relationships in submerged arc welding of pipes, Journal of Material Processing Technology, 168, pp.478-484.

[12] Murugan M (1993); Some Studies on Metal Surfacing, Ph.D. Thesis submitted to the Department of Mechanical Engineering, IIT Delhi, India.

[13] Mostafa NB (1992); Flux cored arc welding of high strength low alloy steel, Ph.D. Thesis submitted to the Department of Mechanical Engineering, IIT Delhi, India.

[14] El-Gendy N, Madian HR and Abu ASS (2013); Design and optimization of a process for sugarcane molasses fermentation by saccharomyces cerevisiae using response surface methodology, International Journal of Microbiology.

[15] Akhnazarova S and Kafarov V (1982); Experiment Optimization in Chemistry and Chemical Engineering, Mir Publishers, Moscow.

[16] Khuri AI and Cornell JA (1987); Response Surfaces–Design and Analysis, Marcel Dekker Inc., New York.

[17] Cochran WG and Cox GM (1963); Experimental Designs, Asia Publishing House, New Delhi.

[18] Hill HE and Prane JW (1984); Applied Techniques in Statistics for Selected Industries, John Wiley & Sons, New York.

[19] Davies OL (1978); The Design and Analysis of Industrial Experiments, Longman Group Limited, New York.

[20] Adler YP, Markov EV and Granovsky YV (1975); The Design of Experiments to Find Optimal Conditions". MIR Publishing, Moscow.

[21] Box GEP, Hunter WG and Hunter JS (1978); Statistics for Experimenters, An Introduction to Design, Data Analysis, and Model Building, John Wiley & Sons New York.