DHĀRANA
INTERNATIONAL JOURNAL OF BUSINESS FROM BHARATIYA VIDYA BHAVAN'S
M. P. BIRLA INSTITUTE OF MANAGEMENT, BENGALURU
Vol.11, #2 (2017) pp 05-12 ISSN 0974-0082

A Cuboid Model for Coverage Processes

T.V. Srinivas[1], T. Srivenkataramana[2], T.V. Raju[3], N.S. Viswanath[4]

Abstract

This paper introduces the concept of a coverage process. It proposes and analyses a cuboid model for coverage processes seeking multi-dimensional expansion. The WHO view of Universal Healthcare is used as a seed to first develop a cube model. The conditions for optimum coverage are derived. The model is then generalized into a cuboid and some of its mathematical properties are investigated. The model has applications in areas like universal insurance, multiple financial inclusion campaign, immunization drives and service quality. This model is forerunner for intrinsic link between organization's objectives with that of service quality delivery for effective relationship between clients & the organization.

Key words and Phrases: *Coverage rate; Cuboid model; Geometric mean; Impact factor; Linear, Geometric and Continuous changes; service quality, Total Service Quality (TSQ) & Universal healthcare.*

1. Introduction

The performance and outreach of growth processes are to be evaluated from time to time. This is true of field of education, economic development, public healthcare, insurance and so on. Several parameters jointly determine the coverage of the targeted population by these programs. The improvements in these measurement parameters are attempted simultaneously through awareness programs and policy directions. We propose a geometric model for this situation and analyze it mathematically, deriving several interesting properties, which have policy implications. The rest of the paper is organized as follows. Section 2 introduces the concept of coverage and proposes a geometric cube model, along with examining its properties. Sections 3 and 4 discuss area of application and a generalization to k dimensions. The final section offers a discussion.

2. Concept of coverage

Growth studies often consider coverage of a target population from several standpoints. The progress on each of these fronts is measured in terms of a coverage parameter. Some examples are in the following table.

1. Research Scholar of Manipal University 2. Professor Emeritus in Statistics & Former Director, Canara Bank School of Management, Erstwhile Bangalore University 3. Director R.V. Institute of Management, Bengaluru 4. Director & Principal, M.P. Birla Institute of Management, respectively.

Table 1: Programs and Parameters

Sl. No.	Base	Parameters	Program
1	Child Health	Proportion of children covered	Immunization
2	Primary Education	Proportions of enrollment, dropout	Universal child education
3	Human Development	Per capita income, Longevity of life, Literacy rate and Quality of life	Programs related income generation, healthcare and Universal education
4	Public Healthcare	Proportions of population, costs and covered ailments	Public sector and PPP healthcare models
5	Maternity Health	Proportion of women covered in child-bearing group, maternity and infant mortality rates	Maternal healthcare packages and related schemes
6	Nutrition for School Children	Proportion of school children covered, Quality and Nutrition value of food	Mid-day meal schemes

In each of the above situations, it is convenient to visualize that a geometric figure is created, which is desired to be *covered optimally.* Thus, with a single parameter there will be a *line segment* and optimization implies a push in just one direction. With two parameters, a rectangle will be created in two dimensions, with the parameters as occupants of the axes. The optimum *coverage* occurs when the *area* of the rectangle is *maximized,* for a given perimeter. This occurs when the rectangle turns into a *square,* calling for equal paced push in both the directions. In the three parameter case, a cube is formed, with the parameters along the three directions. We analyze this case mathematically at some length now. Then consider the *Yeshasvini Health Scheme* of the Karnataka Government, which is a PPP model of health cost coverage for selected ailments and for specified types of expenses for enrolled members of co-operative societies. The scheme can be easily fitted into the framework developed, with the following three parameters *viz.* Proportion of

1. population covered (p1)
2. medical expenses covered (p2)
3. health package covered (p3)

The first parameter is to be improved through awareness drives/ campaigns, while the other two are fallouts of policy decisions.

Working with proportions has a specific in-built advantage that they lie in the interval [0, 1], and hence finally create a geometric figure with each side of length unity. As a result, the figure has length/ area/ volume of magnitude one unit.

We propose a cube model for three-factor coverage situation of Yeshsvini scheme, starting with the definition of Universal Health Coverage of the WHO, as the seed.

2.1 Universal Health Coverage: WHO view

The WHO describes Universal Health Coverage (UHC), aimed to be achieved by the year 2030, as a state where *the health needs of all the citizens are met without any of them experiencing financial hardship.* It displays UHC as a cube with three dimensions – population coverage, service coverage (health package, availability) and cost coverage. It is hoped that this cube is filled up step by step in a phased manner through commitment, consensus and participatory leadership. Each of the three axes must show progressive increments by overcoming electoral populism, personal preferences of medical personnel and hindered flow of public finance. The costs may be covered *via* tax-funding or social insurance. Relevant factors must be integrated into the package design. The healthcare divide must be bridged. The next phase of Indian Economic Revolution (started in 1991) is expected to occur from the services sector (and not manufacturing), which includes health services.

2.2 The UHC cube for Yeshasvini scheme

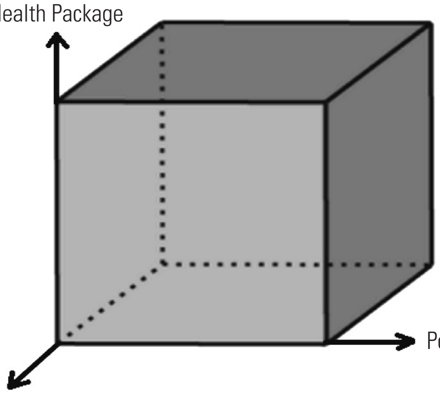

Figure 1: Cube Model for Yeshasvini Scheme

Consider this as a hollow standardized unit cube, that is the maximum in each of the three dimensions (population, health package and cost coverage) is *unity*. As the rates move along the axes, the hollow of the cube gets filled up. Denote the currently reached proportions by (p_1, p_2, p_3), so that the filled up volume is

$$V_3 = p_1\, p_2\, p_3$$

… (1)

This is a good indicator (*impact factor*) of the level (proportion) of health coverage accomplished in the target population, the maximum being clearly unity. This occurs when $p_1 = p_2 = p_3 = 1$. V_{max} has 1 as its value. In general, the volume gets maximized, for given $\sum p_i = q$, when $p_1 = p_2 = p_3 = q/3$. This represents equal values for the three proportions. This is also the point where the geometric mean of the p_i equals the arithmetic and harmonic means. The above result also shows the importance of balanced progresses in each of the three aspects. Alternatively, even if one of the progress directions is *unsatisfactory*, the entire coverage picture becomes murky.

Moving on with this scenario, the correct *average* progress is NOT the arithmetic mean

$$A_3 = (p_1 + p_2 + p_3)/\,3$$

… (2)

but rather the geometric mean

$$G_3 = (p_1\, p_2\, p_3)^{1/3}$$

… (3)

which is nearly zero when *any one* of the proportions is near zero. For instance with 10%, 20% and 40% coverage in the three directions the overall coverage is 20%. But if the 10% coverage is reduced to just 1%, the coverage percentage is now 100 $(0.0008)^{1/3}$ or less than 10%. A coverage of even 40% in one of the directions is hardly able to lift up the overall coverage percentage. Thus what matters most for *optimum* effectiveness is an *equitable coverage* in the three directions. For instance, a 10% coverage in each of the three fronts results in an overall coverage also of 10%, which is better than the percentage triplet (1, 20, 40). This shows the collective role of the three realized directional proportions. The rate G_3 represents the rate at which the cube gets occupied.

For *continuously varying proportions,* the rates of change in the occupied portion of the cube are given by the *partial derivatives* of V_3 with respect to the parameters p_1, p_2 and p_3. These are respectively given by $p_2\, p_3$, $p_1\, p_3$ and $p_1\, p_2$. The overall penetration of the program can be measured in terms of the filled up content of the cube *viz.* V_3, which may be therefore termed as *Total Impact Factor* (TIF). This is maximized, as already mentioned, when the parameters progress at equal pace. Eventually, the hollow cube gets filled up to signal 100% coverage by The Scheme. A generalization of the cube model to *k*-dimensions will be considered in section 4.

Budget allocation for optimum coverage

The filled up volume of the cube is $V_3 = p_1\, p_2\, p_3$ which is maximized, for given $p_1 + p_2 + p_3 = C$, $(0 \le C \le 3)$ when $p_1 = p_2 = p_3$ $(= C/3)$. This is geometrically akin to a rectangle of given perimeter reducing to a square when the area of the figure is to be maximized. This calls for equal paced increases in the p_i for optimum coverage, as mentioned earlier.

Let us now consider the situation with a fixed and given budget C_0 which is to be optimally allocated to the three components in order to maximize the resulting coverage. Let X_i denote the allocation to dimension *i* for *i* = 1, 2, 3. Then the constraint is

$$X_1 + X_2 + X_3 = C_0$$

… (4)

and the objective is to work out the allocation in order to achieve optimal coverage.

The coverage, measured by p_i in direction *i,* clearly depends on the allocation X_i. Thus

$$p_i = f(X_i)$$
... (5)

which represents the functional form of dependence. We examine two particular choices for *f (Xi).*

(a) Proportionality with X_i or $p_i = K_i X_i$

Then

$$V_3 = p_1 p_2 p_3 = (K_1 X_1)(K_2 X_2)(K_3 X_3)$$
... (6)

where the K i are the constants of proportionality. In order to have optimal coverage, we now have the condition

$$(K_1 X_1) = (K_2 X_2) = (K_3 X_3)$$
... (7)

subject to the constraint (4).

Substituting for X_2 and X_3 in (4) in terms of *X1* from (7) leads to

$$X_1 + (K_1/K_2) X_1 + (K_1/K_3) X_1 = C_0$$
$$\text{or } X_1 [1 + (K_1/K_2) + (K_1/K_3)] = C_0$$
$$\text{or } X_{1\,opt} = [(K_2 K_3)/ (K_1 K_2 + K_1 K_3 + K_2 K_3] C_0$$
... (8)

It may be noted that the constants of proportionality (K_i) can be different for the three directions. This allows flexible relations between improvement and cost implication.

The expressions for optimum X_2 and X_3 are similarly written down. It is easily verified that the budget constraint (4) is satisfied. Also for $K_1 = K_2 = K_3 = 1$, we get equal allocation of the budget.

(b) Proportionality with $\sqrt{X_i}$

The improvement is often much slower than the increase in the budget provision. Thus we may take

$$p_i = f(X_i) = K_i X_i^p \text{ for } i = 1, 2, 3$$
... (9)

Though any $p > 0$ may be considered, a choice of p as a fraction is quite realistic. A good choice is, therefore, $p = ½$, so that the coverage improvement is taken to be proportional to square root of the allocation, and

$$p_i = K_i \sqrt{X_i}$$
... (10)

Model (10) incorporates a damping effect on the improvement. For example, in order to double the coverage rate one has to raise the budget allocation four-fold.

For optimal growth now we have the condition

$$K_1\sqrt{X_1} = K_2\sqrt{X_2} = K_3\sqrt{X_3}$$
... (11)

subject to the constraint (4).

A straight forward recasting leads to

$$X_{1\,opt} = [(K_2^2 K_3^2)/ (K_1^2 K_2^2 + K_1^2 K_3^2 + K_2^2 K_3^2)] C_0$$
... (12)

The expressions for X_2 and X_3 are similarly written, noting the cyclic pattern.

Remarks

(1) In practice the constants K_i are to be assessed or estimated empirically.

(2) In the model (5) for p_i, no provision for an intercept (constant) term is made since $p_i = 0$ for $X_i = 0$ or without a budget provision there is no growth!

(3 Other choices for *p,* including different values of *p* for the different directions can also be made. The budget allocations may be worked out algebraically in such cases. Here we get the optimal condition as $K_1 X_1^p = K_2 X_2^q = K_3 X_3^r$, subject to the constraint

$$X_1 + X_2 + X_3 = C_0$$

However this entails the need for assessing three constants of proportionality and an equal number of indices *p, q* and *r.*

(4) The square root cost function is especially convenient since it bypasses assessing the indices *p, q* etc. and at the same time allows coverage to be adequately slower than the budget enhancement. In fact, such a function is known to be robust as a cost model.

(5) Typically the *Ki* are quite small as *pi* are just

proportions. Also the *Xi* may be expressed in larger monetary units like crores of rupees.

3. Areas of applications

These typically include situations where 'coverage' is dependent upon or measured in terms of several factors like the proportion of

1. target population reached
2. geographical area covered
3. facilities provided from a master-list
4. claims/ court cases cleared within a stipulated time etc.

Such examples are plenty in the problems in universal education programs, insurance schemes, healthcare facilities, mass drives of cleanliness, immunization, literacy and so on. It may be noted that India has launched a Universal Health Care project in the year 2010, with the aim of providing affordable, easily accessible healthcare *for all its citizens,* which has three clear dimensions. The model facilitates for making assessment of Total Service Quality in an organizational setting. The vision, mission, objectives of an organization can be organically linked to its performance in terms of meeting expectations of clients (consumers) of the organization. There is, however, need for a generalized setting of the model to expand its use in measuring organizational performance.

4. A generalization: Cuboid model

This concept of measuring net resultant effectiveness can be generalized to any multi-parameter program (for instance the Yeshasvini scheme) in a straight forward manner. With *k* measuring parameters, we can conceptualize the achieved coverage as a *subset* of a *k*-dimensional *cuboid.* Again the scenario of optimum penetration will call for equal progress rate in each of the directions. This is a position where the three mean rates (arithmetic, geometric and harmonic) coincide. This geometric argument establishes the following result:

In a coverage model with k rate parameters p_1, p_2, p_3 ...pk, the optimum (maximum) coverage level is attained, for a given $\sum p_i$, when the parameters move at equal pace.

The content of the above cuboid is given by the product

$$V_k = (p_1\, p_2\, p_3 \ldots p_k) \qquad \ldots (13)$$

In general, average coverage rate will be the geometric mean of the component rates given by

$$G_k = \left(\prod_{i=1}^{k} p_i\right)^{1/k} \qquad \ldots (14)$$

where the p_i are the coverage parameters (rates). This is the rate at which the *cuboid* gets filled up. Ultimate goal is to reach the roof of the cuboid, that is fill it up to the brim, when the hollow cuboid would have become solid. The average rate G_k is, as to be expected, sensitive to smallness of the p_i. As a property of geometric means, when each component rate increases uniformly by a factor of *r,* G_k also goes up by this factor.

The cuboid model is apt for coverage processes over a span of time.

4.1 A new service quality model

The cuboid model can be envisioned to provide an abstract framework for tracing the coverage of *service quality*. In the health sector setup, the dimensions could be empathy for the patient, warmth of doctor-patient relation, adherence to ethical practices, nursing skill and so on. This will pave way for a multiplicative effect of the factors, a consequence of which is the suitability of a geometric average as a summary. Sensitivity to low values of factors is an disadvantage with this measure. However the applicability of this model has to be supported by empirical evidence, unlike in the case of measurable physical coverage rates. This provides a promising opening for further work.

4.2 Effect of change in parameters

It is of interest to examine the effect of parameter variations on the content *Vk* of the cuboid. Let us consider three types of change - *incremental, geometric* and *continuous,* separately.

a) Linear change

Assume that there is an incremental change d_1 in p_1, so that p_1 is to be replaced by (p_1+d_1), other parameters remaining unchanged. Then the consequent change in the cuboid content given by

$$C_{k1} = (V_{k1} - V_k)$$

$$= [(p_1+d_1) \prod_{i=2}^{k} p_i] - [\prod_{i=1}^{k} p_i]$$

$$= d_1 \prod_{i=2}^{k} p_i \qquad \ldots (15)$$

This is simply the product of d_1, the incremental change in p_1, and the other rates. Also the change relative to V_k is

$$RC_k = \text{Change in content/ Initial content}$$

$$= C_{k1} / V_k$$

$$= d_1 / p_1 \qquad \ldots (16)$$

The new geometric mean becomes

$$G_{k1} = (p_1 + d_1)^{1/k} (\prod_{i=2}^{k} p_i)^{1/k}$$

and relative to Gk it is

$$RG_{k1} = G_{k1} / G_k$$

$$= (1 + d_1 / p_1)^{1/k} \qquad \ldots (17)$$

The generic expressions are obtained by replacing 1 with i in the above results.

Similarly, when two parameters p_1 and p_2 get incremental changes of d_1 and d_2 respectively, the content will change by

$$C_{k2} = (p_2 d_1 + p_1 d_2 + d_1 d_2) \prod_{i=3}^{k} p_i \qquad \ldots (18)$$

so that the change relative to Vk is now

$$RC_{k2} = C_{k2} / V_k$$

$$= ((p_2 d_1 + p_1 d_2 + d_1 d_2)/ (p_1 p_2))$$

$$= ((d_1 / p_1) + (d_2 / p_2) + (d_1 d_2 / p_1 p_2)) \qquad \ldots (19)$$

Likewise, the new geometric mean relative to G_k is

$$RG_{k2} = G_{k2} / G_k$$

$$= [(1 + (d_1 / p_1)) (1 + (d_2 / p_2))]^{1/k} \qquad \ldots (20)$$

The generalization of (19) involves higher order terms in the d_i, while (20) can be generalized in the obvious manner.

b) Multiplicative change

In this case evaluating the expressions for the change in cuboid content etc. is straight forward since V_k itself has a multiplicative structure. Thus when p_i changes to $r_i p_i$, the change in V_k is

$$C_k^* = r_i V_k - V_k$$

$$= (r_i - 1) V_k \qquad \ldots (21)$$

The change relative to V_k is $(r_i - 1)$.

When multiplicative changes occur in both p_i and p_j, the change in V_k works out to be

$$C_k^{**} = (r_i r_j - 1) V_k \qquad \ldots (22)$$

and the factor of change is $(r_i r_j - 1)$, and so on. Thus a simultaneous multiplicative change in all the parameters will lead to the content change of

$$(r_1 r_2 \ldots r_k - 1) Vk \qquad \ldots (23)$$

relative change factor being $(r1\ r2 \ldots rk - 1)$. Also the changed geometric average is

$$G\,k\,k = (\prod_{i=1}^{k} ri)\ 1/k\ Gk \qquad \ldots (24)$$

so that the change relative to Gk is

$$R\,G\,k\,k = (\prod_{i=1}^{k} ri)\ 1/k \qquad \ldots (25)$$

which is nothing but the geometric mean of the change factors.

Generalization to more than three dimensions can also be obtained to provide a cuboid model with optimal budget allocation. For instance, under square root cost function and with s dimensions the optimum cost allocation works out to be

$$X1\ opt = [(\prod_{i \neq 1} K i\, 2 + \prod_{i \neq 2} K i2 + .. + \prod_{i \neq s} K i2) / \prod_{i \neq 1} K i\, 2]\ C0$$

etc. which has a cyclic pattern.

c) Continuous change

Since the p_i are proportions which lie in the interval [0, 1], it makes sense to take that these are continuous parameters. Then the rate of change of V_k are simply the *partial derivatives* of V_k with respect to the p_i. Thus the rate of change with respect to p_1 is

$$\partial V_k / \partial p_1 = \partial (\prod_{i=1}^{k} p_i) / \partial p_1 = \prod_{i=2}^{k} p_i$$

and the change relative to V_k becomes $1/ p_1$. Likewise, the rate for simultaneous changes in both

p_1 and p_2 becomes

$$\partial^2 V_k / \partial p_1 \partial p_2 = \prod_{i=3}^{k} p_i$$

with a relative change of $1 / (p_1\ p_2)$. These changes are positive. The expressions lend themselves to straight forward generalization. In the passing, we may note that these results can be obtained for a change in p_1 from (17) and (18) by taking $d_1 = 1$. However this does not carry forward. This is true for change in one parameter at a time. This does not generalize for cases of simultaneous changes in the parameters.

For a comparability with the geometric change case, we have to choose $r_1 = (1 + 1/ p_1)$,

$r_1\ r_2 = (1 + 1/ (p_1\ p_2))$ etc. in the continuous case.

4.3 The direction of change

A change can be an increase or a *decrease.* In the case of cuboid content, this will depend on the *sign* of the first factors in (17), (20), (24) and (25), since the p_i are positive. In brief, a *sufficient* condition for the changes to be positive is that $d_i > 0$ for the linear case, though this condition may not be always necessary. The case $d_i = 0$ implies a situation of no *change*. Thus, as long as the first factors in the expressions are positive, the growth thrust will be positive even if some of the d_i slip down marginally.

For geometric changes in the factors, a sufficient condition for positive changes is $r_i > 1$. Again, this is not *necessary* to keep all the first factors in the expressions positive. Thus a dip below 1 in some r_i can still maintain a positive thrust in V_k. The case of every r_i being 1 corresponds to the scenario of no change.

Finally, every d_i being positive or every r_i exceeding unity ensures increase in the average change, though these conditions may not be *necessary.*

4.4 Remarks

The case of a cube obtains when $k = 3$

The linear and multiplicative cases are algebraically equivalent in the sense that $(p_i + d_i) = pi\,(1 + d_i / p_i)$, which may be taken as $p_i r_i$ with $r_i = (1 + d_i / p_i)$. However the two cases have been treated above separately in order to get explicit forms for clarity of presentation.

In practice, nobody has a direct control over the changes in the parameters, since they are to be *induced* through proactive measures like stepping up the awareness drive, streamlining the delivery systems or realigning the policies.

A positive thrust in each of the *k* dimensions, with equal importance given, is a safe policy for practice.

4.5 Necessary and sufficient conditions

a) With incremental changes d_i in p_i , a necessary and sufficient condition for overall positive increase in the cuboid content is

$$\prod_{i=1}^{k} (1 + d_i / p_i) > 1 \quad \ldots (26)$$

This also ensures an increase in the geometric mean coverage.

b) With geometric coverage rates in pi the necessary and sufficient condition can be expressed as

$$\prod_{i=1}^{k} r_i > 1 \quad \dots (27)$$

It can be easily verified that every d_i being positive implies condition (26), but not *conversely.* A similar statement can be made about each r_i exceeding 1 and the condition (27).

5. Discussion

In a coverage set up, the factors may be envisioned as forming the dimensional axes of a geometrical figure. The number of dimensions may be *one, two, three* or *more than three.* Geometrically, these will respectively create a line segment, a square, a cube or a cuboid. Without loss of generality and in order to allow compatibility, we may take the sides to be of unit length, since any proportion lies between 0 and 1. The coverage process starts with a blank/hollow figure, which gets filled up as the coverage improves. The process eventually terminates when the figure is completely filled up.

The cuboid model gives a clear basis for a coverage process and for analyzing its behavior. The results obtained here explain mathematically the conditions for an increase in overall coverage as well as establish the fact that optimum coverage occurs when all the rates increase at *equal pace,* creeping along the respective axes. This property has important policy implications in several fields.

References

1. http://www.yeshasvini.kar.nic.in/ 1. - retrieved on 30-Nov-14 at 10.34 PM

2. *World Health Atlas* 2011 of WHO.