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# Decentralized robust control for T-S fuzzy bi-linear interconnected system

*This paper presents decentralized fuzzy robust control for a class of nonlinear interconnected large-scale systems which is composed of a number of Takagi-Sugeno (T-S) fuzzy bi-linear subsystems with interconnections. Based on the Lyapunov stability analysis theory and the parallel distribute compensation scheme, some robust stabilization sufficient conditions are derived for the whole close-loop fuzzy interconnected systems. The corresponding decentralized fuzzy controller design is converted into a convex optimization problem with linear matrix inequality (LMI) constraints.*

***Keywords:*** Nonlinear interconnected system; fuzzy bi-linear model; decentralized control; robust stabilization; linear matrix inequality (LMI).

### 1.0 Introduction

Large-scale interconnected systems can be found in many real-life practical applications such as electric power systems, nuclear reactors, economic systems, process control systems, computer networks, and urban traffic network, etc. The properties of interconnected systems have been widely studied and many different approaches have been proposed to stabilize the interconnected linear systems [1][2]. On the other hand, there are a few studies concerning with the stabilization control for the interconnected nonlinear systems [3][4]. Since linearization technique and linear robust control are used, these results are always conservative and only applicable to some special nonlinear interconnected systems. Due to the physical configuration and high dimensionality of interconnected systems, a centralized control is neither economically feasible nor even necessary [5]. Therefore, decentralized scheme is preferred in control design of the large-scale interconnected systems [6]. However, due to the effects of nonlinear interconnection among subsystems, there is still no efficient way to deal with the decentralized control problem of nonlinear interconnected systems.

In recent years, T-S (Takagi-Sugeno) model-based fuzzy control has attracted wide attention, essentially because the fuzzy model is an effective and flexible tool for control of nonlinear systems [7][8]. In this approach, the T-S fuzzy model substitutes the consequent fuzzy sets in a fuzzy rule by a linear model. Local dynamics in different state-space regions are represented by linear models and the overall model of the system is represented as the fuzzy interpolation of these linear models. Just because of this, T-S fuzzy model has been paid considerable attention and is widely used to the control design of nonlinear interconnected systems [9]-[11]. The problem of stabilization of nonlinear interconnected systems was studied in [9], while robust stabilization of a class of multiple time-delay nonlinear interconnected systems was investigated in [10]. The paper [11] designed the $H_\infty$ controller to achieve the decentralized tracking for the nonlinear interconnected systems. The paper [12] proposed a local decentralized law to study the stabilization problem of fuzzy interconnected systems with delay. It is noted that the above nonlinear interconnected systems are all based on T-S fuzzy linear model.

It is known that bi-linear models can be described many physical systems and dynamical processes in engineering fields [13][14]. There are two main advantages of the bi-linear system. One is that it provides a better approximation to a nonlinear system than a linear one. Another is that many real physical processes may be appropriately modelled as bi-linear systems when the linear models are inadequate. A good example of a bi-linear system is the population of biological species described by $\frac{d\theta}{dt}=\theta v$, where $v$ is the birth rate minus death rate, and $\theta$ denotes the population. It is impossible to approximate the aforementioned equation by a linear model [13].

Considering the advantages of bi-linear systems and T-S fuzzy control, the bi-linear fuzzy system based on the T-S fuzzy model with bi-linear rule consequence was attracted the interest of researchers [15]-[17]. Just as the paper [15] pointed out: T-S fuzzy bi-linear model is not to be said to replace the well-known T-S fuzzy model. The T-S fuzzy bi-linear model may be suitable for some classes of nonlinear plants. The robust stabilization for continuous-time fuzzy bi-linear system (FBS) is studied in [15], then the result was extended to the

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FBS with time-delay only in the state [16]. The problem of robust stabilization for discrete-time FBS was investigated in [18]. So far, the decentralized control of nonlinear interconnected systems based T-S bi-linear model has not been discussed.

In this paper, we consider the decentralized robust control of nonlinear interconnected systems based on T-S bi-linear model. Based on the parallel distribute compensation (PDC) scheme, the robust stabilization conditions can be established, and moreover, the decentralized controller design procedure can cast as solving a set of linear matrix inequalities (LMIs).

Notation 1: Throughout this paper, a real symmetric matrix $P>0(P>0)$ denotes $P$ being a positive definite (or positive semi-definite) matrix. In symmetric block matrices, we use an asterisk ($*$) to represent a term that is induced by symmetry and *diag* {....} stands for a block-diagonal matrix. The notion $\sum_{i,j=1}^{s}$ means $\sum_{i=1}^{s}\sum_{j=1}^{s}$. Matrices, if the dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

## 2.0 System description

Consider a nonlinear interconnected large-scale system $\Omega$ composed of $S$ subsystems.

Each fuzzy rule of the subsystem $\Omega_i$ can be represented by a T-S bi-linear model as follows

$$\begin{aligned} R_i^m \;\; & if \;\; \xi_{i1}(t) \;\; is \;\; M_{i1}^m \;\; and \ldots and \;\; \xi_{iv_i}(t) \;\; is \;\; M_{iv_i}^m \\ & then \;\; \dot{x}_i(t) = (A_{im} + \Delta A_{im})x_i(t) + (N_{im} + \Delta N_{im}) \\ & \qquad \times x_i(t)u_i(t) + (B_{im} + \Delta B_{im})u_i(t) \\ & \qquad + \sum_{j=1, j\neq i}^{S} C_{jim}x_j(t) \qquad m = 1,2,\ldots,r_i \end{aligned} \quad \ldots (1)$$

where $r_i$ is the number of the fuzzy rules. $\xi_{ij}(t)$ and $M_{ij}^m$, $j=1,2,\ldots,v_i$ are some measurable premise variables, and fuzzy sets. $x_i(t) \in R^{n_i}, u_i(t) \in R$ are the state vector and control input, respectively. $A_{im}$, $B_{im}$, $N_{im}$ denote the system matrices. $C_{jim}$ represents the interconnection matrix between subsystems. $\Delta A_{im}$, $\Delta B_{im}$ and $\Delta N_{im}$ are real matrices, and are assumed to be of the $[\Delta A_{im}\ \Delta B_{im}\ \Delta N_{im}] = H_{im}F_{im}(t)[E_{i1m}\ E_{i2m}\ E_{i3m}]$ form, where $E_{i1m}$, $E_{i2m}$, $E_{i3m}$, $H_{im}$ are known real constant matrices of appropriate dimension, and $F_{im}(t)$ is an unknown matrix function with Lebesgue-measurable elements and satisfies $F^T_{im}(t)F_{im}(t)<I$ for all $t$.

By using singleton fuzzifier, product inferred, and weighted defuzzifier, the system can be expressed by the following globe model:

$$\begin{aligned} \dot{x}_i(t) = \sum_{m=1}^{r_i} h_{im}(\xi_i(t))[(A_{im} + \Delta A_{im})x_i(t) + (N_{im} \\ + \Delta N_{im})\, x_i(t)u_i(t) + (B_{im} + \Delta B_{im})u_i(t) \\ + \sum_{j=1, j\neq i}^{S} C_{jim}x_j(t)] \end{aligned} \quad \ldots (2)$$

where $h_{im}(\xi_i(t)) \geq 0$ and $\sum_{m=1}^{r_i} h_{im}(\xi_i(t)) = 1$ for all $t$. Based on PDC, the fuzzy controller shares the same premise parts with (1); that is, the $i$th fuzzy controller is formulated as follow

$R^i$ if $(\xi_{i1}(t))$ is $M^m{}_{i1}$ and ... and $\xi_{ivi}(t))$ is $M^m{}_{ivi}$ then

$$\begin{aligned} u_i(t) &= \frac{\rho_i F_{im} x_i(t)}{\sqrt{1 + x_i^{\mathrm{T}} F_{im}^{\mathrm{T}} F_{im} x_i}} \\ &= \rho_i \sin\theta_{im} = \rho_i \cos\theta_{im} F_{im} x_i(t) \end{aligned} \quad \ldots (3)$$

where $F_{im} \in R^{1\times n_i}$ is a local controller gain and $\rho_i > 0$ is a scalar to be assigned.

The overall fuzzy control law can be represented by

$$\begin{aligned} u_i(t) &= \sum_{m=1}^{r_i} h_{im}\rho_i \sin\theta_{im} \\ &= \sum_{m=1}^{r_i} h_{im}\rho_i \cos\theta_{im} F_{im} x_i(t) \end{aligned} \quad \ldots (4)$$

By substituting (4) into (2), the th closed-loop subsystem can be represented as

$$\begin{aligned} \dot{x}_i(t) = \sum_{m,n=1}^{r_i} h_{im}h_{in}[\Theta_{i,mn}x_i(t) \\ + \sum_{j=1, j\neq i}^{s} C_{jim}x_j(t)] \end{aligned} \quad \ldots (5)$$

where $\Theta_{i,mn} = (A_{im} + \Delta A_{im}) + \rho_i \sin\theta_{in}(N_{im} + \Delta N_{im}) + \rho_i \cos\theta_{in}(B_{im} + \Delta B_{im})F_{in}$.

The objective of the paper is to design decentralized fuzzy controllers (4) such that the closed-loop systems (5) is decentralized robust stability.

### 2.1 Main results

Before proceeding with the following theorems, we introduce the following lemmas which will be used in our results.

Lemma 1[11]: Given any matrices $M$ and $N$ with appropriate dimensions such that $\varepsilon > 0$, we have $M^{\mathrm{T}}N + N^{\mathrm{T}}M \leq \varepsilon M^{\mathrm{T}}M + \varepsilon^{-1}N^{\mathrm{T}}N$.

Lemma 2[12]: Let $M$, $N$ and $F(t)$ be real matrices of appropriate dimensions with $F(t)^{\mathrm{T}}F(t) \leq I$. For scalar $\varepsilon > 0$, we have $M^{\mathrm{T}}F(t)N + N^{\mathrm{T}}F^{\mathrm{T}}(t)M \leq \varepsilon M^{\mathrm{T}}M + \varepsilon^{-1}N^{\mathrm{T}}N$.

The following theorem gives the sufficient conditions for the existence of the fuzzy decentralized controller for the interconnected system (5).

Theorem 1 For given scalars $\rho > 0$, $\varepsilon_{1i} > 0$, $\varepsilon_{2i} > 0$, $\varepsilon_{3i} > 0$, $i = 1,2,\ldots, S$, the interconnect system (5) is decentralized robust stability if there exist matrices $P_i > 0$, $i = 1,2,\ldots S$ and $F_{im}$, $i = 1,2,\ldots, S$; $m = 1,2,\ldots, r_i$ such that the following inequality (6) is satisfied.

$$\Xi_{i,mm} < 0, \quad i = 1,2,\ldots,S; m = 1,2,\ldots,r_i \quad \ldots (6a)$$

$$\Xi_{i,mn} + \Xi_{i,nm} < 0, \; i = 1,2,\ldots,S; \; 1 \leq m < n \leq r_i \quad \ldots (6b)$$

where $\Xi_{i,mn} = \phi_{i,mn} + \sum_{j=1, j\neq i}^{S} P_i C_{jim} C_{jim}^{\mathrm{T}} P_i + (S-1)\mathrm{I}$;

$$\begin{aligned} \phi_{i,mn} = A_{im}^{\mathrm{T}}P_i + P_i A_{im} + (\varepsilon_{1i} + \varepsilon_{3i}\rho_i^2)P_i H_{im} H_{im}^{\mathrm{T}} P_i \\ + \varepsilon_{2i}\rho_i^2 P_i P_i + \varepsilon_{1i}^{-1} E_{i1m}^{\mathrm{T}} E_{i1m} + \varepsilon_{2i}^{-1} N_{im}^{\mathrm{T}} N_{im} \\ + \varepsilon_{2i}^{-1}(B_{im}F_{in})^{\mathrm{T}}(B_{im}F_{in}) + \varepsilon_{3i}^{-1} E_{i2m}^{\mathrm{T}} E_{i2m} \\ + \varepsilon_{3i}^{-1}(E_{i3m}F_{in})^{\mathrm{T}}(E_{i3m}F_{in}). \end{aligned}$$

Proof: Take the Lyapunov function candidate as

$$V(t)=\sum_{i=1}^{S}V_i(t)=\sum_{i=1}^{S}x_i^{\mathrm{T}}(t)P_ix_i(t) \qquad \text{... (7)}$$

The time derivatives of $V(t)$, along the trajectory of the system (5) is given by

$$\begin{aligned}\dot V(t)=&\sum_{i=1}^{S}\sum_{m,n=1}^{r_i}h_{im}h_{in}[x_i^{\mathrm{T}}(t)(\Theta_{i,mn}^{\mathrm{T}}P_i\\&+P_i\Theta_{i,mn})x_i(t)+\sum_{j=1,j\neq i}^{S}x_j^{\mathrm{T}}(t)C_{jim}^{\mathrm{T}}\\&\times P_ix_i(t)+x_i^{\mathrm{T}}(t)P_i\sum_{j=1,j\neq i}^{S}C_{jim}x_j(t)]\end{aligned} \qquad \text{... (8)}$$

Considering :

$$\begin{aligned}\Theta_{i,mn}^{\mathrm{T}}P_i+P_i\Theta_{i,mn}=&(A_{im}+\Delta A_{im})^{\mathrm{T}}P_i+P_i(A_{im}\\&+\Delta A_{im})+(\rho_i\sin\theta_{in}(N_{im}+\Delta N_{im}))^{\mathrm{T}}P_i\\&+P_i(\rho_i\sin\theta_{in}(N_{im}+\Delta N_{im}))+(\rho_i\cos\theta_{in}\\&\times(B_{im}+\Delta B_{im})F_{in})^{\mathrm{T}}P_i+P_i(\rho_i\cos\theta_{in}(B_{im}\\&+\Delta B_{im})F_{in})\end{aligned} \qquad \text{... (9)}$$

Applying Lemma 1 and Lemma 2, we get

$$\Theta_{i,mn}^{\mathrm{T}}P_i+P_i\Theta_{i,mn}\leq\phi_{i,mn} \qquad \text{... (10)}$$

Similar, applying Lemma 1, we have

$$\begin{aligned}&\sum_{i=1}^{S}\sum_{m=1}^{r_i}h_{im}[\sum_{j=1,j\neq i}^{S}x_j^{\mathrm{T}}(t)C_{jim}^{\mathrm{T}}P_ix_i(t)\\&\quad+x_i^{\mathrm{T}}(t)P_i\sum_{j=1,j\neq i}^{s}C_{jim}x_j(t)]\\&\leq\sum_{i=1}^{S}\sum_{m=1}^{r_i}h_{im}h_{in}[x_i^{\mathrm{T}}(t)P_i\sum_{j=1,j\neq i}^{S}C_{jim}C_{jim}^{\mathrm{T}}\\&\quad\times P_ix_i(t)+(S-1)x_i^{\mathrm{T}}(t)x_i(t)]\end{aligned} \qquad \text{... (11)}$$

Substituting (10) and (11) into (8) yields

$$\begin{aligned}\dot V(t)\leq&\sum_{i=1}^{s}\sum_{m=1}^{r_i}h_{im}^2x_i^{\mathrm{T}}(t)\Xi_{i,mm}x_i(t)\\&+\sum_{i=1}^{s}\sum_{1=m<n}^{r_i}h_{im}h_{in}x_i^{\mathrm{T}}(t)(\Xi_{i,mn}+\Xi_{i,nm})x_i(t)\end{aligned} \qquad \text{... (12)}$$

Therefore, it is noted that (6) implies $\dot V(t)<0$, so the interconnected system (5) is robust stability. Thus, we complete the proof.

The matrix inequality (6) leads to bi-linear matrix inequality (BMI) optimization, a non-convex programming problem. Non-convexity implies the existence of local minima and the BMI problems are NP-hard. In the following theorem, we will derive a sufficient condition such that the matrix inequality (6) can be transformed into an LMI problem.

Theorem 2 For given scalars $\rho>0$, $\varepsilon_{1i}>0$, $\varepsilon_{2i}>0$, $\varepsilon_{3i}>0$, $i=1,2,\ldots,S$, the interconnect system (5) is decentralized robust stability if there exist matrices $Z_i>0$, $i=1,2,\ldots,S$ and $G_{im}$, $i=1,2,\ldots,S$; $m=1,2,\ldots,r_i$ such that the matrix inequality (13) is satisfied. Moreover, the feedback gains are given by $F_{im}=G_{im}Z^{-1}$, $i=1,2,\ldots,S$; $m=1,2,\ldots,r_i$.

$$\begin{bmatrix}\varphi_{i,m} & * & * & * & * & * & *\\ Z_i & -\frac{I}{s-1} & * & * & * & * & *\\ E_{i1m}Z_i & 0 & -\varepsilon_{1i}I & * & * & * & *\\ N_{im}Z_i & 0 & 0 & -\varepsilon_{2i}I & * & * & *\\ B_{im}G_{im} & 0 & 0 & 0 & -\varepsilon_{2i}I & * & *\\ E_{i2m}Z_i & 0 & 0 & 0 & 0 & -\varepsilon_{3i}I & *\\ E_{i3m}G_{im} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{3i}I\end{bmatrix}<0,$$
$$i=1,2,\ldots,S;m=1,2,\ldots,r_i \qquad \text{... (13a)}$$

$$\begin{bmatrix}\varphi_{i,m}+\varphi_{i,n} & * & * & * & * & * & * & * & * & * & * & *\\ 2Z_i & -\frac{2I}{s-1} & * & * & * & * & * & * & * & * & * & *\\ E_{i1m}Z_i & 0 & -\varepsilon_{1i}I & * & * & * & * & * & * & * & * & *\\ E_{i1n}Z_i & 0 & 0 & -\varepsilon_{1i}I & * & * & * & * & * & * & * & *\\ N_{im}Z_i & 0 & 0 & 0 & -\varepsilon_{2i}I & * & * & * & * & * & * & *\\ N_{in}Z_i & 0 & 0 & 0 & 0 & -\varepsilon_{2i}I & * & * & * & * & * & *\\ B_{im}G_{in} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{2i}I & * & * & * & * & *\\ B_{in}G_{im} & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{2i}I & * & * & * & *\\ E_{i2m}Z_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{3i}I & * & * & *\\ E_{i2n}Z_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{3i}I & * & *\\ E_{i3m}G_{in} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{3i}I & *\\ E_{i3n}G_{im} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{3i}I\end{bmatrix}<0,$$
$$i=1,2,\ldots,S;\ 1\leq m<n\leq r_i \qquad \text{... (13b)}$$

Proof: letting $P_i=Z_i^{-1}$ and noting $M_{im}=F_{im}Z$.

Then, pre-multiplying and post-multiplying $diag\{P_i,\overbrace{I,I,\ldots,I,I}^{11}\}$ and $diag\{P_i,\overbrace{I,I,\ldots,I,I}^{6}\}$ to (13a) and (13b), respectively, results in

$$\begin{bmatrix}\bar\varphi_{i,m} & * & * & * & * & * & *\\ I & -\frac{I}{s-1} & * & * & * & * & *\\ E_{i1m} & 0 & -\varepsilon_{1i}I & * & * & * & *\\ N_{im} & 0 & 0 & -\varepsilon_{2i}I & * & * & *\\ B_{im}F_{im} & 0 & 0 & 0 & -\varepsilon_{2i}I & * & *\\ E_{i2m} & 0 & 0 & 0 & 0 & -\varepsilon_{3i}I & *\\ E_{i3m}F_{im} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{3i}I\end{bmatrix}<0,$$
$$i=1,2,\ldots,S;m=1,2,\ldots,r_i \qquad \text{... (14a)}$$

$$\begin{bmatrix}\bar\varphi_{i,m}+\bar\varphi_{i,n} & * & * & * & * & * & * & * & * & * & * & *\\ 2I & -\frac{2I}{s-1} & * & * & * & * & * & * & * & * & * & *\\ E_{i1m} & 0 & -\varepsilon_{1i}I & * & * & * & * & * & * & * & * & *\\ E_{i1n} & 0 & 0 & -\varepsilon_{1i}I & * & * & * & * & * & * & * & *\\ N_{im} & 0 & 0 & 0 & -\varepsilon_{2i}I & * & * & * & * & * & * & *\\ N_{in} & 0 & 0 & 0 & 0 & -\varepsilon_{2i}I & * & * & * & * & * & *\\ B_{im}F_{in} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{2i}I & * & * & * & * & *\\ B_{in}F_{im} & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{2i}I & * & * & * & *\\ E_{i2m} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{3i}I & * & * & *\\ E_{i2n} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{3i}I & * & *\\ E_{i3m}F_{in} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{3i}I & *\\ E_{i3n}K_{im} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{3i}I\end{bmatrix}<0,$$
$$i=1,2,\ldots,S;\ 1\leq m<n\leq r_i \qquad \text{... (14b)}$$

$$\overline{\varphi}_{i,m} = A_{im}^{\mathrm{T}} P_i + P_i A_{im} + (\varepsilon_{1i} + \varepsilon_{3i}\rho_i^2) P_i H_{im} H_{im}^{\mathrm{T}} P_i + \varepsilon_{2i}\rho_i^2 P_i P_i + \sum_{j=1, j\neq i}^{s} P_i C_{jim} C_{jim}^{\mathrm{T}} P_i .$$

where applying the Schur complement to (15a) results in the condition (6a). Similar, the (15b) is equivalent to (6b). According to Theorem 1, the interconnected system (5) is robust stable. Thus the proof is completed.

### 2.2 Simulation examples

In this section, the proposed approach is applied to the following example to verify its effectiveness. We consider a fuzzy bilinear interconnected system, which is composed of two subsystems as follows

*Subsystem 1*

$$R_1^1: \ if \quad x_{11} \quad is \quad M_{11}^1$$
$$then \quad \dot{x}_1(t) = (A_{11} + \Delta A_{11}) x_1(t) + (N_{11} + \Delta N_{11}) \times x_1(t) u_1(t) + (B_{11} + \Delta B_{11}) u_1(t) + C_{211} x_2(t);$$

$$R_1^2: \ if \quad x_{11} \quad is \quad M_{11}^2$$
$$then \quad \dot{x}_1(t) = (A_{12} + \Delta A_{12}) x_1(t) + (N_{12} + \Delta N_{12}) \times x_1(t) u_1(t) + (B_{12} + \Delta B_{12}) u_1(t) + C_{212} x_2(t);$$

*Subsystem 2*

$$R_2^1: \ if \quad x_{21} \quad is \quad M_{21}^1$$
$$then \quad \dot{x}_2(t) = (A_{21} + \Delta A_{21}) x_2(t) + (N_{21} + \Delta N_{21}) \times x_2(t) u_2(t) + (B_{21} + \Delta B_{21}) u_2(t) + C_{121} x_1(t);$$

$$R_2^2: \ if \quad x_{21} \quad is \quad M_{21}^2$$
$$then \quad \dot{x}_2(t) = (A_{22} + \Delta A_{22}) x_2(t) + (N_{22} + \Delta N_{22}) \times x_2(t) u_2(t) + (B_{22} + \Delta B_{22}) u_2(t) + C_{122} x_1(t);$$

Where

$$A_{11} = \begin{bmatrix} -55 & -17 \\ -23 & 28 \end{bmatrix}, A_{12} = \begin{bmatrix} -62 & 19 \\ -32 & -28 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} -10 & 4 \\ -5 & -10 \end{bmatrix}, A_{22} = \begin{bmatrix} -32 & 49 \\ -45 & 28 \end{bmatrix};$$

$$B_{11} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}, B_{12} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, B_{21} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, B_{22} = \begin{bmatrix} 1 \\ 1 \end{bmatrix};$$

$$N_{11} = N_{12} = \begin{bmatrix} -3 & 5 \\ 1 & -3 \end{bmatrix}, N_{21} = N_{22} = \begin{bmatrix} -4 & 6 \\ 4 & -4 \end{bmatrix};$$

$$C_{211} = \begin{bmatrix} 1 & 1 \\ 5 & 1 \end{bmatrix}, C_{212} = \begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix}, C_{121} = \begin{bmatrix} -1 & 3 \\ 5 & 1 \end{bmatrix},$$

$$C_{122} = \begin{bmatrix} -1 & 1 \\ 5 & 3 \end{bmatrix}; E_{111} = \begin{bmatrix} 1 & 7 \\ 1 & 3 \end{bmatrix}, E_{112} = \begin{bmatrix} 6 & 1 \\ 1 & 8 \end{bmatrix},$$

$$E_{211} = \begin{bmatrix} 5 & 0 \\ 2 & 1 \end{bmatrix}, E_{212} = \begin{bmatrix} 5 & 1 \\ 3 & 1 \end{bmatrix}; E_{121} = \begin{bmatrix} 4 & -1 \\ 1 & 6 \end{bmatrix},$$

$$E_{122} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, E_{221} = E_{222} = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}; E_{131} = \begin{bmatrix} 2 \\ 1 \end{bmatrix};$$

$$E_{132} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}; E_{231} = \begin{bmatrix} 1 \\ 0.1 \end{bmatrix}, E_{232} = \begin{bmatrix} -1 \\ 0.2 \end{bmatrix};$$

$$H_{11} = H_{12} = H_{21} = H_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The membership functions are chosen as $\mu_{F_{11}^1}(x_{11}) = \mathrm{Sin}(-2x_{11}), \mu_{F_{11}^2}(x_{11}) = 1 - \mu_{F_{11}^1}(x_{11})$ and $\mu_{F_{21}^1}(x_{21}) = \mathrm{Sin}(-2x_{21}),\ \mu_{F_{21}^2}(x_{21}) = 1 - \mu_{F_{21}^1}(x_{21})$.

By letting $\rho_1 = 0.5, \rho_2 = 0.3,\ \varepsilon_{11} = \varepsilon_{12} = 1.3,\ \varepsilon_{21} = \varepsilon_{22} = 0.7$, $\varepsilon_{31} = \varepsilon_{32} = 1$, solving LMIs (14) gives the following feasible solution:

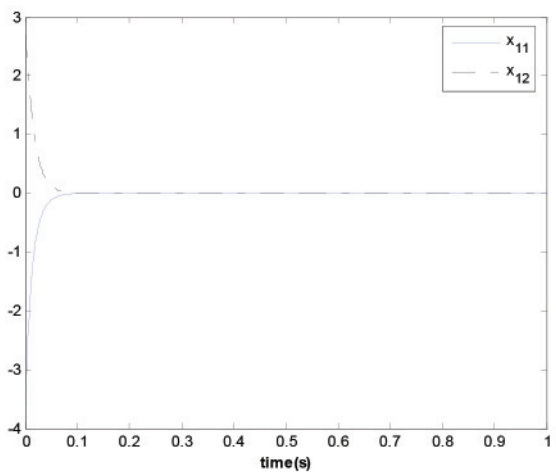


Fig.1: State responses of subsystem 1

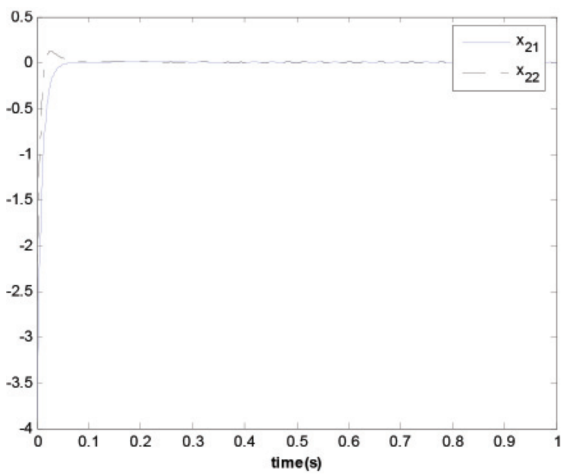


Fig.2: State responses of subsystem 2

$F_{11} = [-3.4454 \quad -2.0463];$

$F_{12} = [-2.5552 \quad -3.0671];$

$F_{21} = [-3.4405 \quad -4.2641];$

$F_{22} = [-1.4401 \quad -2.2894].$

Initial condition is assumed to be $x_{10} = [-3.6 \quad 2.7]^T$ and $x_{20} = [-3.9 \quad -2.6]^T$. The simulation results are shown in Figs.1 and 2 show the state responses of two subsystems and control trajectory is shown in Fig.3. It can be seen that with the decentralized fuzzy control law the closed-loop system is robust stable. The simulation results show that the fuzzy controller proposed in this paper is effective.

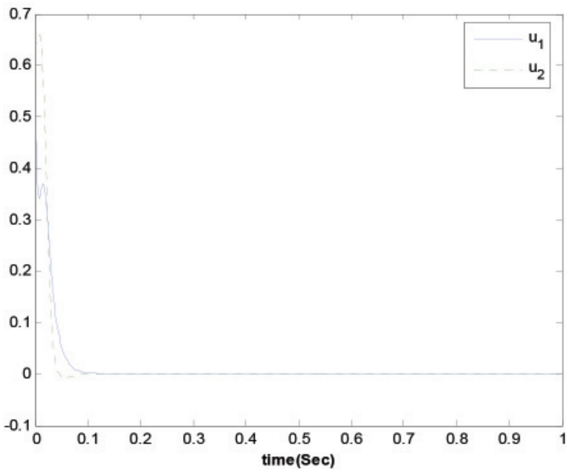


Fig.3: Control trajectory

### 5.0 Conclusions

In this paper, a T-S fuzzy bi-linear model is proposed to study the robust control problems for nonlinear interconnected systems using fuzzy decentralized control. Based on the Lyapunov criterion, the sufficient conditions for robust stabilization of the interconnected system are presented. The decentralized controllers designing problems can be formulated as a convex optimization problem with LMI constraints. A simulation example is included to show the effectiveness of the proposed approach.

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