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Long-term open pit mine production scheduling with variable cut off grade for cost optimization

Selection of cut off grade in long-term open pit mine planning is a tough research challenge now-a-days. The subsequent operational planning for the selected cut off grade decides the economic factor in mine production scheduling. The distribution of grade, sequence of mining operation, economic parameters, the capacities of mining operations are influencing points for deciding the model. In any given period of time the dynamic cut off grade is a function of the availability of ore and the capacity of stockpile as well as the process plant. The extraction sequence and cut off grade strategy should be considered simultaneously in order to achieve the optimum result. By keeping these points in first row, various attempts have been made to develop an electronic technique for the extraction sequence of open pit mines. Because of the numerous variables involved for getting the optimum result, different approaches have been made is not sufficient to widespread acceptance. A new model has therefore been proposed to overcome this shortcoming. The optimum sequences of extraction in each period are recognized by optimum processing decisions. To examine the applicability of the model developed, a case study is offered to validate.

Keywords: *NPV, production scheduling, economic loss, mining sequence, cut off grade.*

1. Introduction

The optimal cut off grade plays an important role for considering economic viewpoint in mineral resource industries, which leads the maximization of NPV. It is the most difficult way to choose the cut off grade for deciding the sequence of extraction in open pit mining process. The economic viability of mining operations depends on the cut off grade decision during a project's life. Higher cut off grade is responsible for getting higher overall NPV for a given mining project. In the starting phase of mining sequence, higher average grade is required to meet the higher cut off grade demand. The grade distribution of the mining deposit plays a vital role for realizing the higher cut off grade (Dagdelen 1993). The mining sequence is dependent on the cut off grade which indicates the quantity of mineral mined, transported to the process plant, processed and finally refined for salable product in the market (Lane 1988; Mohammad 2002). The selection of cut off grade is very important for getting a mining project a successful one. Therefore, the optimization of cut off grade is vital in course of time during the life of the mine.

The idea of cut off grade development methods and the subsequent formulation of algorithms have been analyzed in this present study by visualizing their merits. As a result of these studies, a constant cut off grade has been replaced by the variable cut off grade in a given period of time. Numerous variable cut off grade procedures have also been particularized based on the optimum cut off grade algorithms. The concept of economic cut off grade was given by (Mortimer 1950). However, the maximization of NPV is very difficult for the breakeven cut off grade throughout the mining operation. The optimization of the cut off grade was given by Lane's theory (1964, 1988), which was the initial approach to showcase. The structure of a function in order to maximize the NPV of cash flow is formulated by this theory. It also includes different constraints on the capacities i.e. mine, process plant and refinery in the mining operation.

Researchers have given their views by taking different constraints, such as Shinkuma and Nishiyama (2000), Crains and Shinkuma (2003), Ataei and Osanloo (2003, 2004) and Gholamnejad (2009). Dual cut off grades were framed by Halls and John (1969), it indicates all estimated costs (mining, dressing, refining and selling) and has shown good result. Another approach was made by Taylor (1972), according to him there is some differences between planning and operational cut off grades. Based on his statement, "the constant cut off grade and maximum NPV both are mismatched with each other". The idea behind the establishment of different stockpiles according to their values were highlighted by Taylor (1985). The outcome of these studies was more focused on the limits of capacities of any mining, processing and refining stages but not on the sequence of mining for optimization of cut off grade. Hypothesis has been made for knowing the mining sequence in advance by this method of the study hence the cut off

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grade calculation gives some error due to this assumption. Consequently, different approaches were made to develop a computerized procedure for cut off grade optimization, taking into account the sequence of mining as well 4D-Network Relaxation (Akaike and Dagdelen 1999; Mogi et al. 2001), and Dynamic Programming (Wang et al. 2008), Lagrangian Relaxation (Dagdelen 1985; Dagdelen and Johnson 1986; Kawahata 2007), (Johnson 1968, 1969). Unfortunately, the response was not very good for its difficulty and large size of the model.

For deciding the optimum cut off grade, mining sequence plays an important role. By avoiding the changes in the mining sequence lead an impractical mine design. The study offered a new binary integer programming model for solving the difficulties of extraction sequence. The present study draws an attention on the effect of cut off grade strategy which depends on the mining sequence. The paper concludes economic loss assessment of each and every block is considered by taking each alternative processing decisions as well as the distribution of orebody grade. The probabilities distribution of orebody grade and economic loss assessment are combined together with different constraints in a binary integer programming model. For giving the best mining sequence while optimizing the cut off grade in each period of mine life during scheduling was the main intension of the framed model.

2. Cut off grade optimization in production scheduling

The following questions are most important for deciding the cut off grade during planning stage.

1. What is the present market value of the material to be mined from the deposit?
2. What should be the further step once the material is mined?

It is very difficult to maximize profits in different situation by considering one cut off grade throughout the mining operation process (Gershon 1983). Cut off grade is calculated in order to manage the material flow from the extraction source to the appropriate destination. Decision criteria based on the cut off grade is shown in the Fig.1.

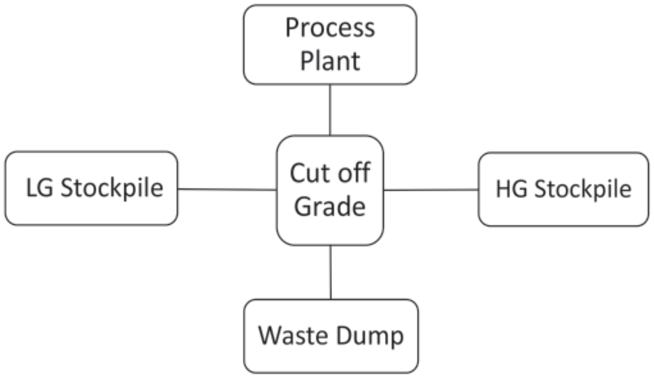

Fig.1 Criteria for taking decision based on cut off grade

Different parameters which are involved to calculate the cut off grade i.e. mining cost, transportation cost from mine to destination, processing cost, process plant capacity, capacity of mining operation, sequence of mining, deposit grade distribution, cash flow and the metal price consideration. The changes of mining sequence depend on the processing decision for a given block of ore. The changes in mining sequence lead to a huge disturbance on cut off grade during mining operation. Due to this disturbance on cut off grade, the declination of NPV is observed during the mine's life. Therefore, the optimization of cut off grade is required during the mining sequence in order to bridge the gap (Johnson 1968). So, optimization of cut off grade plays an important role in mining operation, because:

1 Processing material is placed at appropriate destination (i.e. process plant, stockpile, waste dump) according to the decision taken on optimization of cut off grade for getting maximum economic profit.

2 The improvement of mining sequence can be achieved by optimization of cut off grade during the life of the mine.

2.1 Mining sequence considering a variable cut off grade

Extraction of each block of ore is carried out on the basis of cut off grade with respect to mining sequence. Classification of blocks generally depends on the cut off grade for the block. Classification of block gives an idea about the settlement of further process in the mining sequence. Proper classification of block saves cost of recovery of mineral and the block economic value of the deposit. Block economic value of each block may tend to wrong, if proper block classification is not done. In line with that, Richmond (2001) coins the term "economic loss", which is used to differentiate ore and waste block. The settlement of mined ore, g(g = 1, 2, ..., G ordered from lowest grade to highest grade) which associated with actual economic loss is the difference between the potential value of the ore block and the recovered value after selling, which is given by the equation 1:

$$K^{g}_{xyz} = \left[(SP - SC)\delta_{xyz}\, R^{g} - C^{g}_{p} - C^{g}_{m}\right] - \left[(SP - SC)\delta_{xyz}\, R^{g'} - C^{g'}_{p} - C^{g'}_{m}\right] \quad \text{... (1)}$$

SP: Unit selling price of the metal

SC: Unit selling cost of the metal

xyz: Block identification number

δ_{xys}: Average grade value of block xyz

R^{g}: Total metal recovery of material if treated as type 'g'

C_{p}^{g}: Processing cost of the block xyz if treated as type 'g'

C_{m}^{g}: Mining cost of the block xyz if treated as type 'g'

g: Actual settlement type for block xyz

g': Selected settlement type for block xyz

For a particular block xyz, if it is processed by actual settlement type then the economic loss is zero. In production scheduling, the application of conditional simulation techniques can optimize the cost. The conditional simulation technique is created number of similarly realizations of block grades. The projected financial cost (PFC) is calculated by using the probabilities distribution of block xyz and the average grade for each type of settlement caused from independent realization as described in the equation 2:

$$PFC^{g}_{xyz} = \sum_{g=1}^{G}[P^{g}_{xyz} \mid O]\ [K^{g}_{xyz}] \qquad ...(2)$$

$P^{g}_{xyz}|O$: Probability distribution of block xyz if treated as type 'g'

The projected financial cost (PFC) is minimized for the optimal processing type of the block xyz within scheduling time frame i.e.

$$K(opt)^{t}_{xyz} = Min\left[PFC^{g}_{xyz}\right] \qquad ...(3)$$

2.2 CONSTRUCTION OF INTEGER PROGRAMMING MODEL

As per the explanations and assumptions made above, the mathematical programming model of the mining sequence is prepared according to the integer decision variables. It decides, in which period the particular block is extracted and at the same time to determine its destination. This model is optimized the block extraction sequence and the cut off grade strategy simultaneously. The objective function of the model can be written mathematically in the equation 4:

$$Minimize\ Z = \sum_{xyz\,\in\,\varphi}\ \sum_{t}^{T} \frac{K(opt)^{t}_{xyz}\ *\ b^{t}_{xyz}}{(1+r)^{t}} \qquad ...(4)$$

Maximization of NPV is not performed by the objective function in equation (4) but it, optimizes the feasible extraction sequencing and confirms a desired cut off grade as well. Afterward, the extracted block materials destination is quite straight forward. The feasible extraction sequences and the amount of ore having the desired grade and quality are to be sent to the process plant in a priority basis. Consequently, the objective function mentioned above indirectly leads to a maximum NPV which is optimal. Else, in theory the generated NPV would only be optimal but not in actual mining practice. In this present model the integration of suitable ore blocks having good grade values has been taken into consideration for maximizing the throughput to the process plant. This model fulfills the gap between the production and requirement of desired values during production scheduling phase in mining operation. Minimization of projected financial cost and to know the feasible extraction sequences result in maximum NPV. The proposed model suggests a unique cut off grade policy with optimizing the mining cost. Proper management of cash flow is required in order to maximize the NPV by considering different risk factor based on the possible variations in production from the mining process during the life of operation. The framed model in equation (4) contains a series of constraints in equations 5-12:

$$\sum_{xyz\,\in\,\varphi} (\delta_{xyz} - H^{t}_{\alpha}) * M^{o}_{xyz} * b^{t}_{xyz} \le 0 \quad for\ all\ t \qquad ...(5)$$

$$\sum_{xyz\,\in\,\varphi} (\delta_{xyz} - I^{t}_{\alpha}) * M^{o}_{xyz} * b^{t}_{xyz} \ge 0 \quad for\ all\ t \qquad ...(6)$$

$$\sum_{t=1}^{T} b^{t}_{xyz} = 1 \quad for\ all\ xyz \in \varphi \qquad ...(7)$$

$$\sum_{xyz\,\in\,\varphi} M^{o}_{xyz} * b^{t}_{xyz} \le H^{t}_{o}\ for\ all\ t \qquad ...(8)$$

$$\sum_{xyz\,\in\,\varphi} M^{o}_{xyz} * b^{t}_{xyz} \ge I^{t}_{o}\ \ for\ all\ t \qquad ...(9)$$

$$\sum_{xyz\,\in\,\varphi} (M^{o}_{xyz} + M^{w}_{xyz}) * b^{t}_{xyz} \le H^{t}_{w\ \&\ o}\, for\ all\ t \qquad ...(10)$$

$$\sum_{xyz\,\in\,\varphi} (M^{o}_{xyz} + M^{w}_{xyz}\ ) * b^{t}_{xyz} \ge I^{t}_{w\ \&\ o}\ for\ all\ t \qquad ...(11)$$

$$b^{t}_{k} - \sum_{w}^{W} \sum_{r=1}^{t} b^{r}_{w} \ \le\ 0 \quad for\ all\ t\ and\ k \qquad ...(12)$$

where:

$K(opt)^{t}_{xyz}$: The optimal settlement type for block xyz in period 't'

T: Total number of scheduling periods

t: Index of scheduling periods, t = 1, 2, 3, $\cdots$T

φ: Total numbers of blocks to be scheduled.

r: Rate of discount in each scheduling period.

b^{t}_{xyz}: The binary variable equal to

$$= \begin{Bmatrix} 1\ \ if\ block\ xyz\ is\ extracted\ at\ period\ t \\ 0\ Otherwise \end{Bmatrix}$$

M^{o}_{xyz}: Total tonnes of ore in block xyz

M^{w}_{xyz}: Total tonnes of waste in block xyz

δ_{xyz} : The average grade of block xyz

$H_{\alpha}{}^{t}$: The higher bound average grade of material sends to the process plant in period t.

$I_{\alpha}{}^{t}$: : The inferior bound average grade of material sends to the process plant in period t.

H^{t}_{o}: The higher bound total tonnes of ore processed in period t.

Table 1: The features of each mineralized block class for limestone ore deposit.

Particulars	Unit No.	Waste dump	Stockpile (LG)	Stockpile (MG)	Process plant
Processing type		1	2	3	4
Grade range	Percentage	≤ 35	35-40	40-45	≥ 45
Average grade	Percentage	27	39	44	49
Metal recovery	Percentage	0	58	70	94.5
Selling cost	USD/tonne of limestone	0.4	0.4	0.4	0.4
Mining and processing cost	USD/tonne of ore	0.6	1.85	2.2	3.0
Limestone price	USD/tonne of limestone	7.8			

I_o^t: The inferior bound total tonnes of ore processed in period t.

$H_{w\&o}^t$: The higher bound total amount of material (waste and ore) to be mined in period t.

$I_{w\&o}^t$: The inferior bound total amount of material (waste and ore) to be mined in period t

W: Total numbers of blocks overlaying block ‘k’.

k: The index of block considered for extraction in period ‘t’.

W: The counter for the ‘W’ overlaying blocks.

The average grade value of the material directed to the process plant up to a certain value is limited by constraints (5) and (6). The block is removed in one period only which is enforced by constraint (7). The capacity of process plant is ensured by constraints (8) and (9). For securing a smooth feed or ore to the process plant, the higher and inferior bounds are necessary to get the good result. The actual available capacity of the equipment for each period is ensured by constraints (10) and (11). These higher and inferior bounds are the total amount of material (ore and waste) to be mined in period. The bench slope restriction is ensured by constraint (12) on the basis of ‘W’ constraints for each block per period.

The framed model provides a tool for evaluating substitute approaches as a part of feasibility studies at the long-term production scheduling stage. In the other hand, selection of resultant cut off grade throughout the scheduling period is a risk-oriented job for the decision maker. This selection procedure may affect the major investments for utilization of mineral resources. In order to overcome such circumstances, variable cut off grade policy stands tall for the sustainability of both investments and better utilization procedure at long-term scheduling stage.

3. Application of framed model in a limestone deposit

The section shows the efficacy of the designed model for better result in long-term production scheduling. The proposed model was implemented on a real limestone block to validate and ensure the effectiveness of block extraction sequence as well as cut off grade strategy. A limestone mine sends its mineralized products to four different destinations: waste dump, low grade stockpile, high grade stockpile, or process plant. The features of these four classes are itemized in Table 1.

Equal probable realization concept was brought for obtaining the result more reliable. One hundred equally probable realizations of the limestone orebody grades were produced using Sequential Gaussian Simulation. The results of the simulation for a given limestone ore block is shown in Table 2.

Table 2: Simulation result of a given block of limestone deposit

Grade range	≥ 45	40-45	35-40	≤ 35
No. of realization	30	25	15	12
Corresponding probability (%)	30	25	15	12

Taking into consideration for a given block xyz, for medium grade stockpile the correct mining destination is (‘g’ = 3), but it is incorrectly directed to the waste dump (‘g’’ = 2), then the economic loss valuation due to this misclassification can be calculated from equation 1 in the following way:

$$L^3_{xyz} = [(7.8–0.4) * 0.44 * 0.70–2.2]$$
$$–[(7.8–0.4) * 0.44 * 0.58–1.85] = 0.041$$

Table 3 shows the results of the economic loss for the other values of ‘g’ and ‘g’’. If the class ‘g’=2 is selected from block xyz, then the misclassification which affects the projected financial cost can be achieved from equation 2 as follows:

$$PFC^2_{xyz} = [(0.091 * 0.30) + (0 * 0.25) + (0.041 * 0.15)$$
$$+ (0.174 * 0.12)] = 0.054$$

According to Table 3, the optimum projected financial cost is 0.054; therefore, the optimum destination of this block is ‘g’=2, meaning that it is better to send this block to low grade stockpile.

Sequence of extraction plays a vital role for strengthening the long-term production scheduling. Extraction of each block for a given period depends on its economic value of the same block in that period for deciding the sequence of extraction. In contrast, economic value of the blocks varies due to the changes in price and costs during this particular period of time. It may therefore be possible that the optimum classification of block in one period can be different from the

TABLE 3: RESULTS OF ECONOMIC LOSS OF THE DIFFERENT BLOCK CLASSIFICATION

Selected mining destination	Actual mining destination				Projected financial cost
	1	2	3	4	
1	0	0.424	0.679	1.027	0.331
2	0.091	0	0.041	0.174	0.054
3	0.201	0.004	0	0.088	0.072
4	0.502	0.082	0.002	0	0.171

optimum classification of block in other periods due to this economic value variation. Hence, the loss function can be calculated for block xyz in period 't' as in equation 4 for getting the idea about the variation. Because of the operational requirements for long-term mine production scheduling, the minimization of the objective function depends on the available constraints as in equations 5 through 12. In the meantime, the iterative steps of optimization mechanism are tedious and time taking. So as to overcome such difficulties, an Excel spreadsheet was developed to ease the performance of the calculations. For solving the presented model in case of limestone mine, an input file for the block model which includes characteristics of the counters for each block, tonnage, grade and ore content of each block, and the projected financial cost of each block using Excel software was provided. Decision variables and available constraints related to the type of block in the model are considered.

4. Conclusion

This paper has offered a mathematical model which is based on binary integer programming for open pit limestone mines. The combination of sequential mining operation and cut off grade tactic has considered in getting the optimum cost for long-term production scheduling. Considering different processing types, the proposed technique develops a practical and achievable scheduling procedure while minimizing the projected financial cost through satisfying different system constraints. Even if the proposed model is not established to maximize NPV directly, it provides a comprehended NPV, which gives optimum result by assuming the sequence of mining and cut off grade approach considerations. In fact, NPV is increased subject to determine the probability distribution of the ore blocks, because minimization of PFC is suggested in the proposed model. More high-grade ore blocks are required to mine in earlier stage in order to satisfy the model. This study gives important value to the financial cost by taking it as objective function. Undoubtedly, the concept of financial cost method is very effective for determining the optimum processing type of the excavated material. The limitation of conventional methods is succeeded by the model developed in this present study and it includes certain outcome such as the following:

- Number of required variables are reduced and, afterward, managing the available variables and constraints in a shorter period of time.
- The ability to consider several types of processing into account.

The planned model was applied to a limestone ore deposit. In the present case study, it shows flexibility at the commencement of planning stage including production scheduling for assessment of different alternatives. This study safeguards the optimum resource utilization by reducing the financial cost as well as the time for extraction sequence. The framed model precise the economic decisions with respect to major mining investments.

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