HE WEI
LI MENGYING
and
SHUYI GUO

Damage identification for metal beam structure based on curvature difference and frequency perturbation

With the development of metallurgical technology, a lot of large spanmetal beam structures appear in mining, machinery and construction. Structural damage identification is extremely important in engineering. The technology of structural damage identification based on dynamic characteristics has been one of the hot issues in current engineering research. Based on the absolute curvature difference modal analysis method and perturbation theory, the finite element model of a simply supported beam structure was established by ANSYS software, so as to study the damage localization and damage quantification. Under the condition of single damage and multiple damage of simple beam structure, reducing the elastic modulus of each element in the model by means of numerical simulation, and analyzing the change of curvature mode to enable study the damage localization. Then, based on the perturbation theory, the curvature difference mode method is used to analyze the damage quantitatively. The results are of great reference value for damage detection of large spanbeam structures.

Key words: *Metallurgical, damage identification, metal beam structure, absolute curvature difference, perturbation.*

1. Introduction

It is the important link of transportation, and the health of the bridge is related to the smooth road and the safety of people's life and property. Due to the design and construction defects or overloading serious, bridge structure will produce different degrees of damage in the actual operation of the bridge. If these injuries cannot be detected in time and properly repaired, damage will rapidly increase until the collapse, bringing significant losses in the people's lives and property. Therefore, it is a hot and difficult problem to study a kind of highly accurate and feasible method for the damage detection at home and abroad.

Over the past decade, domestic and foreign scholars have been looking for the overall damage assessment method that can be applied to complex structure. The research results in the domestic and international trends are mainly concentrated in the following aspects: (1) Damage early warning; (2) Damage localization; (3) Damage quantification. In 1991, A.K. Pandsy et al. first proposed the curvature mode modal index. By studying the relationship between the change of curvature mode and damage firstly, the absolute value of the curvature change of the mode shape is used to judge the damage location. At the same time, the degree of damage can be calculated [1]. Wahab used the curvature modal analysis method to detect the bridge damage [2]. Based on the first order curvature mode derivation, the fitting polynomial can estimate the degree of damage, and it has a good reference value for the damage detection of the structure [3].The sensitivity curvature difference can identify damage and diagnose the damage of the support near the beam and frame node [4].

In 2007, Wei Jinhui et al. introduced the concept of rate of change of curvature modal based on the curvature modal theory. The structural displacement mode data are extracted by ANSYS, and the rate of change of curvature mode for the simply supported beam structures with different damage conditions is analyzed [5]. Du siyi et al. established the finite element model and the definition of damage identification unit parameters in 2010. By combining the perturbation theory with the vibration theory, the first and second order perturbation equations of structural Vibration Eigen Value are derived. And an optimization algorithm for the structural damage diagnosis parameters of the two equations in un-determined situation is given, which avoids the errors caused by the use of modal shapes to identify structural damage and low accuracy or lack of degrees of freedom [6]. Under the introduction of random errors, the low-order modal information can accurately and easily identify the structure of the damage location. And the recognition effect shows that, the method can effectively estimate the damage degree and provide a reference for future structural damage identification [7].Taking a twin tower cable-stayed bridge as an example, the modal frequency index is used as the input parameter of the neural network. The recognition method based on modal frequency and BP neural network can identify damage

Messrs. He Wei and Li Mengying, School of Civil Engineering and Communication and Shuyi Guo, School of Mechanical Engineering, North China University of Water Resources and Electric Power, Zhengzhou 450 045, Henan, China

location and damage degree of bridges [8].

Based on the advantages of curvature mode sensitivity to structural damage, the absolute curvature difference modal index is used to simulate the damage of the simply supported bridge model: Firstly, the first five displacement modes of the simply supported beam model under different damage conditions are extracted by the finite element software ANSYS. Then the corresponding order curvature modes are obtained by the difference method. Finally, the damage identification is performed by using absolute curvature difference modal damage index, and the damage is quantitatively analyzed based on the curvature difference formula of perturbation theory.

2. Basic theory

2.1. Curvature modal method

When the damage identification of the simply supported beam bridge is analyzed by the curvature mode method, the mechanical model considered is bending deformation and the influence of shear deformation and moment of inertia on the beam is not considered. That is the Euler-Bernoulli beam model. The vibration differential equation of the bending vibration of the beam without considering the viscous damping is

$$m\frac{\partial^2 y(x,t)}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 y(x,t)}{\partial x^2}\right]=p(x,t) \quad \text{...} \quad (1)$$

Formulas, $y(x,t)$ is transverse vibration displacement; x coordinates along the beam length; t is time function; $EI(x)$ is bending rigidity of the beam; m is the mass per unit length; $p(x,t)$ is plus incentives.

According to the knowledge of differential calculus, the curvature of any point on the plane curve of beam structure can be written approximately

$$\frac{1}{\rho(x)}=\frac{\partial^2 y(x,t)}{\partial x^2}=\sum_{s=1}^{\infty} q_s(t)\varphi_s''(x) \quad \text{...} \quad (2)$$

Formulas, $r(x)$ is the radius of curvature at the section; $1/\rho(x)$ is the curvature at the section; $q_s(t)$ is generalized mode coordinate and a function of time; $\varphi_s''(x)$ is main mode function; s is modal order.

In the actual engineering structure, because there is no sensor which can measure structural curvature mode directly at home and abroad, it can measure the structure of the displacement mode firstly and then by the difference approximation to calculate the curvature modal value

$$\varphi''_{sk}\approx\frac{\varphi^s_{k+1}-2\varphi^s_k+\varphi^s_{k-1}}{l^2} \quad \text{...} \quad (3)$$

Formulas, s is modal order; k is the k-th node; l is the distance between two adjacent measuring points; φ''_{sk} is the curvature modes of the s-th order, k-th node of the beam; φ^s_k is modal shapes of the s-th, the k-th node of the beam; φ^s_{k+1} is modal shapes of the s-th, the k+1-th node of the beam; φ^s_{k-1} is modal shapes of the s-th, the k-1-th node of the beam.

According to the abrupt change of absolute curvature curve before and after structural damage, the damage position can be located

$$\Delta_{sk}=\left|\varphi''_{skd}-\varphi''_{sku}\right| \quad \text{...} \quad (4)$$

Formulas, D_{sk} is damage location index; φ''_{sku} , φ''_{skd} are the curvature values of the s-th order, the k-th node before and after the structural damage, respectively.

2.2. The basic theory of unit matrix perturbation[9]

The dynamic control equation of the structure without damping is

$$[M]\{\ddot{x}\}+[K]\{x\}=\{p(t)\} \quad \text{...} \quad (5)$$

The characteristic equation is

$$([K]-\lambda[M])\{\varphi\}=\{0\} \quad \text{...} \quad (6)$$

Formulas, $[M]$, $[K]$, $\{p(t)\}$, $\{x\}$, $\{\ddot{x}\}$, λ, φ are the overall mass matrix of the structure, total stiffness matrix, excitation load, velocity, acceleration, eigen value, eigen vector, respectively.

For the eigen values of structural vibration without heavy roots and dense roots, the derivation is as follows:

Assuming the structure has n degrees of freedom, r units damage and assuming that the rate of change of some parameters of the i-th damage unit is $\varepsilon_i(0\le\varepsilon_i<1)$, the overall stiffness matrix and mass matrix of the structure are all functions of small parameters ε_i , the natural frequencies and modal shapes determined are also a function of the small parameters ε_i. Therefore, which can be Taylor series expansion in the neighbourhood of $\varepsilon_i = 0$, and because ε_i is a small parameter, ignoring the second order above driblet was:

Quality matrix:

$$[M]=[M_0]+\sum_{i=1}^{r}[M_{1,i}]\varepsilon_i+\sum_{i=1}^{r}\sum_{j=1}^{i}[M_{2,ij}]\varepsilon_i\varepsilon_j \quad \text{...} \quad (7)$$

Stiffness matrix:

$$[K]=[K_0]+\sum_{i=1}^{r}[K_{1,i}]\varepsilon_i+\sum_{i=1}^{r}\sum_{j=1}^{i}[K_{2,ij}]\varepsilon_i\varepsilon_j \quad \text{...} \quad (8)$$

Eigen values:

$$\lambda^s=\lambda_0^s+\sum_{i=1}^{r}\lambda^s_{1,i}\varepsilon_i+\sum_{i=1}^{r}\sum_{j=1}^{i}\lambda^s_{2,ij}\varepsilon_i\varepsilon_j \quad \text{...} \quad (9)$$

Feature vector:

$$\{\varphi^s\}=\{\varphi^s_0\}+\sum_{i=1}^{r}\{\varphi^s_{1,i}\}\varepsilon_i+\sum_{i=1}^{r}\sum_{j=1}^{i}\{\varphi^s_{2,ij}\}\varepsilon_i\varepsilon_j \quad \text{...} \quad (10)$$

Formulas, s is the modal order; $[M]$, $[K]$, l^s, $\{\varphi^s\}$ are the total mass matrix, the stiffness matrix, the s-th order eigen value, the feature vector after the structural damage, respectively; $[M_0]$, $[K_0]$, λ_0^s, $\{\varphi_0^s\}$ are the total mass matrix, the stiffness matrix, the s-th order eigen value, feature vector before the structural damage, respectively; $[M_{1i}]$, $[K_{1i}]$, $[M_{2i}]$, $[K_{2i}]$ are the first order perturbation and the second order perturbation of the mass matrix and stiffness matrix, respectively; are the first order perturbations and the second order perturbations of eigen values and feature vector, respectively. Among them,

$$\lambda_{1,i}^s = \frac{\partial \lambda^s}{\partial \varepsilon_i}, \lambda_{2,ij}^s = \frac{1}{1+\delta_{ij}} \frac{\partial^2 \lambda^s}{\partial \varepsilon_i \partial \varepsilon_j}; \varphi_{1,i}^s = \frac{\partial \{\varphi^s\}}{\partial \varepsilon_i},$$

$$\varphi_{2,ij}^s = \frac{1}{1+\delta_{ij}} \frac{\partial^2 \{\varphi^s\}}{\partial \varepsilon_i \partial \varepsilon_j}, \delta_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}.$$

3. Simple supported beam model

Taking a simple supported beam for example, length of which is 3 m, the cross-sectional area b×h = 0.2m×0.3m, the moment of inertia 0.00045 m^4, the elastic modulus of the material 210 GPa and density is 7850 kg/m^3. The model has 31 nodes, 30 units, and each unit is equally spaced, units and nodes number are shown in Fig.1.

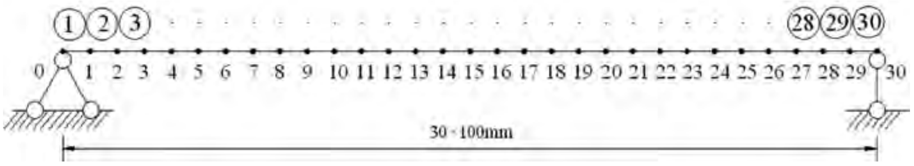

Fig.1 Simple supported beam model

4. Absolute curvature difference curve damage localization

4.1. Single injury

Taking the working condition: 1/2-span (15 element) damage as example, is shown in Table 1.The curves of the first five absolute curvatures of case 1 to case 6 are shown in Fig.2 to Fig.6.

From Fig.3 to Fig.6, it can be seen that the first five orders absolute curvature difference curves can obviously locate the damage. Because the damage unit is a mid-span unit, the absolute curvature difference curves of the second and the fourth order are just the vibration mode nodes near the damage unit. But it can be seen that the injury is still sensitive.

Table 1: 1/2-span damagecondition

Damage condition	Location and degree
Condition 1	1/2-span (15element) damage 5%
Condition 2	1/2-span (15element) damage 10%
Condition 3	1/2-span (15element) damage 15%
Condition 4	1/2-span (15element) damage 20%
Condition 5	1/2-span (15element) damage 25%
Condition 6	1/2-span (15element) damage 30%

The effect of different absolute curvature difference curves on the damage location of the simply supported beam structure is different. The first order and third order are better than the fourth and fifth order. The 14th node and the 15th node corresponding to the span-intermediate damage unit appeared obvious spike-shaped mutation on the curve of absolute curvature modal difference. The nodes corresponding to the uninjured units are relatively gentle in the first, third and fifth absolute curvature curves. The second-order and fourth-order absolute curvature difference curves on the left side fluctuate largely, but it does not affect the damage location. The change rule of the absolute value of abrupt peak in the curvature mode curve is, when the modal order is the same, the absolute value of the mutation peak increases with the increase of damage degree.

4.2. Two element damage

Take condition: 1/4-span (7unit) damage 30%, 1/2-span (15unit) damage 15% as an example. The first five order curvature modes and the first three absolute curvature curves are shown in Fig.7 to Fig.10.

It can be seen from Fig.8 to Fig.10, which the corresponding node of the damage unit in the absolute curve of curvature difference appeared obvious mutation, as with the single damage. And the corresponding nodes of the uninjured units show a gentle curve. Since the 15th element is in the vicinity of the mode node in the second-order displacement mode, the damage position cannot be identified by the second-order absolute curvature difference curve in the two-element damage. Both the first and third absolute curvature difference curves can clearly locate the damage. The change rule of the absolute value of abrupt peak in the curvature mode curve is: In the same condition, the absolute value of the mutation peak increases with the increase of the modal order.

5. Damage quantitative analysis

The purpose of structural damage identification is to locate and quantify. In summary, the damage localization has been basically completed. And the quantitative analysis, based on the curvature difference model perturbation theory, is carried out as following. The nodes referred to the table are all the identification point.

5.1. Quantitative analysis of single element damage

It can be seen from the formula that only one node can be used to quantitatively analyze the damage degree. Considering the influence of mode nodes, the 10th and 15th are selected. When the damage occurs, the change of

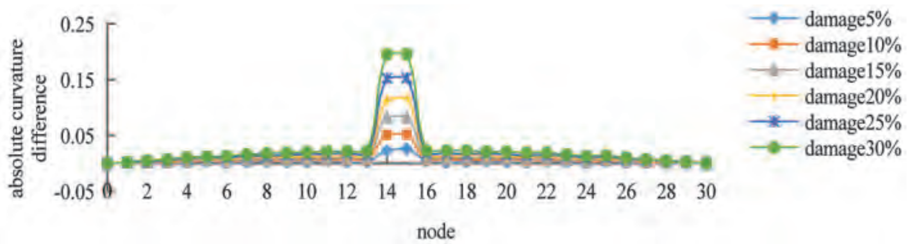

Fig.2 The first-order absolute curvature difference curve

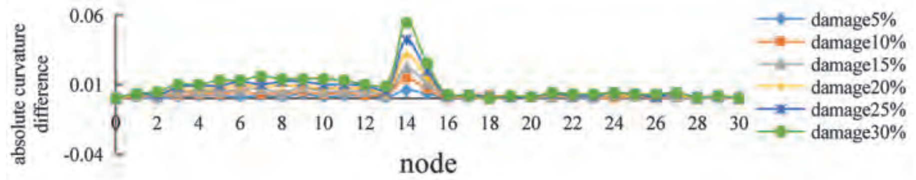

Fig.3 The second-order absolute curvature difference curve

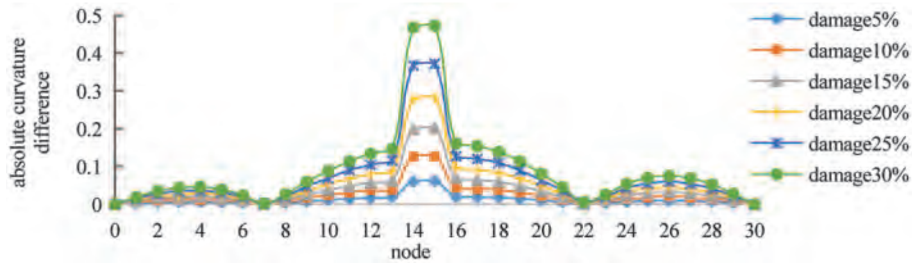

Fig.4 The third-order absolute curvature difference curve

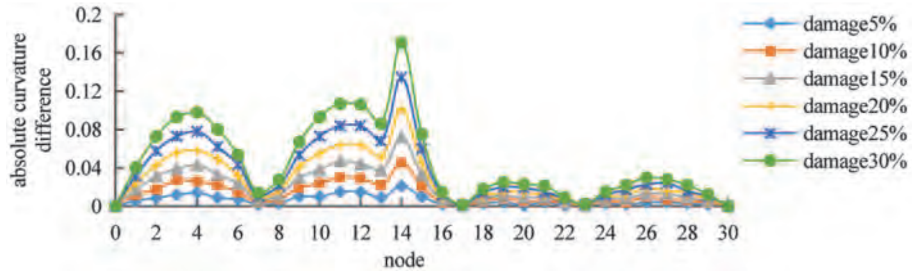

Fig.5 The fourth-order absolute curvature difference curve

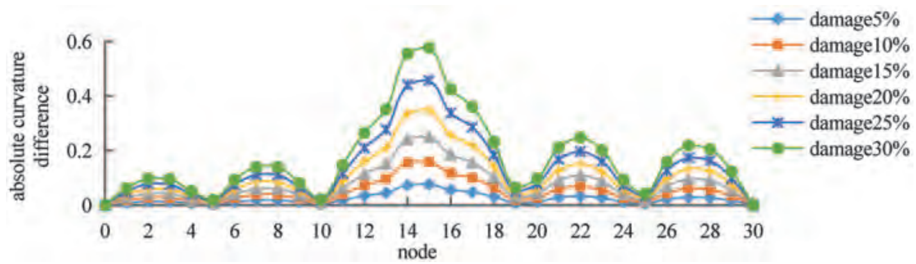

Fig.6 The fifth-order absolute curvature difference curve

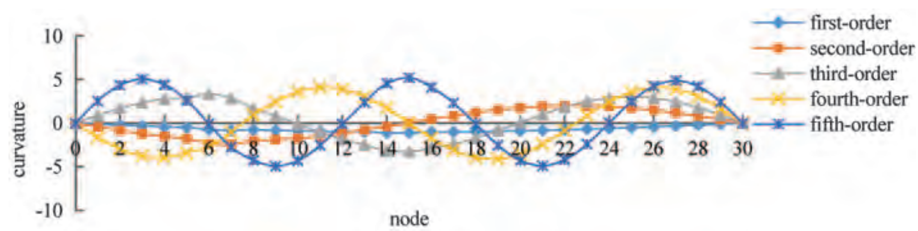

Fig.7 The first five order curvature mode

displacement of the 10th node on the second-order vibration mode and the15th node on the third-order vibration mode are not much. At the same time, considering the damage location or the corresponding nodes nearby, such as node 5, node 1 and the nodes with large variation, such as the 15th node on the first vibration mode and the 25th node on the second vibration mode.

As can be seen from Table 2, if the selected recognition point is in the vicinity of the node corresponding to the damage unit, the damage quantification is ideal and the relative error is also small in single element damage. When the injury position and damage degree are the same, the identification points quantitative damage on the third order are better than those of the first and second order. For the same damage unit, the effect of quantitative damage is getting worse with the degree of damage increasing, that is, the greater the degree of injury, the greater is the relative error.

5.2. Quantitative analysis of two elements damage

Taking condition: 1/4-span (7unit) damage 30%, 1/2-span (15unit) damage 15% as an example, and the results are shown as following.

As can be seen from Table 3, if the node number combination is different, the damage quantification value will be different in the two elements damage. But the relative error will not vary widely. When the injury position and injury degree is certain, the second-order recognition point is more accurate than the first-order recognition point. In the same condition, the larger is damage degree of the unit, the relative error of damage quantitative is also larger. That is, the greater the degree of damage, the greater is the relative error.

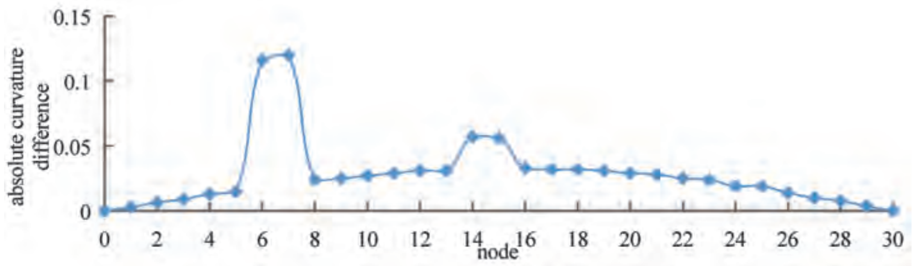

Fig.8 The first-order absolute curvature difference curve

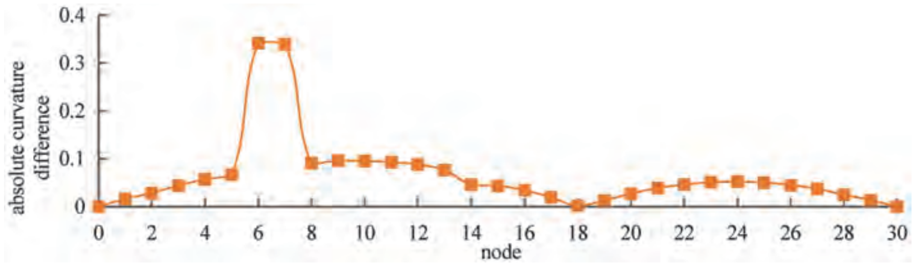

Fig.9 The second-order absolute curvature difference curve

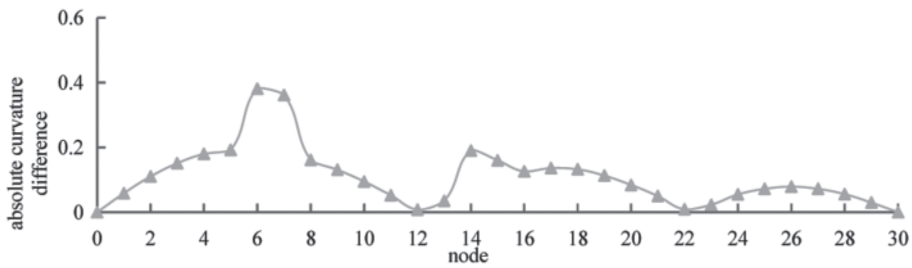

Fig.10 The third-order absolute curvature difference curve

TABLE 2: 1/2-SPAN DAMAGE IDENTIFICATION

	Node	Perturbation recognition value					
		First order (%)	Relative error (%)	Second order	Relative error (%)	Third order	Relative error (%)
7	5	6.00	20.00	4.92	1.60	5.50	10.00
	10	4.76	4.80	4.50	10.00	4.78	4.40
	15	4.74	5.20	4.77	4.60	4.71	5.80
	25	4.00	20.00	4.40	12.00	4.90	2.00
8	5	8.67	13.30	9.57	4.30	9.20	8.00
	10	9.13	8.70	9.50	5.00	9.10	9.00
	15	8.97	10.30	9.07	9.30	8.90	11.00
	25	8.00	20.00	8.00	20.00	8.87	11.30
9	5	14.00	6.67	13.25	11.67	14.00	6.67
	10	12.77	14.87	12.86	14.27	12.98	13.47
	15	12.91	13.93	12.90	14.00	12.82	14.53
	25	12.00	20.00	12.67	15.53	13.42	10.53
10	5	15.00	25.00	15.68	21.60	16.33	18.35
	10	16.71	16.45	16.93	15.35	16.41	17.95
	15	16.24	18.80	16.27	18.65	16.54	17.30
	25	17.00	15.00	15.14	24.30	16.67	16.65
11	5	20.00	20.00	18.86	24.56	21.33	14.68
	10	18.79	24.84	18.78	24.88	19.39	22.44
	15	19.20	23.20	19.18	23.28	19.66	21.36
	25	23.00	8.00	20.29	18.84	19.80	20.80
12	5	20.80	30.67	22.00	26.67	22.86	23.80
	10	21.46	28.47	21.60	28.00	21.92	26.93
	15	21.47	28.43	21.62	27.93	22.26	25.80
	25	29.00	3.33	20.00	33.33	22.79	24.03

6. Conclusions

Taking a simple supported beam bridge model as the research object, and using ANSYS software to build finite element model, and extracting the former five-order mode shapes of simply supported beam, and using the difference method to obtain the curvature modal graph. Then, damage localization was performed with absolute curvature difference modal damage identification index, and the damage matrix was quantified by the perturbation theory of element matrices. The main conclusions are as follows:

(1) When the structure damage occurs, the curvature mode will be mutated at the corresponding node of the damage unit. The absolute value of the abrupt peak of the curvature mode corresponds to the degree of injury at the damage site. For the simply supported beam structure, absolute curvature mode differences show different peaks in the different degrees of damage. Therefore, the absolute curvature difference mode curves can effectively locate the structural damage, and recognizing multiple lesions.

(2) The low-order mode can accurately determine the damage location when damage identification is performed. However, the difference of curvature difference is small, when the damage occurs at a certain vibration mode node in the simulation results. It cannot identify the damage location, and need mutual control higher-order to analyze the damage case. For the more difficulty to measure the actual location of damage parameters of the structure, the damage location can be deduced by computing the nodes near the damage location. Therefore, it has a good prospect that using absolute curvature difference modal damage index to structural damage identification.

TABLE 3: THE TWO ELEMENTS DAMAGE IDENTIFICATION

Damage condition	Node number combination		Perturbation recognition value							
	First order	Second order	First order (%)		Relative error (%)		Second order (%)		Relative error (%)	
19	6,7	7,20	20.64	15.91	31.20	6.07	22.19	15.21	26.03	1.40
	6,15	7,24	22.06	13.92	26.47	7.20	22.16	16.30	26.13	8.67
	10,19	9,17	21.69	12.85	27.70	14.33	21.80	14.85	27.33	1.00
	14,15	11,22	25.37	19.61	15.43	30.73	22.13	17.01	26.23	13.40
20	10,11	9,10	15.71	18.58	21.45	25.68	16.14	18.95	19.30	24.20
	10,24	9,24	18.06	20.97	9.70	16.12	16.70	19.56	16.50	21.76
	17,25	23,24	15.26	18.74	23.70	25.04	15.60	19.80	22.00	20.80
	9,24	8,20	17.90	21.05	10.50	15.80	16.69	19.65	16.55	21.40
22	6,7	6,7	14.30	22.46	4.67	25.13	15.00	25.00	0.00	16.67
	6,23	6,23	7.83	1.75	47.80	94.17	13.02	22.22	13.20	25.93
	16,23	7,24	8.92	20.77	40.53	30.77	12.80	21.94	14.67	26.87
	7,15	23,24	13.04	21.57	13.07	28.10	14.50	21.54	3.33	28.20
23	6,23	6,20	21.46	12.89	28.47	14.07	22.34	11.09	25.53	26.07
	7,15	8,18	21.00	10.33	30.00	31.13	19.96	12.44	33.47	17.07
	7,10	8,24	20.98	10.33	30.07	31.13	22.59	13.28	24.70	11.47
	13,15	13,20	21.44	12.44	28.53	17.07	23.43	15.89	21.90	5.93

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