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Selection of optimum cut-off grade for surface mining of metalliferous deposits: a review

One of the challenging problems for surface mining operation optimization for metalliferous deposits is choosing the optimum cut-off grade while doing mine production planning, which is a very important component of mine design process. Many researchers have done significant approaches towards it but till date no full proof solution to this problem has been found. An overview on the selection of optimum cut-off grade, as suggested by various researchers has been presented in this paper.

Keywords: Cut-off grade, net present value, optimization, dynamic programming.

1.0 Introduction

Selection of optimum cut-off grade for metalliferous deposits is a fundamental problem faced by the mine planners because of the variation in costs of operation, metallurgical characteristics and stripping ratios (open pit) for the different deposits. Cut-off grade optimization theory supports the fulfillment of ultimate objective of a mining operation through maximizing the Net Present Value (NPV) (Asad, 2007).

In case of any metalliferous deposit there could be different limiting cut-off grades depending on the limitation of mine capacity, mill capacity and smelting and refinery capacity. If the throughput is limited by the mining capacity, the opportunity costs have to be distributed per unit material mined and the corresponding cut-off grade is called the mine limited cut-off grade. If the mill is limiting the throughput, the opportunity costs are distributed per unit of ore and the corresponding cut-off grade is called the mill limited cut-off grade. Similarly if the final product is limited by the market constraint, the opportunity costs are distributed per unit weight of the product and the corresponding cut-off grade is known as refinery limited cut-off grade. However the optimum cut-off grade at any moment may not be necessarily one of the above three limiting cut-off grades, as it is decided on the basis of the interaction of mine, plant and refinery capacities (Lane, 1964).

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The main objective of each optimization of mining operation is to maximize the net present value of the whole mining project, but this approach without consideration of environmental issues during planning is not really an optimum design (M. Osanloo *et al.*, 2008).

The importance of determination of cut-off grade in surface mine planning of metalliferous deposits are as follows (B. Mishra *et al.* 2004):

1. Cut-off grade decides the amount of waste and ore in the body of the deposit.
2. Cut-off grade affects the mine costs. By changing the cut-off grade the stripping ratios change, thereby affecting the mine equipment cost for producing a given number of tonnes of ore per day.
3. The cut-off grade decides the amount of material to be handled economically in the different stages – mining, concentrating and smelting and refining during the mining process.
4. Cut-off grade affects tailing and dump costs.
5. The decision about cut-off grade is of major economic significance and cannot be taken by the application of simple cost formulae.

2.0 Determination of cut-off grade

Choosing the best cut-off grade that maximizes the economic outcome for a given deposit has been a major topic of research since the 1960s, and many researchers have made their contributions in devising various methods and algorithms.

2.1 Method proposed by Henning (1963)

Henning was the first to present a methodology for calculation of cut-off grade. He considered two objectives for the computation of cut-off grade.

Objective-1

(a) To maximize the spread between the present value of annual operating profit and investment cost.
(b) To maximize the spread between the sums of the annual working profits during the lifetime of the plant and the investment cost.

Objective-2

- This objective is similar to objective 1, but with the condition that for some reason the life of the property is fixed.
- Objective 1(a) is an adaptation of principle of maximum profit and one would assume that in free market economy this is one of the most commonly applied method.
- Objective 2 embraces parameters other than purely economic motives. It also considers the part of the ore which can actually be mined at loss.

2.1.1 Determination of the optimum cut-off grade

The mathematical treatment for determination of optimum cut-off grade is as follows:

Let,

X	cut-off grade (%)
k_1	variable operating costs (\$/tonne)
k_2	fixed plant operating cost (tonne/ year)
p	annual production (tonne/year)
n	life expectancy of plant (years)
a_1	plant capital costs proportional to capacity (\$/tonne annual capacity)
a_2	plant capital costs independent of capacity (\$)
m, x	value of one tonne ore with grade x (\$/tonne)
q	1 + interest rate/100
D	Difference between the cost of mining marginal ore and the average mining cost. The cost of mining marginal ore is often less because no further development is required or because certain set up costs disappear.
i	Average ore grade (%)
m.i	value of 1 tonne of ore of grade i (\$/tonne)

For disseminated ore bodies, following relationship is often valid:

$Rx = C.\ e^{a-bx}$ and $R = n.\ p$ ... 1

Where,

R_x - Ore reserves (tonne)

e - Natural logarithm base,

C - Ore reserves with the cut-off grade a/b (tonne); a, b - Constants.

For disseminated ore bodies where this function holds good, the average grade is a function of the cut-off grade, given by the following equation, $i = x + 1/b$

Calculation of optimum cut-off grade for maximizing the spread between the sums of the annual working profits during the lifetime of the plant and the investment cost–

Operation costs = $k_1 + k_2/p$

Annual working profit = $p\ (i.m - k_1 - k_2/p)$

Total profit, $V = n.p\ (i.m - k_1 - k_2/p) - a_1p - a_2$ ... 2

$\Rightarrow V = R\ (i.m - k_1 - k_2/p) - a_1p - a_2$ ... 3

$\Rightarrow V = C.\ e^{a-bx}\ \{m(x + 1/b) - k_1 - k_2/p\} - a_1p - a_2$ (putting $R = C.\ e^{a-bx}$) ... 4

Differentiating V with respect to x and putting dv/dx = 0,

$m.x = k_1 + k_2/p$ ... 5

If the mining costs for the marginal ore are not same as the average mining costs for ore, then including the difference (= D) in the above formula, the formula changes to –

$m.x = k_1 + k_2/p - D.$ ... 6

Calculation of optimum annual production with respect to annual working profits during the lifetime of the plant and the investment cost

$V = R\ (i.m - k_1 - k_2/p) - a_1p - a_2$

Differentiating V with respect to p,

$dV/dp = R.k_2/p^2 - a_1$

V_{max} is obtained when $dV/dp = 0$, i.e. $\Rightarrow p = (R.k_2)/a_1)^{1/2}$

Using 'p' value we finally got,

$V_{max} = R\ (i.m - k_1) - 2\ (R.k_2.a_1)^{1/2} - a_2$ ... 7

Calculation of the optimum cut-off grade with the objective to maximize the spread between the present value of annual operating and investment cost.

The following argument was based on the results obtained from the discussions of the above. If mining of 1 tonne marginal ore now plus mining of 1 tonne ore with average grade at the conclusion of mining operations give the same economical result as mining of 1 tonne of average grade ore now, then the grade of the marginal ore is the optimum cut-off grade. This is valid at any given time during the life of the mine. Taking the interest rate into account, this can be formulated as:

$m.x - (k_1 + k_2/p - D) + 1/q^n\ [m(x+1/b) - (k_1 + k_2/p)]$

$= m(x + 1/b) - k_1 + k_2/p$

$\Rightarrow D + 1/q^n\ [m(x+1/b) - (k_1 + k_2/p)] = m/b$

$\Rightarrow x = q^n\ (1/b - D/m) + (k_1 + k_2/p)/m - 1/b$ ... 8

Where, n means the remaining life of the plant at any given time.

Hence, the break-even cut-off grade varied in this connection with time. With n = 0 it reduces to equation (6).

Calculation of the optimum cut-off grade with an objective of getting maximum profit in free market economy and with fixed property life.

He considered also the part of ore which will be mined at loss, this can be formulated as: Using $Rx = C.\ e^{a-bx}$ and $R = n.\ p$

$V = \{(q^n - 1)/\ q^n\ (q - 1)\}.p.\ [m.\ (x + 1/b) - k_1 - k_2/p] - a_1p - a_2$ ... 9

$$=> V= \{(q^n – 1)/q^n (q – 1)\}. (C/n). e^{a – bx} [m. (x + 1/b) – k_1] – k_2 \{(q^n – 1)/ q^n (q – 1)\} – a_1.C/n. e^{a – bx} – a_2 \quad ... 10$$

Differentiating V with respect to x and putting dV/dx = 0,

$$m.x= k_1+ a_1. \{(q^n – 1)/ (q – 1)\} \quad ... 11$$

With q = 1 (i.e., interest rate = 0%) equation (11) changes to– $=> m.x = (k_1 + a_1/n)$ ... 12

2.1.2 Shortcomings of this method

- The constants (a, b) values used here to get the optimum result cannot be determined with confidence, so the result is on a limited side and practically undetermined.
- Fixed costs are irrelevant in determination of the cut-off grade, and that maximization of profits implies a descending structure of cut-off grades during the project's life.
- Another limitation of this approach is to restrict the solution to constant production levels, which may not feasible for mine life's condition, production and economics.
- It is having several operational parameters constant during the life of the project, and does not consider the possibility of capacity additions.

2.2 METHOD PROPOSED BY LANE (1964, 1988)

Lane presented a model for calculation of cut-off grade, which become one of the standardized approach for choosing the optimum cut-off grade. Lane identified three stages in mining operation: 1. Mining 2. Concentrating 3. Refining

Each stage has its own associated costs and a limited capacity. The operation as a whole incurs fixed cost. Lane selected maximum present value as the most important economic criteria, which can be applied for calculation of cut-off grade.

MODEL

Material ↓ Mine
Ore ↓ Concentrator
Concentrate ↓ Refinery
Product ↓ Market

Maximum capacity	Unit costs	Other factors
M	m	Fixed costs-f
C	c	Selling price-s
R	r	Recovery-y

Lane derived optimal cut-off grades based on limiting capacities, and confirmed Henning's (1963) result that optimal cut-off grades would be descending over time. His approach maintains several operational parameters constant during the life of the project, and does not consider the possibility of capacity additions. First three cut-off grades were derived by supposing that each of the three stages alone limits the total capacity of the operation. These grades, may be called as the limiting economic cut-off grades, depend directly on the price and indirectly on the actual grade distribution of the deposit.

In the second stage three balancing cut-off grades are defined. These were the grades, which just balance the capacities of each pair of stages. Stages were independent of economics altogether being directly determined by the grade distribution. Also they were dynamic in nature. In an irregular body they can vary rapidly as mining progresses. Finally the best choice of cut-off grade is always one of these six cut-off grades – three limiting economic cut-off grades and three balancing cut-off grades - determined as explained below.

2.2.1 Determination of optimum cut-off grade

1) Determination of optimum cut-off grade considering limited capacity for any one stage

The basic profit expression is,

$$P = (s – r) Q_r – cQ_c – mQ_m – f T \quad ... 13$$

Where,

Q_m- Quantity of material to be mined, Q_c - Quantity of material to be concentrate.

Q_r - Quantity of material to be refined yearly. $\bar{g}$ - is the average grade.

Limited mine capacity: Calculation of cut-off grade assuming that the mining rate is the governing constraint.

If the mining capacity M is the applicable constraint, then the time needed to mine material Q_m is

$$\text{Time, } T = T_m = Q_m/M \quad ... 14$$

So, on differentiating profit (P) with respect to grade (g),

$$dP/ dg = (s – r) dQ_r /dg – cdQ_c /dg – (Q_m + Q_m / M) dQ_m / dg \quad ... 15$$

Equation (15) reduces to

$dP/dg = (s – r) dQ_r /dg – cdQ_c /dg$, as Q_m is constant with respect to grade

For maximizing the value of P, dP/dg = 0

$$=> (s – r) dQ_r /dg = cdQ_c /dg \quad ... 16$$

Again

$$Q r = \bar{g} .y.Q_c, \quad ... 17$$

Differentiating equation (17) with respect to grade (g),

$$dQ_r /dg = \bar{g} .y. dQ_c / dg + y.Qc.d. \bar{g} /dg \quad ... 18$$

So, the optimum cut-off grade is, $g_m= \bar{g} = c/y(s–r)$ ... 19

Limited concentrator capacity: Calculation of cut-off grade assuming that the concentrating rate was the governing constraint.

If the concentrator capacity C was the controlling factor in the system, then the time required to mine and process a Q_c block of material was

Time, $T = T_c = Q_c / C$ ... 20

Substituting equation (20) into (13) gives,

$P = (s - r) Q_r - cQ_c - mQ_m - fQ_c / C$

$= (s - r) Q_r - (c + f/C) Q_c - mQ_m$ ... 21

Differentiating P with respect to g and setting the result equal to zero, yields

$dP/dg = (s - r) dQ_r /dg - (c + f/C) dQ_c /dg - mdQ_m /dg$

$= 0$ ... 22

As before,

$dQ_m /dg = 0$

$dQ_r /dQ_c = gy$

Thus,

$\Rightarrow (s - r) dQ_r /dg = (c + f/C) dQ_c /dg$ ... 23

$\Rightarrow dQ_r /dQ_c = (c + f/C) / (s - r) = gy$ ... 24

So, the optimum cut-off grade when the concentrator is the constraint,

$gc = (c + f/C) / y (s - r)$ ... 25

Limited refinery capacity: Calculation of cut-off grade assuming that the refining rate is the governing constraint.

If the capacity of the refinery (or the ability to sell the product) is the controlling factor then the time is given by,

$T_r = Q_r / R$ ... 26

Substituting equation (25) into (13) gives,

$P = (s - r) Q_r - cQ_c - mQ_m - f Q_r /R = (s - r - f/R) Q_r - c Q_c - mQ_m$ ... 27

Differentiating P with respect to g and setting the result equal to zero, gives

$dP/dg = (s - r - f/R) dQ_r /dg - cdQ_c /dg - m dQ_m /dg$

$= 0$ ... 28

As before,

$dQ_m /dg = 0$

$dQ_r /dQ_c = g y$

Simplifying and rearranging gives,

$\Rightarrow (s - r - f/R) dQ_r /dg = cdQ_c /dg$ ... 29

$\Rightarrow dQ_r /dQ_c = c / (s - r - f/R) = gy$ ... 30

So, the optimum cut- off grade when the refinery is the constraint,

$gr = c/y (s - r - f/R)$ ... 31

2) Determination of optimum cut-off grade considering limited capacity for any two stages

None of the three limiting cut-off grades were necessarily the optimum cut-off grade to employ in cases when the capacities of two stages are limited.

Under such situations, the concept of balancing cut-off grades can usefully be introduced. These were cut-off grades which cause each pair of stages to be just in balance at their maximum capacities.

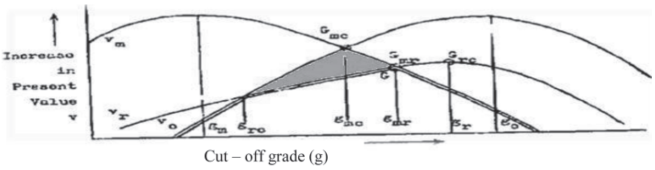

Fig.1 Increase in present value (v) v/s cut-off grade (g)

Balancing cut-off grades may be calculated directly from the grade distribution of the next Qm of material to be mined. Three ratios were required as functions of the cut-off grade – ore: material, product: material, and product: ore. The balancing cut-off grades were then defined as follows:

g_{mc}, ore : material = C : M

g_{mr}, product : material = R : M

g_{rc}, product : ore = R : C

The balancing cut-off grades were independent of economic factors entirely. They were also dynamic in the sense that they depend upon the grade distribution of the material ahead and can vary widely through an irregular body.

3) Determining the overall optimum of the six cut-off grades:

Finally, calculating profits based on cut-off grades satisfying different constraint conditions, a graph had been obtained to finalize the optimum value of the cut-off grade considering mining, concentrating and refining constraints. The corresponding cut-off grade for each pair (G_{mc}, G_{mr}, G_{rc}) were selected using the following rules:

Mine and concentrator $G_{mc} = g_m$ if $g_{mc} < g_m$

$= g_c$ if $g_{mc} < g_c$

$= g_{mc}$ otherwise.

Mine and refinery $G_m = g_m$ if $g_{mr} < g_m$

$= g_r$ if $g_{mr} . g_r$

$= g_{mr}$ otherwise.

Mine and concentrator $G_{rc} = g_r$ if $g_{rc} < g_r$

$= g_c$ if $g_{rc} < g_c$

$= g_{rc}$ otherwise.

2.2.2 Shortcomings of this method

- For the profit maximization, differentiating with respect to grade (g), the equation (18) is $dQ_r /dg = \bar{g} .y.dQ_c / dg + y.Q_c.d \bar{g} /dg$, but for the determination of optimum cut-off grade Lane had considered this part only $\rightarrow \bar{g}$ .y.dQ_c /dg and left this part $\rightarrow$ y.Q_c.d $\bar{g}$ /dg without any justification.

- Lane had considered '$\bar{g}$' as a constant and differentiated on that basis and ultimately that constant ($\bar{g}$) becomes the optimum cut-off grade $\rightarrow \bar{g}$ = c/(s–r).y, which he was actually defined as the average grade before, which is a contradictory and not possible at the same point of time and makes the calculation really unexplainable.
- Using the above equations, Lane had plotted the graph for the determination of optimum cut-off grade; the optimum cut-off grade may lie anywhere in the shaded area enclosed by G_{mc}, G_{mr} and G_{rc} as shown in the Fig.1, but it would not provide any specific point on this shaded area to determine the optimal solution. So, the determination of optimum cut-off grade as has been proposed may not be optimum always.

Lane further developed his ideas in his book: "The Economic Definition of Ore" (Lane K., 1988). Here he described the problem of cutoff grade optimization as dependent on the opportunity cost of deferring the processing of higher grade ore. He explains that if low quality material was processed today, in essence deferring the processing of any available higher quality ore is deferred to some later date. Since the longer these potential profits were deferred into the future, the more they were discounted and then the act of processing marginal ore imposes an opportunity cost due to the time-value of money. This opportunity cost can be minimized by operating at higher cutoff grades and processing higher quality material sooner. With this understanding, the cutoff grade optimization problem becomes a balancing act between reducing the opportunity cost against the wasting of the resource. In order to solve such a problem, Lane outlined a dynamic programming approach that was able to ensure that an optimal cutoff grade is chosen for all points in time, assuming future prices are known (Lane K., 1988).

However, the future prices can only be estimated through forecasting involving the element of uncertainty.

2.3 Method proposed by Dowd (1976)

Dowd used dynamic programming to determine the sequence of annual cut-off grades with the objective to maximize the net present value, but limiting themselves to the operation stage, not considering capacity additions. His examples are constrained by the mill which remains full in all the cases presented, and his work showed only cut-off grade optimization. An expression of dynamic programming that has been used in his approach is as shown below.

Let

$$R (x_1, y_1; x_2, y_2; \ldots \ldots x_n, y_n) = g_1 (x_1, y_1) + g_2 (x_2, y_2) + \ldots \ldots g_n (x_n,y_n)$$

R, being the return from producing x_1 tonnes at a grade y_1% in period one plus the return from producing x_2 tonnes at y_2% in period two, etc. The objective is to maximize the return R subject to the constraints.

$$xi > 0 \text{ and } \sum_{i=1}^{n} xi = x$$

Where x is the total ore reserve. The basic functional equation for this maximization is:

$$f_n (x) = \max_{x_n,\, y_n} [g_n (x_n, y_n)+ f_{n-1}(x - x_n)]$$

Where, f_n (x) is the benefit obtained from the optimal policy for the n periods and x_n and y_n represent optimal strategies.

Dowd had also used stochastic programming to deal with the variation in the prices and the costs that will vary over time and their prediction is subject to uncertainty, but if sufficiently accurate estimates are possible they can be forecasted in terms of probability. It was supposed that all costs and prices can assume any one of the given set of values with certain specified probabilities. Furthermore, it was supposed that at the beginning of any period price, and/or cost can move from one of these permissible values to another. This change is not deterministic, but assumed to be stochastic, ruled by transition matrix P = (p_{ij}).

A stochastic programme for the prices can be formulated as:

$$f_n (x) = \max_{xn,\ yn} [\sum_{j=1}^{M} g_n (x_n,y_n) p_j (n) + f_{n-1}(x - x_n)]$$

$$\text{Where } p_j (n) = \sum_{j=1}^{M} p_i (n-1)p_{ij} = 1, 2, 3 \ldots M$$

As p_{ij} is the probability of a change in price from value i to value j. It is clear that

$$0 < p_{ij} < 1 \text{ and } \forall\ i, j$$

$$\Sigma U\ pij = 1\ V\ i, j$$

Dowd had developed a stochastic distribution of metal prices, with Bayesian probabilities of getting a metal price j in year n+1 given a metal price i in year n. He developed and solved two examples, one for a deposit with a homogenous grade-tonnage distribution throughout, and one where five production levels are defined, each with its own grade-tonnage distribution throughout.

2.3.1 Shortcoming of this method

- There is a serious lack of background in relation to concentrator operation.
- He recognized that his model was oversimplified, but that it served the purpose of introducing a general model to generate a solution.
- Dynamic programming models are robust and best fitted, but they require long computer hours when state variables are more than few.

2.4 Method proposed by Rudenno (1979)

Rudenno favoured a variable cut-off grade concept with the help of quadratic expressions for each variable in the determination of cut-off grades, which represented the contribution to the rate of return (or values) of changes in the respective variable. Adding up all the contributions with the initial value of the rate of return, he obtained an expression for the rate of returns that can be maximized. He proposed linear programming techniques to perform this maximization. This formulation is complex and still not extensively used due to various limitations.

2.5 Method proposed by Dagdelen (1992)

Dagdelen stated that breakeven cut-off grades in open-pit mine planning may maximize the undiscounted profits. Therefore, the cut-off grade can be introduced as a function of economic parameters and the grade distribution within the deposit, together with mining, milling, and refinery capacity limitations. This methods is good for specific mine production and management, but suffer in their generalized acceptance with the low robustness.

2.6 Method proposed by Whittle and Wharton (1995)

Whittle and Wharton proposed the idea of opportunity cost utilization in cut-off grade optimization in monometallic deposits by introducing two pseudo costs, named delay and change costs.

2.7 Method proposed by Cairns *et al.* (2003)

Majorly, Cairns, and Shinkuma (2003) presented a model to determine the influence of certain economic factors, such as price net of interest rate on cut-off grade and grade distribution. For this purpose, they incorporated optimal choices and their allocations at the margin during the model.

2.8 Method proposed ASAD (2005b and 2007)

The major efforts belong to Asad's contributions during 2005 and 2007. First, Asad (2005a) modified Lane's algorithm for cut-off grade optimization of two-mineral deposits with an option to stockpile. Subsequently, he improved the algorithm by considering dynamic metal prices and cost escalation (Asad, 2005b and 2007).

2.9 Method proposed by Mishra *et al.* (2006)

Mishra followed the lane approaches and developed a different method to find the optimum cut-off grade. The mining operation consists of three stages, i.e., mining, concentrating, and refining. The cut-off grade decides the amount of material to be handled economically in three stages. He concluded the result with only one case with keeping all constraints in all the three stages of mining, concentrating and refining.

2.10 Method proposed by Bascetin *et al.* (2007)

From a different viewpoint, he modified Lane's algorithm by adding an optimization factor on the basis of the generalized reduced gradient algorithm, and presented a Windows© based programme to determine the maximum NPV.

2.11 Method proposed by Osanloo *et al.* (2008)

Osanloo found environmental issues to be crucial parameter in cut-off grade optimization. Hence, they modified Lane’s algorithm on the basis of maximization of NPV simultaneously with minimization of environmental costs. They claimed that their algorithm is more effective in long-term production planning.

2.12 Method proposed by Gholamnejad (2009)

Gholamnejad incorporated rehabilitation cost into cut-off grade optimization. In his study, the authors have modified and rearranged Lane’s algorithm through the inclusion of variable capacities for the mine, processing plant (concentrator), and refinery. All results are compared to the original algorithm and with a modified version suggested by Gholamnejad.

2.13 Method proposed by Aditya Mishra *et al.* (2009)

Dynamic programming had been introduced by Aditya Mishra *et al.* (2009), for the determination of optimum cut-off grade (pit cut-off and mill cut-off). The conventional method (following Lane’s algorithm) for different constraint conditions has number of limitations. In an effort to overcome these limitations, an approach based on dynamic programming technique has been used to compute the same easily and at a faster rate with high level of accuracy. A computer package has been developed that integrates eight numbers of modules to represent the eight numbers of different possible constraint conditions. The package has been tested and validated with the help of available data.

Model no.	Mine capacity	Mill capacity	Smelter and refinery capacity
Model no.1	Unlimited	Unlimited	Unlimited
Model no.2	Limited	Unlimited	Unlimited
Model no.3	Unlimited	Limited	Unlimited
Model no.4	Unlimited	Unlimited	Limited
Model no.5	Limited	Limited	Unlimited
Model no.6	Limited	Unlimited	Limited
Model no.7	Unlimited	Limited	Limited
Model no.8	Limited	Limited	Limited

The software (cut-off grade optimizer) had been developed to fulfill the needs of the industry. The software incorporates all the costs incurred during mining, milling and refining process as well as incorporates the time value of money. Software also takes into consideration the different capacity constraints (mine, mill/plant and smelter and refinery) that may be faced during planning of a surface metalliferous mine.

Validation of the software was done using some available data. The results generated by the software were tallied with manually calculated results and errors were found to be within permissible limits. Hence after thorough testing and validation, it may be concluded that the computer package is very much functional, reliable and can be used for calculation of optimum cut-off grade in case of mine.

2.14 Method proposed by Shiwei Yu *et al.* (2011)

This research is based on an integrated intelligent system based on the genetic algorithm, random simulations, and neuron networks. The relationship among the cost, revenue, grade, and metal recovery rate is highly complex and nonlinear. The research was aimed at obtaining the optimized combination of cut-off grade and milling grade to maximize the profit of production process. The major feature of this research includes:

- Establishment of a model to optimize cost, revenue, grade, and metal recovery rate in production process. This model includes 3 unknown functions related to cut-off grade and milling grade.
- Design of a simple random simulation technique to obtain the cut-off grade related to the loss rate.
- Efficient integration of some soft computing methods through a neural network. The inner layer of the network carries out the local approximation while the outer layer carries out the global search. The integration of these two layers can avoid the common problem of the local minimization of neural network.

2.15 Method proposed by Asad *et al.* (2011)

In this model, Asad demonstrated the combined impact of introducing economic parameters, escalation and stockpiling options into the cut-off grade optimization model and how the optimum cut-off grade policy maximizes the net present value (NPV) of an open pit mining operation subject to the mining, processing, and refining capacity constraints. The optimum cut-off grade policies indicated that the impact of escalation on the objective function could be enormous in cases where operating and fixed costs were escalating at higher rates. This may change some of the economic open pit operations to an uneconomic scenario.

He proposed a case study, considering a hypothetical copper deposit divided into 3 pushbacks to demonstrate the developed model. It may be observed that the operation presented in the case study becomes unprofitable during later years at an escalation rate of 6 per cent per year in the operating and fixed costs. Therefore, long term mining plans should always include escalation of economic parameters to establish the feasibility of the mining venture.

The results also reflected that the creation of stockpiles scheduled approximately 55 million tonnes of additional ore for processing, which facilitated in neutralizing the effect of escalating economic parameters through enhancement of the life of operations along with NPV. However, it was clear that allowing the creation of long-term stockpiles was a strategic decision, and exercising this option depends exclusively upon the operating conditions of an open pit mining operation.

The proposed cut-off grade optimization model considered the escalation of economic parameters, and fixed costs, with a vision that mine planning activity was focused on survival strategies under harsh economic situations. As such, it was a contribution to the mine planning community in terms of facilitating the evaluation of different economic alternatives, ultimately ensuring the optimum utilization of resources coupled with appropriate policy formulation for making major mining investments.

The proposed methodology is limited in its application to the metallic ores; therefore, while making this important decision, one must give a serious consideration to issues such as material deterioration and compaction during long exposure to the environment. Similarly, loss of values may take place due to leaching, and oxidation may introduce processing complexities and result in reduced recoveries. The model does not consider uncertainty in economic parameters, especially, the uncertainty associated with the metal price. Also, it is limited to the creation of long-term stockpiles. Therefore, the development of cut-off grade optimization models taking into account metal price uncertainty and allowing the processing of stockpile material during mine life are some of the areas for future research.

2.16 Method proposed by E. Bakhtavar *et al.* (2012)

This new 'variable capacities-based algorithm' was attempted to reset all capacities according to the selected cut-off grade for each stage. An initial assumption was that the refinery capacity was equal to the market demand, and mine and concentrator capacities are then considered as a function of cut-off grade and the refinery capacity. A computer programme had been developed using Microsoft Excel for implementation of the variable capacities based algorithm given by Gholamnejad (2009) which was developed on Lane's original algorithm by adding rehabilitation costs to the equation. In the example given here, this rehabilitation cost was assumed to be equal to 0.5 \$/tonnes. Originally, in Gholamnejad's algorithm the rehabilitation cost was incorporated into determination of the optimum cut-off grade by Lane's algorithm. According to the developed Lane's algorithm, Equation (32) was rewritten as Equation (33) in order to find the total profit (P).

$$P = (s–r)\ Q_r – cQ_c – mQ_m – f\ T = (s–r).\ \bar{g}\ .y.\ Q_c – m.\ Q_m – cQ_c – f\ T \quad ...\ 32$$

$$NPV = P/T\ [(1+d)^T – 1/d.(1+d)^T] \quad ...\ 33$$

A computer-based programme was developed in Microsoft office excel as given below.

Step 1 - Start

Step 2 - Determination of the number of loops using equation (34),

$N = (g_{max} - g_{min}) / \lambda = (1 - 0)/0.01 = 100$ ... 34

Where,

N - is number of loops

g_{max} - is the maximum possible grade (considered here to be equal to 1)

g_{min} - is the minimum possible grade (considered here to be equal to 0)

λ - is an increment amount added to the grade value after each loop (considered here to be 0.01)

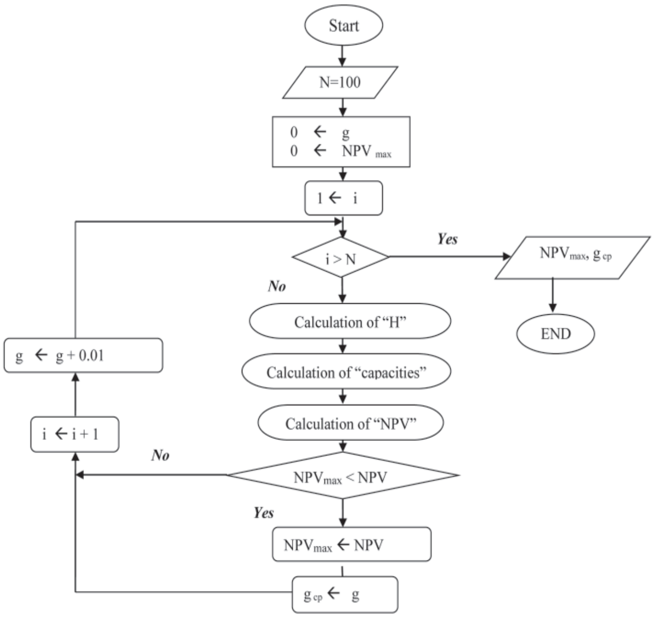

Fig.2 Flow chart of the variable capacities-based algorithm

Step 3 - Cut-off grade was considered to vary between minimum (0) and maximum (1) in the loop as mentioned above. In first pass (i = 1), cut-off grade is assumed to be equal to 0. It is increased by adding the amount of 0.01 during next loop (i + 1).

Step 4 - Using an initial cut-off grade equal to 0, Furthermore, waste to ore ratio (H) is calculated using equation (35) ,

$H = (Q_m - Q_c)/ Q_c$ ... 35

Step 5 - Calculation of mining, processing and refinery capacities as variable parameters. To find out the maximum efficiency of investment that can be obtained. Where, R – is the maximum refinery capacity (pounds per year).

Amount of material recovered from the processing,

$Q_r = y.\,\bar{g}\,.Qc$ ... 36

Maximum concentrator capacity (tonnes per year),

$C = R/y.\,\bar{g}$ ... 37

Assuming that refinery capacity equals the market demand, which is also a variable,

$M = C.\,(1+H)$, is the maximum mining capacity (tonnes per year). ... 38

Therefore, an amount of material (C) can be sent to the processing plant, expressed as a fraction of the total material extracted annually from the mine (M) according to the assigned cut-off grade and considering the maximum efficiently utilization of the investment.

So, amount of waste rock can be determined using equation (37), $W = M - C$ ... 39

The waste to ore ratio can be rewritten as,

$H = (Q_m - Q_c)/Q_c = (M - C)/C$ ... 40

Hence, this relation is derived to calculate waste rock amount, $W = H.C$ ... 41

Step 6 - Calculation of NPV using equation (33),

For this purpose, total profit (P) is first calculated by Equation (32), and Equation (33) used to estimate the mine life (T) which is required for NPV calculation.

$T = \max (Q_m/M, Q_c/C, Q_r/R)$ ... 42

Step 7 - Checking for maximum NPV in each loop. If the calculated NPV in loop 'i' is greater than the previous iteration, it was considered as the new maximum NPV (NPV_{max}) and the associated cut-off grade was taken into account as the optimum grade (g_{op}) and then step 8 was followed.

Otherwise, step 9 must be followed.

Step 8 - The initial value assumed as zero for cut-off grade was increased by adding the value of 0.01and it is replaced to the initial value for next loop i+1.

Step 9 - Checking for all loops to be considered. If all loops (i >N) greater are taken into account, the NPVmax and its related optimum cut-off grade (gop) are considered the final solution, otherwise, step 4 is repeated.

Step 10: End.

According to the developed Lane's algorithm, equation (32) was rewritten as equation (43) in order to find the total profit. Where, h is the rehabilitation cost.

Finally profit, $P = (s - r)\, Q_r - (c - h).\, Q_c - (m + h).Q_m - f\,T$ ... 43

Overall, results from the above algorithm proposed here are considered preferable, and the algorithm is found to be robust and better in determination of the optimum cut-off grade than both versions of Lane's algorithm.

Case example

For a deposit having 1200 tonnes of ore and waste rocks within in its pit outline, having grade intervals of 0.1. The veracity of the new algorithm proposed in this paper can be judged by comparison with results from both Lane's original algorithm and a modified version proposed by Gholamnejad (2009) employing the data given in Tables I and II, and the

TABLE I: MATERIAL CATEGORIES ACCORDING TO THE CASE EXAMPLE (GHOLAMNEJAD, 2009)

Grade (lb/tonne)	Quantity (tonnes)
0.0 – 0.1	130
0.1 – 0.1	145
0.2 – 0.1	115
0.3 – 0.1	140
0.4 – 0.1	110
0.5 – 0.1	170
0.6 – 0.1	110
0.7 – 0.1	90
0.8 – 0.1	125
0.09 – 0.1	65
Total	1200

TABLE II: DESIGN PARAMETERS ACCORDING TO THE CASE EXAMPLE (GHOLAMNEJAD, 2009)

Parameter	Value
Maximum capacity of mine (tonnes/year)	100
Maximum capacity of concentrator (tonnes/year)	50
Maximum capacity of refinery (lb/year)	40
Cost of mining ($/tonne)	1
Cost of milling ($/tonne)	2
Cost of refinery ($/tonne)	5
Recovery	100%
Constant cost ($/year)	300
Selling price ($/lb)	25
Discount rate	12%

result obtained in Table III. In the example given here, this rehabilitation cost was assumed to be equal to 0.5 ($/tonne).

Overall, results from the algorithm proposed here are considered preferable, and the algorithm is found to be robust and better in determination of the optimum cut-off grade than both versions of Lane's algorithm.

In this study, Lane's original algorithm had been modified by considering variable capacities in the determination of the optimum cut-off grade in open-pit mines. The results of this new algorithm show that the optimum cut-off grade and maximal NPV was determined to be less than that obtained from previous algorithms, and it was suggested that the proposed algorithm was more capable in determination of the optimum cut-off grade. It was also concluded that mine, processing plant, and refinery can all work with full capacities during this mining project.

The results indicated that the variable capacities-based algorithm was preferable for utilization in optimum cut-off grade determination. In order to facilitate use of the new proposed algorithm, a computer-based programme was developed in Microsoft Office Excel, but again its results were limited and do not include the dynamicity of the mining, milling & refinery constraints with time.

2.17 METHOD PROPOSED BY KHODAYARI *ET AL.* (2012)

Objective of this research was to determine the optimum cut-off grade of processing plant in order to maximize output rate. For performing this optimization, an Operations Research (OR) model had been developed. The object function of this model was output rate that must be maximized. This model had two operational constraints namely mining and processing restrictions. For solving the model a heuristic method had been developed. In this paper, determination of the optimum cut-off grade in order to maximize the output rate had been investigated.

In the model, Khodayari had stated that the objective function was maximizing the amount of product that can be produced per year, which in contrast with Lane's model, while it was a functional goal and not an economical objective. The results of this research revealed that in this case, optimum cut-off grade was equal to the balancing cut-off grade of mining and processing operations, which depends on mining, processing capacities and tonnage - grade distribution of the ore in the pit.

2.18 METHOD PROPOSED BY ASAD *ET AL.* (2013)

In 2013, Asad presented an extension of Lane's heuristic approach to cut-off grade optimization. The proposed modification enhances the application of Lane's original model not only to open pit mining operations with multiple ore processing facilities, but also accounts for geological uncertainty. The model utilizes multiple simulated or equally probable grade – tonnage curves derived from equally probable simulated realizations of the ore body model.

However, irrespective of considering multiple grade – tonnage curves, the heuristic approach suggests a unique cut-off grade policy and looks into the risk of maintaining the cash flows to maximize NPV based on the possible variations in production from mine, processes, and market/refinery during the life of operation. Apart from considering multiple ore processing streams, the proposed heuristic approach is limited to single mine or material source and market/refinery (i.e., it sums the quantities of metal in concentrate from

TABLE III. RESULTS OBTAINED THROUGH THREE DIFFERENT ALGORITHMS

Algorithm	Cut-off (lb/ton)	M (t)	C (t)	R (lb)	Qc (t)	Qr (lb)	T (year)	Profit ($)	NPV ($)
Lane (1964)	0.40	100	50	40	670	450	13.4	2440.0	1185.07
Gholamnejad (2009)	0.375	100	50	40	715	472	14.3	2277.5	1064.75
Variable capacities-based	0.20	90.95	70.11	40	925	527.75	13.19	3546.88	1737.99

individual processing streams).

A mining complex with multiple mines and limited capacity of individual market or refining processes presents a very complex supply chain system under geological uncertainty, and a heuristic procedure may not capture the details of such a complex optimization problem. As such, future studies require the development and implementation of Stochastic Integer Programming (SIP) based mathematical formulations to solve such a complex optimization problem.

2.19 Method proposed by Azimi *et al.* (2013)

An uncertainty based multi-criteria ranking system to investigate the problem of cut-off grade strategy (COGS) selection considering metal price and geological uncertainties has been developed by the researchers. The system aims at selection of the best COGS among technically feasible alternative cut-off grade strategies under uncertainty circumstances. Their developed system was based on integrating metal price and geological uncertainties as well as operating flexibility to close the mine early. This model incorporated this operating flexibility into the proposed system using a Monte Carlo based real options (RO) valuation model. For this purpose, in addition to the expected value, other risk criteria are considered to rank the alternatives. These risk criteria include abilities of strategies in producing extra profits, minimizing losses, and achieving the predefined goals of the production. In this study, the technically possible cut-off grade strategies are generated using the Lane comprehensive algorithm. To demonstrate the effectiveness of the proposed system, they utilized data of an Iranian gold mine. Results show that the proposed system outperforms conventional methods in the sense that it shows significantly lower average mis-ranking than the other methods and also selects a strategy with a higher value. The sensitivity analysis of the proposed system relative to the gold price shows that the system was highly dependent on the parameters of the stochastic process used to model the evolution of the metal price. Therefore, special consideration should be given in estimating stochastic process parameters.

2.20 Method proposed by Rahimi *et al.* (2014)

An optimum cut-off grade modelling was developed with the objective function of net present value (NPV) maximization in this research work. The costs of processing methods and associated environmental costs were also involved in the model. Next, limiting and balancing cut-off grades of processing methods were calculated through Lagrange multiplier optimization method. In this paper, an applicable iterative algorithm based upon NPV maximization was done to determine the optimum concentration and leaching cut-off grades. This procedure is an approach of determining the optimum concentration and leaching cut-off grades, which is adversely affected by all environmental pollutants in the sequences of copper production. The results indicated that the optimum concentration and leaching cut-off grades decrease during mine life.

In addition, it had demonstrated that low-grade ores treatment by leaching method can led to improvement on NPV, although waste management system would be greatly complicated. On the other hand, it was realized that processing of low-grade ores by heap leaching methods did not only results in more profit but also the adverse effects of waste dumping decreases. The calculation proved that the environmental costs of leaching process were less than those of the concentration method. Furthermore, it was also observed that the environmental costs of mine per unit of final product which has only used concentration method was significantly more than the mine used by both leaching and concentration methods. The results showed that the concentration and leaching optimum cutoff grades policy can make an improvement on overall NPV by 35% in comparison with the traditional approaches of cut-off grades determination. The adverse environmental impacts of low-grade ores dumping were also reduced by using hydrometallurgical methods. The most important advantages of hydrometallurgical methods are their low operating and investment costs that make it possible to economically extract low-grade and small copper deposits (Dreisinger 2006; Watling 2006).

2.21 Method proposed by Thompson *et al.* (2014)

In this method, the researchers developed a stochastic dynamic model for cut-off grade optimization for use in mining valuation and optimal operation that accurately describes the engineering realities. The model did not require an a-priori block ordering but can incorporate any feasible mining schedule. Only those inputs that were already produced by current mining industry software and are familiar to mining professionals are needed. The optimization problem was formulated and solved as a system of nonlinear partial differential equations (PDE) that can be solved with a specially designed numerical implementation. The PDE simultaneously solved for the value, optimal cut-off strategy, and the hedging statistics for every possible future price scenario. This information can subsequently be used to simulate optimal mine operation through time in order to assess and measure the operational, market and margin risk of mining firms.

The impact of forward price uncertainty on the optimal cut-off policy of mines had been illustrated by employing a new real options model for determining the optimal cut-off grade of ore under stochastic prices. The first was that in the presence of market uncertainty, cut-off grades were lower than those predicted using traditional deterministic models. This implies that the traditional view may be leading to far greater resource waste than was optimal. The second important insight was that when valuation time horizons were longer, cut-off values were lower. Since governments typically consider longer time horizons than mining firms with finite land lease agreements, the extrinsic value of ore will be higher to a government than to a mining firm if both were using a stochastic framework. Governments therefore have an interest in encouraging the use of stochastic cut-off models in the mining industry in order to reduce resource waste and increase valuations.

Future metal price uncertainty had significant and profound impacts on cut-off grade determination, which was one of the most fundamental operational decisions in mining, motivates the need for further research into mining models that account for cut-off strategies in the face of price uncertainty.

Limitation of the study:

- The above model assumed only one stochastic factor, and considered only that factor was responsible for the fluctuations in the future price expectations.
- This formulation considered only a single processing mode. Often mines will have more than one processing method such as a mill and leach bed for instance.
- The option of stockpiling below cut-off grade ore for future processing are not done, such metals receive the discounted futures price associated with the expected time of processing.

2.22 Method proposed by Qing-Hua *et al.* (2014)

In this method, the equivalent grade and net present value was adapted to establish cut-off grade optimization, especially considering dynamic recovery rate. They had proposed that multiple metal cut-off grade optimization model was set using the objective function to maximize the net present value (NPV). The method of equivalent coefficient calculation was proposed to converting multi metals cut-off grade optimization to a single metal cut-off grade optimization.

- Cut-off grade optimization of multi-metal is proposed based on two stages: mining and processing in this paper. Considering grade-tonnage distribution concentrate market price and recovery rate, the model of multi-metals cut-off grade optimization was constructed using equivalent factor and NPV. Objective function was expressed as one variable function by equivalent factor. Multi-metal cut-off grade was converted into a single metal cut-off grade.
- A verification example of molybdenum and wolfram was presented for confirming the cut-off grade optimization model in this study. The results of optimal cut-off grade and break even cut-off grade were compared in the paper. It provides advice for production of multi-metals mine.
- he cut-off grade is calculated with many parameters such as concentrate price, discount rate and costs. Those economic parameters are always variable as market changes. So the cut-off grade is optimal with those parameters and this period. Average value and statistical analysis are usually used to eliminate this influence in this study.

2.23 Method proposed by Moosavi *et al.* (2014)

In this work a new mathematical model based on binary integer programming for solving the extraction sequence problem in open pit mines, which can combine reasonable mining operations and cut-off grades strategy into one has been presented. It asserts that the dynamic cut-off grade during any given period was a function of the ore's availability and the needs of the mill in that period considering minimizing economic loss as the objective function.

This model had solved the problem in three steps:

- The actual economic loss associated with each type of processing for each block was determined;
- The probability distribution and average grade for each type of processing were computed from independent realization;
- Each block with its expected economic loss (EEL) was developed as a binary integer programming model.

The formulation was based on an economic loss assessment of a block considering each of the alternative processing decisions and the probabilities distribution of the ore body grades. The proposed procedure developed a model that generated a practical and feasible schedule considering processing types while satisfying system constraints and minimizing EEL. Although the proposed model was not set up to directly maximize NPV, it provides a realized NPV, which was optimum given the mining sequencing and cut-off grade strategy considerations. As a matter of fact, that NPV can be increased by determining the probability distribution of the blocks, because the model tends to minimize the EEL. This leads to more high-grade blocks being mined in earlier periods.

So, the economic loss function method was an efficient technique to determine the optimum processing type of the material in each period with certain innovation:

- Reducing the required number of variables and, subsequently, managing the available variables and constraints in a short time.
- The capability of taking various types of processing into account.

2.24 Method proposed by Yasrebi *et al.* (2015)

In this work the optimum cut-off grade (COG), the objective function, was calculated using a computer-based algorithm with utilisation of non-linear programming based on the Lane algorithm by using LINGO optimisation software in order to obtain a maximised profit value (PV). However, further optimization methods require excessive computational time to find the optimum point for COG. It has been found that the proposed algorithm can be intended when there was not a trending model between coordinates and ore grade (grade values in terms of X, Y and Z direction of voxels within a block model) within a deposit, such as a porphyry copper deposit. For a better use of the proposed algorithm, it may be suggested that the grade distribution within a studied deposit should be normalised for more convenience in order to achieve cut-off grade.

2.25 Method proposed by Rahimi *et al.* (2015)

An optimization technique used to obtain concentration and heap leaching optimum cut-off grades has been presented in this paper. Maximization of NPV was considered as the objective function. It was observed that using heap leaching method in processing of low grade copper ores not only can have effects on the NPV of the mines but also can decreases the complexity of wastes management and the adverse environmental impacts of mining projects. The results indicated that the copper price changes had an essential effect on optimum cut-off grades for leaching and concentration in case mining capacity was the applicable constraint. In the case study presented, optimum cut-off grades for leaching and concentration were found to decrease as copper price rises. However, the rate of leaching cut-off grades reduction was less than concentration cut-off grades.

2.26 Method proposed by Mohammadi *et al.* (2015)

In this study, the golden section search (GSS) method and the imperialist competitive algorithm (ICA) were used to optimize the cut-off grade in mine No. 1 of the Golgohar iron ore mine. For the purpose, in the first step, the objective function was developed by considering three types of salable products in this iron ore mine. Since the objective function of this problem was non-linear, three methods can be used to solve it: analytical, numerical, and meta-heuristic. In order to do so, at first, the Lane's method was modified, and the objective function for determination of the optimum cut-off grade based on maximizing the net present value (NPV) for future cash flows was defined. Then the golden section search (GSS) method and imperialist competitive algorithm (ICA) were used. Consequently, the optimum cut-off grades were calculated between 40.5% and 47.5% using both of these methods. NPVs obtained by the GSS method and ICA were 18487 and 18142 billion Rials respectively, and thus the value for the GSS method was higher. To solve the problem, the number of iterations in the GSS method for each year was equal to 18, and that in ICA was less than 18. Also the process of programming and computing in GSS was much easier than that in ICA. Thus, in this problem, GSS had a priority higher than ICA.

2.27 Method proposed by Almasi *et al.* (2015)

The issue addressed in this work has been the analysis of the fundamental economic concepts in achieving optimal cut-off grade. The leaching cutoff grade sorts the ores in two sections: the high-grade, to be processed in the mineral processing plant and the low grade, to be processed in the leaching plant. The second group processing can be done in a mineral processing plant also, but it would be more profitable at the leaching plant surely. The data used in this investigation was adapted from Sarcheshmeh copper mine. According to the computation at this mine, the leaching cut-off grade equals to 0.354%, but this cut-off grade is supposed to be mill cut-off grade and the ores with grade between 0.15% and 0.354% is the leaching cut-off grade, then only it should be transferred to the heap leaching unit.

2.28 Method proposed by Rahimi *et al.* (2015)

An analysis of the bioheap leaching method and their associated environmental considerations on optimum cut-off grades policy has been presented in this work. This novel algorithm was simply applied to specify optimum cut-off grades in Sungun copper mine. This research work had a remarkable novelty because the outcomes and its modelling embrace capital cost and processing recovery variation for changing the optimum cut-off grades. This work considered environmental parameters and accessibility to sustainable development besides technical and economical problems. This increased the novelty of the current research subject. The results obtained from the current research led to the idea that the investment costs play a key role in determining optimum cut-off grades. This subject had been ignored by many of the previous researchers. Furthermore, the effect of recovery changes had reflected similar results. Comparing this algorithm to the similar ones, it was concluded that this new one was the most complete one for the sake of considering several technical parameters (capacity limitations, ore tonnages and by products, grade-recovery variations), economical parameters (capital costs, investment costs, operation costs and opportunity costs) and sustainable development parameters (social costs and environmental costs of different processing methods).

The outcomes of the new algorithm and its comparison to other research works imply that it was essential to involve investment costs in optimum cut-off grades calculations so that it was directed to real NPV determination. While studying Sungun copper mine, this issue indicates 16% reduction in NPV in comparison to research which hadn¡¦t obtained any investment costs.

The presented algorithm considered the effect of average grade variations on processes recoveries and optimum cut-off grades. Thus, grade reduction on Sungun copper mine led to the recovery decrease of concentration method up to 9% and also NPV to 13% which directly affects the optimum cut-off grades. Holistically, this study proved that the environmental cost of mines practicing leaching and concentration methods is less than mines using only concentration method. This problem was for the sake of less adverse environmental and social impacts of hydrometallurgical methods than pyrometallurgical ones. So, applying hydrometallurgical processes for processing low grade copper ores pave the way to access sustainable development by performing novel algorithm.

Analyzing the changes of commodity price on cut-off grade policy and accessing to sustainable development were regarded as important issues supplying this algorithm in future. The results of the research will be more exact if the discount rate is contemplated in construction phase and investment costs. This requires a new algorithm expansion.

Limitation of the study

- The most difficult parts of this method were calculation and determination of the sustainable development factors including social and environmental issues and their not being quantative.
- From practical aspects, calculation of the recovery of different processing methods and determination of grade-recovery curve were identified as the time and cost consuming part of the model.

2.29 Method proposed by Narrei *et al.* (2015)

This work described the process to develop the cut-off grade based on Lane's algorithm with regard to reducing the most important undesirable environmental effects. The main modification was considering the costs and revenues caused by reclamation of pit, waste dump and tailing dams. In addition, this model incorporate the revenues from reuse of waste materials or tailings (for building materials, concrete, sand lime bricks, glass-ceramics etc.) to the Lane's algorithm.

Due to the importance of selecting an appropriate reclamation plan for the mined out land, a combined multi-attribute decision-making approach (MADM) has also been proposed to evaluate the post-mining land use plan with the use of effective and major criteria with respect to the user's preference orders. The developed model had been verified by the data gathered from Gol-Gohar iron ore mine of Iran and results indicated that there was some important improvement in mine project goal.

2.30 Method proposed by Zhang *et al.* (2016)

This research work had established a theoretical two-stage model shown in Fig.3, examining the processing of the stockpiled materials, and its impacts on the optimal mining rate, cut-off grade strategy, and mine's profit. Profit from the stockpiled material cannot be ignored, which may account as much as15% of the mine's total profit.

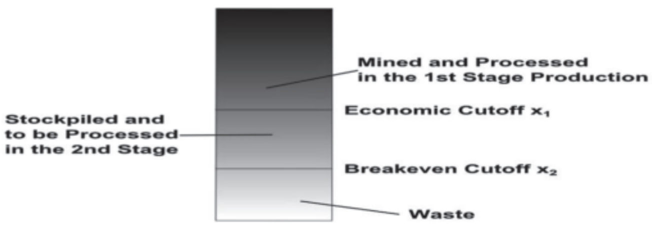

Fig.3. A two-stage production model with cut-off grades and processing he stockpile material.

This research also measures how responsive the optimal mining rate was to the changes in the input variables, with the calculations of their pseudo-elasticities.

Considering that the mine¡¦s processing plant was always operating in full capacity during the production years, it was found that:

- Total material quantity, metal price, recovery rate, highest grade in deposit, processing capacity and discount rate prove to positively impact the optimal mining rate;
- The processing cost was perfectly inelastic with the optimal mining rate;
- Mining cost and the capital cost of the mining equipment negatively impact the optimal mining rate;
- The effects of several variables were diminishing as their per cent of change increases;
- The processing cost was perfectly inelastic to the optimal mining rate.

In addition, the consideration of the stockpiled material significantly impacts the mining strategy (mining rate and cut-off grade) of the deposit. The two-stage model was also beneficial to policy makers and mineral property owners, when assessing the impacts of a policy promoting the stockpile management and strategic decision.

Limitation of the study

- A constant marginal capital cost was assumed in the model, which did not cover the effect of "economics of scale".
- The stockpiling was not necessarily to be processed after depletion, which was examined in this work and can be processed any time before depletion.
- Some ores such as sulphuric ones cannot be stockpiled for a long time due to problems of degradation and oxidization.
- The location of stockpiles was very important. For simplicity, this research assumes that the stockpiling was done just on ground besides the processing facility with no degrading or additional capital expenditure.
- The economic cut-off grade considering the opportunity cost was usually decreasing as the mine exploration goes on; however, this research assumes that the economic cut-off grade remains constant during the mine's life.

3.0 Conclusion

A review on various methods and algorithms for the determination of optimum cut-off grade that had been used by different researchers has been presented here. The cut-off grade affects the economy of the mining operation to a great extent. In an effort to determine the optimum cut-off grade, different authors had suggested different methods but the methods suffered from various shortcomings. It may be kept in mind that the results presented by various authors, to some extent, dependent on the specified operating conditions of the concentrating and refining plants, and to this extent the result may not truly be called 'general'. In addition, these operations will normally have a well-defined operating range, which may vary in actual situations. Many factors, such as the ability of the operation to respond to changes, have not been considered.

Moreover, it is critical that the material classified as waste today could become economical to be processed in future depending on mining and economic conditions.

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