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Circular failure analysis for slopes made up of homogeneous material – a review

Circular failure analysis for slopes aims at determining critical factor of safety for an existing slope and/or designing of a new slope. There are various analytical and numerical techniques for calculation of factor of safety. Each technique has its own limitations, applicability conditions and a priori assumptions. An effort has been made to review the various analytical methods and to identify suitable ones for stability analysis of homogeneous slopes, and the outcomes have been presented in this paper.

Keywords: *Critical factor of safety, critical slip surface, homogeneous slope.*

Introduction

In India, rapid increase in production of various minerals has intensified opencast mining activities. This has resulted in the opencast mines going deeper and deeper day by day with maximum stripping ratio up to 1:15 (Coal/ mineral : Overburden), at a depth of about 500 m (Kumar and Villuri, 2015). As a direct consequence, this will increase the risks of high wall slope and dump failures tremendously, which can give rise to a significant economic losses and safety impact. In such a scenario slope stability analysis becomes an integral part of the life cycle of the opencast mining projects (Kumar and Prakash, 2015).

In high wall slopes made up of homogeneous material and spoil dumps common mode of failure is circular one. There are various methods of circular failure analysis which can broadly be classified into two – analytical and numerical techniques. In this paper discussions are limited to various analytical methods. To guarantee the stability of a homogeneous slope, the circular arc method is traditionally used for stability analysis (Xiao et al., 2015). The two objectives of slope stability analysis are calculating factor of safety (FOS) for a given slip surface and determining the critical slip surface (CSS) for a given slope (Kalatehjari, 2014). The critical slip surface for a given slope is one which is having minimum factor of safety. And therefore, circular failure analysis essentially involves optimization techniques to assess the stability of a slope.

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Methods of circular failure analysis

In the analysis of slope stability, the primary task is to calculate the factor of safety (Ugai and Leshchinsky, 1995; Zheng, 2006; Shen et al., 2013; Zhou and Cheng, 2013; Lin and Cao, 2014). There are many methods of slope stability analysis for the quantitative assessment of the safety or stability of the slopes. However, they can be classified into two approaches: deformation approach and limit equilibrium approach (John, 1990). The deformation approach relies on stress-strain characteristics of soil and needs a suitable analytical technique (Finite Element method, Distinct Element method, Boundary Element method, etc.) to determine the deformation of the slope and the measure of stability. Because of some of their limitations in practical application engineers still choose the limit equilibrium methods for slope stability analysis (John, 1990).

Limit equilibrium methods are the primary methods that are being used for decades in terms of determining the factor of safety(Ho, 2014). These methods are much simpler in comparison with other methods (Sen, 1994; Chiwaye and Stacey, 2010; Lin et al., 2014; Mathews et al., 2014) and are not as expensive as finite element methods are(Duncan, 1996). Limit equilibrium methods consume less CPU time to calculate factor of safety compared to finite element methods (Lin et al., 2013) and the results obtained by these methods are in close agreement with those obtained by the methods based on different numerical techniques (Sen, 1994).The main limitations are determination of slip surface and assumption of side forces (Lam and Fredlund,1993; Griffith and Lane, 1999; Cheng and Yip, 2007; Nian et al., 2012; Zhang et al., 2013), and inability to consider strains and deformations.

Various methods based on limit equilibrium theory to calculate factor of safety may, in general, be divided into two groups:

1. MASS PROCEDURE

In this procedure, the mass of the material above the slip surface is treated as a single unit. This procedure is useful for homogeneous slope, which is hardly the case in most natural and man-made slopes (Das, 2005).

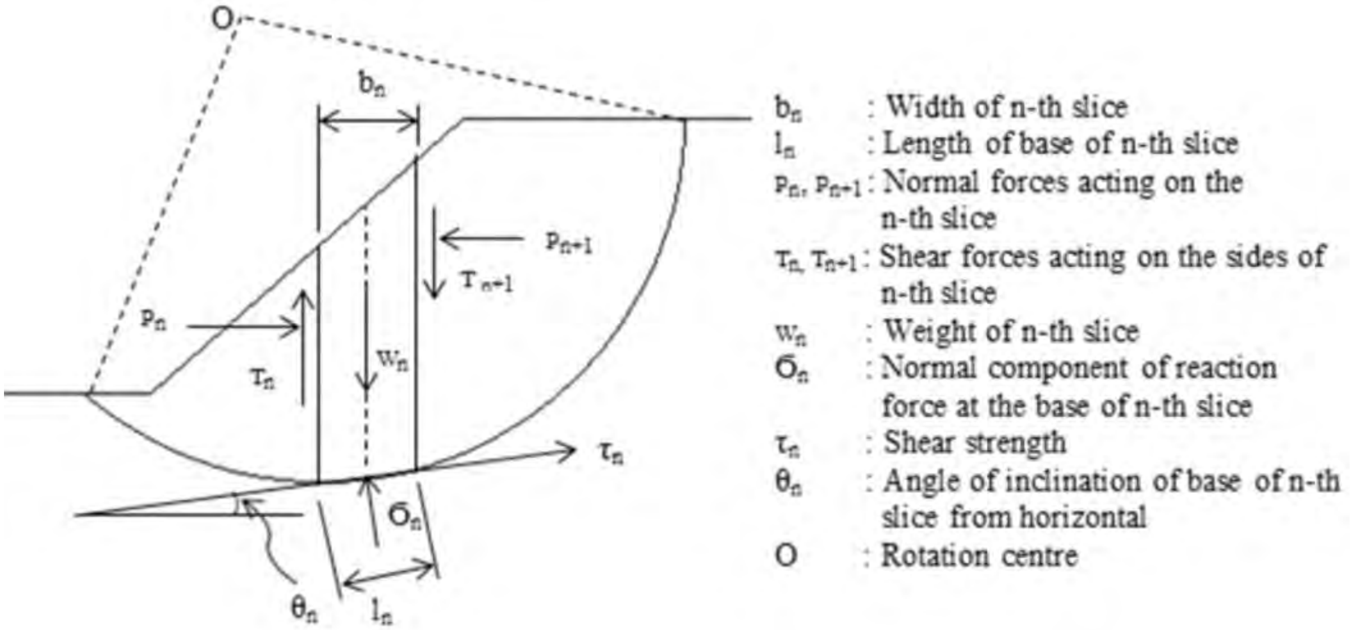

Fig.1 Schematic of the method of slices showing rotation center and forces acting on a slice

2. Method of slices

In this case, the material above the slip surface is divided into a number of parallel slices and each individual slice is affected by a general system of forces, as shown in Fig. 1. The limit equilibrium method of slices is purely based on the principle of statics in which the force and/or moment equilibrium have to be satisfied (Xu et al., 2011 and Burman, et al., 2015). The stability of each slice is determined separately. Though the stress-strain relation of material is not taken into consideration in methods of slices, it provides a good estimate for the factor of safety without the knowledge of initial conditions (Cheng et al., 2007).

For a system where material above the slip surface is divided into n number of slices, there are (6n - 2) unknowns, as listed in Table 1. But 4n equations that can be written for the system, as listed in Table 2, makes the solution statically indeterminate. However, to make the solution determinate {(6n - 2) - 4n} or (2n - 2) unknowns has to be reduced by making some simplifying assumptions. These assumptions generally categorize the available methods of analysis (Sharma and Lovell, 1983).

There are many limit equilibrium methods based on various combinations of assumptions that are discussed below.

Table 1: Unknowns associated with the method of slices (adopted fromAbramson et al., 2002)

Unknowns	Variable
1	Factor of safety
n	Normal force at base of each slice
n	Location of normal force
n	Shear force at base of each slice
n - 1	Interslice force
n - 1	Inclination of interslice force
n - 1	Location of interslice force (line of thrust)
6n - 2	Total number of unknowns

Table 2: Equations associated with the method of slices (adopted from Abramson et al., 2002)

Equations	Condition
n	Moment equilibrium for each slice
2n	Force equilibrium in two directions for each slice
n	Mohr-Coulomb relationship between shear strength and normal effective stress
4n	Total number of equations

Ordinary method of slices

Ordinary method (Fellenius, 1936) of slices ignores all interslice forces and fails to satisfy force equilibrium for both the entire sliding mass and the individual slices (Abramson et al., 2002). This method satisfies only moment equilibrium (Ho, 2014) and is very convenient for hand calculations but less accurate for effective stress analyses with highpore water pressure(Duncan and Wright, 2005). Factor of safety calculated by this method is often slightly less than that by Bishop Method (Zhou, 2010).

Janbu's simplified method

Janbu (1954a, 1954b, 1973) also assumes interslice forces as zero. This method satisfies both vertical force equilibrium and the overall horizontal force equilibrium for the entire mass, but does not satisfy the moment equilibrium. However, Janbu introduced a correction factor to account for this incompleteness.

Janbu's generalized method

The Janbu's (1954a, 1954b, 1973) generalized method includes the effect of interslice forces by making assumption regarding the point at which the interslice forces act. As the position of the normal stress on the last uppermost slice is not used the moment equilibrium is not satisfied for this slice (Sarma,1979). However, this method suggests that the actual location of the thrust line is an additional unknown, and equilibrium can be satisfied rigorously if the assumption selects the correct thrust line.

Bishop's simplified method

Bishop (1955) considers circular sliding surfaces and utilizes the rotational equilibrium of the entire sliding mass for the calculation of factor of safety. In its simplified form, it assumes the interslice forces to be horizontal. The vertical components of the interslice force are assumed to be equal to be zero by Bishop (1955). This method satisfies the vertical force equilibrium for each slice and the overall moment about the center of the circular slip surface (Ho, 2014).

Bishop's rigorous method

It is an extension of Bishop's Simplified method where (n -1) interslice shear forces are considered for calculation of factor of safety. Bishop (1955) introduces an additional unknown by suggesting that there exists a unique distribution of the interslice resultant force, out of possible infinite number, that will rigorously satisfy the equilibrium equations (Abramson et al., 2002).

Lowe and Karafaith's method

This method calculates the factor of safety from force equilibrium equations. Lowe and Karafaith (1960) assume that the inclination of interslice force is equal to the average of the ground surface and slice base angle. This assumption leaves (4n - 1) unknowns and fails to satisfy moment equilibrium.

Morgenstern-Price method

Morgenstern-Price (1965) method is similar to Spencer's method. However, this method is silent about the inclination of the interslice resultant force or its point of application. Although the application of this method is quite cumbersome because of its complexity, it provides the most rigorous limit equilibrium solution (Ho, 2014).

Spencer's method

Spencer (1967) proposes that the resultant of the side forces on each side is at the mid-height of each slice, but does not mention anything about its inclination. This method satisfies both force and moment equilibrium. Factor of safety calculated by this method is more accurate than that by Bishop's method. Jiang and Yamagami (2004) proposed an extended Spencer's method for 3-D slope stability analysis.

Corps of Engineers method

The Corps of Engineers (1970) calculates the factor of safety from force equilibrium equations and assumes the inclination of interslice force as either parallel to the ground surface or equal the average slope angle between the left and right end points of the failure surface. This method satisfies both vertical and horizontal force equilibrium but does not satisfy moment equilibrium for entire mass above slip surface.

Sarma's method

Sarma's method (1973) used to assess the stability of slopes under seismic conditions.This method determines slope stability by applying a horizontal acceleration to the material above the failure surface and calculating the factor of safety the soil mass has to the applied force. The soil strength parameters are reduced until a zero horizontal acceleration is required for failure. That is until the factor of safety has the value of just 1.0. It can also be used for static conditions by neglecting the horizontal force. The method can analyze a wide range of slope failures and it is not restricted to planar or circular failure surfaces. All equilibrium conditions are satisfied by this method. It may provide information about the factor of safety or about the critical acceleration required to cause collapse. This method considers slices parallel to slope face.

Table 3 lists the common methods of analysis and the conditions of static equilibrium that are satisfied in calculating the factor of safety.

All the limit equilibrium methods are applicable for homogeneous as well non-homogeneous slopes. The reliability of most of these methods is within 3% (Miller et al., 1979). As far as shape of the slip surface is concerned any shape is suitable for all the methods except for ordinary method of slice and Bishop's simplified method. These two methods are suitable for circular slip surface only.

Table 3: Static equilibrium conditions satisfied by limit equilibrium methods (adapted from Abramson et al., 2002)

Method	Force equilibrium		Moment equilibrium
	Vertical	Horizontal	
Ordinary method of slices	No	No	Yes
Janbu's simplified	Yes	Yes	No
Janbu's generalised	Yes	Yes	No
Bishop's simplified	Yes	No	Yes
Bishop's rigorous	Yes	Yes	Yes
Lowe and Karafaith's	Yes	Yes	No
Morgenstern-Price	Yes	Yes	Yes
Spencers's	Yes	Yes	Yes
Corps of Engineers	Yes	Yes	No
Sarma's	Yes	Yes	Yes

Ordinary slice method estimates most conservative value of factor of safety amongst all the limit equilibrium methods (Burman, 2015). Other limit equilibrium methods like Bishop's method, Spencer's method and Morgenstern and Prices's method aim at estimating a more realistic interslice forces which may develop in reality and this leads to somewhat higher estimation of factor of safety (Burman, 2015).This method is not suitable for the progressive failure of slopes (Chiwaye, 2010; Ho, 2014).The limit equilibrium method of slices is based purely on the principles of statics. The key piece of missing physics in limit equilibrium method is that it does not take into consideration the stress, strain and displacements, and this is what creates many difficulties with limit equilibrium methods (Burman, 2015).

The simplified Bishop method which is regarded as one of the non-rigorous limit equilibrium methods of slices has been widely accepted as one of the best methods of limit equilibrium for calculating the factor of safety of circular slip surface (Zhu,2008). The value of factor of safety of circular slip surfaces determined by this method are in good agreement with those determined by other rigorous methods of slices such as Morgenstern-Price method and Spencer method (Zhu,2008). For circular failure surfaces, Spencer and Morgenstern-Price methods are at least six times as expensive to run as the Bishop Simplified method and the difference in result is not more than 1% (Fredlund and Krahn, 1977).The simplified Bishop method has been regarded as the accurate method of slices, though it does not satisfy all the limit equilibrium conditions (Duncan, 1996). Even results obtained with slices methods and finite element method are in close agreement. Duncan (1996) reported that the difference between various methods was less than 6%.

Conclusions

The assumptions and mechanics underlying each method based on the concepts of limit equilibrium are studied thoroughly in available literature and the following conclusions are drawn:

1. All the methods are found suitable for stability analysis for homogeneous slope.
2. Bishop simplified method is preferable for circular failure analysis over other methods because of the ease of computation, simplicity and accuracy.
3. The determination of factor of safety does depend only in a minor way on the method used, while a fundamental is an appropriate choice of the parameters involved. Again the accuracy of this method depends on the selection of searching technique for critical slip surface.
4. Bishop's method coupled with robust searching technique is adequate for stability analysis for homogeneous slope.

References

1. Abramson, L. W., Lee, T. S., Sharma, S. and Boyce, G. M. (2002): "Slope stability concepts-slope stabilization and stabilisation methods," Second edition, published by John Willey & Sons, Inc., pp. 329-461.
2. Bishop, A. W. (1955): "The use of slip circles in stability analysis of slopes," *Geotechnique*, Vol. 5, No. 1, pp. 7-17.
3. Burman, C., Acharya, S. P., Sahay, R. R. and Maity, D. (2015): "A comparativestudy of slope stability analysis using traditional limit equilibrium method and finite element method," *Asian Journal of Civil Engineering (BHRC)*, Vol. 16, No. 4, pp. 467-292.
4. Cheng, Y. M., Lansivaara, T and Wei, W. B. (2007): "Two-dimensional slope stability analysis by limit equilibrium and strength reduction methods," *Computers and Geotechnics*, Vol. 34, pp. 137-150.
5. Cheng, Y. M. and Yip, C. J. (2007): "Three-dimensional asymmetrical slope stability analysis extension of Bishop's, Janbu's, and Morgenstern-Price's techniques," *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, Vol. 133, No. 12, pp. 1544-1555.
6. Chiwaye, H. T. and Stacey, T. R. (2010): "A comparison of limit equilibrium and numerical modeling approaches to risk analysis for open pit mining," *The Journal of the Southern African Institute of Mining and Metallurgy*, Vol. 110, pp. 571 - 580.
7. Corps of Engineers (1970): "Slope stability manual," EM 1110-2-1902, Washington, DC: Department of the Army, Office of the Chief Engineers.
8. Das, B. (2005): "Fundamentals of geotechnical engineering," Brooks/Cole Publishers, pp. 351-367.
9. Duncan, J. M. (1996): "State of the art: limit equilibrium and finite element analysis of slopes," *Journal of Geotechnical Engineering*, Vol. 122, No. 7, pp. 577-596.
10. Duncan, J. M. and Wright, S. G. (2005): "Soil strength and slope stability," Wiley, New Jersey.
11. Fellenius, W. (1936): "Calculations of the stability of earth dams," Proceedings of the 2nd. Congress of Large Dams, Washington D. C., Vol. 4, pp. 445-463.
12. Fredlund, D. G. and Krahn, J. (1977): "Comparison of slope stability methods of analysis," *Can. Geotech. Journal*, Vol. 14, No. 3, pp. 429-439.
13. Griffiths, D. V. and Lane, P. A. (1999): "Slope stability analysis by finite elements," *Géotechnique*, Vol. 49, No. 3, pp. 387-403.
14. Ho, I-Hsuan (2014): "Parametric studies of slope stability analysis using three - dimensional finite element technique: geometric effect," *Journal of GeoEngineering*, Vol. 9, No. 1, pp. 33-43.
15. Janbu, N. (1954a): "Stability analysis of slopes with dimensionless parameters," Thesis for the Doctor of Science in the Field of Civil Engineering, Harvard University Soil Mechanics Series, No. 46.
16. Janbu, N. (1954b): "Application of composite slip surface

for stability analysis," European Conference on Stability of Earth Slopes, Stockholm.

17. Janbu, N. (1973): "Slope stability computations," In: Embankment Dam Engineering, Casagrande Memorial Volume, Eds. Hirschfield and Poulos, John Wiley, New York, pp. 47-86.

18. Jiang, J. C. and Yamagami, T. (2004): "Three-dimensional slope stability analysis using an extended spencer method," *Soils and Foundations*, Vol. 44, No. 4, pp. 127-135.

19 Bowders, Jr., John J. and Lee, S. C. (1990): "Guide for selecting appropriate method to analyze the stability of slopes on reclaimed surface mines," Mining and Reclamation Conference and Exhibition, Charleston, West Virginia.

20. Kalatehjari, R., Ali, N., Kholghifard, M. and Hajihassani, M. (2014): "The effects of method of generating circular slip surfaces on determining the critical slip surface by particle swarm optimization," *Arabian Journal of Geosciences*, Vol. 7, No. 4, pp. 1529-1539.

21. Kumar, A. and Villuri, V. G. K. (2015): "Role of mining radar in mine slope stability monitoring at open cast mines," *Procedia Earth and Planetary Science*, Vol. 11, pp. 76-83.

22. Kumar, V. and Prakash, V. (2015): "A model study of slope stability in mines situated in South India," *Advances in applied Science Research*, Vol. 6, No. 8, pp. 82-92.

23. Lam, L. and Fredlund, D. G. (1993): "A general limit equilibrium model for three-dimensional slope stability analysis," *Canadian Geotechnical Journal*, Vol. 30, No. 6, pp. 905-919.

24. Lin, H. and Cao, P. (2014): "A dimensionless parameter determining slip surfaces in homogeneous slopes," *KSCE Journal of Civil Engineering*, Vol.18, No.2, pp.470-474.

25. Lin, H., Xiong, W. and Cao, P. (2013): "Stability of soil nailed slope using strength reduction method," *European journal of Environmental and Civil Engineering*, Vol. 17, No. 9, pp. 872-885.

26. Lin, H., Zhong, W., Xiong, W. and Tang, W. (2014): "Slope stability analysis using limit equilibrium method in nonlinear criterion," *The Scientific World Journal*, Volume 2014, pp. 1-7.

27. Lowe, J. and Karafiath, R. V. (1960): "Stability of earth dam upon drawdown," Proceedings of the first Pan American Conference on Soil Mechanics and Foundation Engineering, Maxico City pp. 537?52.

28. Mathews, C., Farook, Z., Arup and Helm, P. (2014): "Slope stability analysis – limit equilibrium or the finite element method?," *Ground Engineering*, pp. 22-28.

29. Miller, R. P., Douglass, P. M. and Robinson, R. A. (1979): "Surface mine spoil stability evaluation – Interior coal Province Vol. 1 and 2," – *USBM - OFR* 78 (1) & (2) -80.

30. Morgenstern, N. R. and Price, V. E. (1965): "The analysis of the stability of general slip surfaces," *Geotechnique*, Vol. 15, No. 1, pp. 77-93.

31. Nian, T. K., Huang, R. Q., Wan, S. S. and Chen, G. Q. (2012): "Three- dimensional strength-reduction finite element analysis of slopes: geometric effects," *Canadian Geotechnical Journal*, Vol. 49, No. 5, pp. 574-588.

32. Sarma, S. K. (1973): "Stabilityanalysis of embankment and slopes," *Geotechnique*, Vol. 23, No. 3, pp.423-433.

33. Sarma, S. K. (1979): "Stability analysis of embankments and slopes," *Journal of the Geotechnical Engineering Division, ASCE*, Vol. 105, No. GT-5, pp. 1511-1524.

34. Sen, P. (1994): "Computer aided design of openpit slopes," Thesis for the Doctor of Philosophy in Mining Engineering, Indian School of Mines, Dhanbad.

35. Sharma, S. and Lovell, C. W. (1983): "Strengths and weakness of slope stability analysis," Proceedings of the 34th Annual Highway Geology Symposium, Atlanta, Georgia, pp. 215-232.

36. Shen, J., Karakus, M. and Xu, C. (2013): "Chart-based slope stability assessment using the Generalized Hoek-Brown criterion," *International Journal of Rock Mechanics and Mining Sciences*, Vol.64, pp.210-219.

37. Spencer, E. (1967): "A method of analysis of the stability of embankments, assuming parallel interslice forces," *Geotechnique*, Vol. 17, pp. 11?26.

38. Ugai, K.and Leshchinsky, D. (1995): "Three-dimensional limit equilibrium and finiteelement analyses: acomparison of results," *Soils and Foundations*, Vol.35, No.4, pp.1-7.

39. Xiao, S., Li, K., Ding, X. and Liu, T. (2015): "Numerical computation of homogeneous slope stability," *Computational Intelligence and Neuroscience*, pp. 1-10.

40. Xu, Y., Chatterjee, J. and Amini, F. (2011): "A comparative slope stability analysis of new orleans levee subjected to hurricane loading," *EJGE*, Vol. 16, pp. 325-336.

41. Zhang, Y., Chen, G., Zheng, L., Li, Y. and Zhuang, X. (2013): "Effects of geometries on three-dimensional slope stability," *Canadian Geotechnical Journal*, Vol. 50, pp. 233-249.

42. Zheng, H., Tham, L. G. and Liu, D. (2006): "On two definitions of the factor of safety commonly used in the finite element slope stability analysis," *Computers and Geotechnics*, Vol.33, No. 3, pp. 188-195.

43. Zhou, X. P. and Cheng, H. (2013): "Analysis of stability of three-dimensional slopes using the rigorous limit equilibrium method," *Engineering Geology*, Vol.160, pp.21-33.

44. Zhou, Y., Tang, X., Zhu, X and Liu, E. (2010): "Slope slip band based on circular slip surface," *EJGE, Bund. B*, Vol. 14, pp.1-8.

45. Zhu, D. Y. (2008): "Investigation on the accuracy of the simplified Bishop method," In: Proceedingsof the 10th. Internal Symposium on Landslides and Engineered Slopes, China, pp. 1055-1057.