SHIHUI SUN
TIE YAN
ZHAOKAI HOU
JINYU FENG
and
GUOQING YU

Numerical simulation of wellbore gas-liquid phase transition based on Lattice Boltzmann method

Whenever gas kick occurs, gas flows in mixture with the flowing drilling fluid or migrate upwardly when drilling fluid is suspended. When wellbore temperature and pressure are higher than the critical temperature and pressure of natural gas, natural gas is in a supercritical state. In the process of gas migration along the wellbore, gas volume will gradually increase due to the decrease of wellbore temperature and pressure. At the critical point, phase behavior changes and gas volume increases very rapidly, which can bring great harm to well control; what is worse, it may lead to well blow out. Therefore, it is of great significance to analyse the effects of phase change characteristics of the supercritical fluid on well control safety. Due to the huge advantages and strong adaptability of Lattice Boltzmann method (LBM) to solve gas-liquid two-phase flow problem with complicated and changeable phase interface, wellbore gas-liquid phase transition based on inter-particle interaction of LBM method is researched in this paper. Simulation results show that different initial densities of mixed fluid have significant influence on gas and liquid phase distribution after phase transition. Continuous gas is formed in phase change position of a wellbore when initial density of mixed fluid is less than the critical density. And gas change in phase behaviour migrates along the wellbore as bubbles when initial density of mixed fluid is greater. Research results not only improve the understanding of gas-liquid phase transition mechanism, but also can provide some valuable attempts to promote the application of LBM method in wellbore gas-liquid two-phase flow.

Keywords: *Gas kick, gas-liquid two-phase flow, phase transition, LBM*

1.0 introduction

During drilling, when wellbore temperature and pressure are higher than the critical temperature and pressure of natural gas, natural gas is in a supercritical state. In the process of supercritical fluid migrate along the wellbore gas volume will gradually increase due to the decrease of wellbore temperature and pressure. At the critical point, phase behavior changes and gas volume increases very rapidly, which can bring great harm to well control; what is worse, it may lead to well blowout. Numerous researches related to gas-liquid phase transition have been carried out in the past several years.

Most studies on gas-liquid phase equilibrium are based on thermodynamic equilibrium [1-2]. It is believed that when the gas-liquid two-phase equilibrium is reached, the fugacity of each component of the system in each phase is equal. From the point of solubility, it is believed gas dissolution rate in the liquid phase continuously decreases and liquid phase evolution rate increases with time as gas solubility increases. And when gas dissolution rate is equal to liquid phase evolution rate, the concentration of each component in the phases are stable. That is phase equilibrium is achieved [3]. Henry's law elucidates gas-liquid equilibrium from the perspective of gas partial pressure [4-5]. It regards gas partial pressure equal to the partial pressure generated by solute when gas and liquid phase are in equilibrium. Researches on phase transition of gas-liquid two-phase flow in wellbore focus on gas solubility test in water, base oil, oil-based drilling fluid and synthetic-based drilling fluid by experimental method [6-9]. The influence of wellbore temperature, pressure, and composition of drilling fluid on gas solubility in different drilling fluid systems are analyzed [10-12]. Numerical simulation is an important method to study gas-liquid two-phase flow. The current numerical simulation methods for gas-liquid mass transfer problems mainly include Volume of Fluid (VOF) method [13-14]. Level Set Method (LSM) [15-18] and Front Tracking Method (FT) [19-20] are all based on the Navier-Stokes equation (N-S), irrespective of VOF, LSM or FT method has its own limitations. Interface reconstruction of

VOF method is complex, and VOF method is difficult to deal with complex topology changes such as gas bubbles coalescence and collapse. LSM method is difficult to ensure quality conservation. FT method is difficult to achieve, poor in quality conservation, and have certain difficulties in processing problems with complex topological structure changes. Moreover, VOF, LSM, and FT are all based on N-S equations. The pressure-Poisson equation needs to be solved in the calculation process. The pressure-Poisson equation is slow to solve, low in parallel efficiency, and unsuitable for large-scale parallel computing. Compared to the traditional numerical methods which are based on the N-S equation, LBM has peculiar advantages, such as: easy treatment of boundary conditions, high efficiency of computation, suitable for flow with large distortion and the nature of parallel computation [21]. In this paper, wellbore gas-liquid phase transition based on antiparticle interaction of LBM method is researched.

2.0 Gas-liquid phase transition model based on LBM

Lattice Boltzmann model for single-component gas-liquid two-phase flow can be divided into color model, pseudo-potential model and free energy model based on different inter-phase interaction modes [22]. Among them, the pseudo-potential models first proposed by Shan and Chen are most widely used because of their simplicity and stability [23-24]. Pseudo-potential model is used to study gas-liquid two-phase separation, droplets and bubbles integration in this paper. The evolution equation is the standard evolutionary equation of Lattice Bhatnagar-Gross-Krook (LBGK), and can be expressed by:

$$f_i\left(\boldsymbol{x}+\boldsymbol{c}_i\Delta t, t+\Delta t\right) - f_i\left(\boldsymbol{x},t\right) = \frac{1}{\tau}\left[f_i^{eq}\left(\boldsymbol{x},t\right) - f_i\left(\boldsymbol{x},t\right)\right] \quad \ldots (1)$$

f_i^{eq} is equilibrium function, the general form can be written by:

$$f_i^{eq} = \rho w_i\left[A + B\boldsymbol{c}_i\cdot\boldsymbol{u}^{eq} + C\left(\boldsymbol{c}_i\cdot\boldsymbol{u}^{eq}\right)^2 + D\left(\boldsymbol{u}^{eq}\right)^2\right] \quad \ldots (2)$$

Where, f_i is discrete velocity distribution function; c_i is discrete velocity; τ is relaxation time, which is related to fluid viscosity; u^{eq} is equilibrium velocity, A, B, C and D are constant, which can be determined by mass and energy conservation law; ρ is density, which can be obtained from:

$$\rho = \sum_{i=0}^{n} f_i^{eq} \quad \ldots (3)$$

Where, n is the total number of squares.

In order to achieve gas-liquid phase change and phase separation, inter-particle interaction force F is introduced. Pseudo-potential model describes the effects of particle interactions by changing equilibrium velocity. Correction equilibrium velocity can be written by:

$$u^{eq} = u + \Delta u \quad \ldots (4)$$

Interaction force correction velocity Δu and macro velocity u are calculated respectively by:

$$\boldsymbol{u} = \frac{\Sigma\boldsymbol{c}_i f_i}{\rho}$$

$$\Delta\boldsymbol{u} = \Delta t\frac{\boldsymbol{F}}{\rho} \quad \ldots (5)$$

Substituting u^{eq} into equilibrium distribution function, which means inter-particle interaction force is introduced to evolution equation, then obtain the following equation:

$$f_i^{eq} = \rho w_i\left[1 + \frac{\boldsymbol{c}_i\boldsymbol{u}^{eq}}{c_s^2} + \frac{\left(\boldsymbol{c}_i\boldsymbol{u}^{eq}\right)^2}{2c_s^4} - \frac{\left(\boldsymbol{u}^{eq}\right)^2}{2c_s^2}\right] \quad \ldots (6)$$

The lattice model of D2Q9 is adopted in this paper, which is shown in Fig.1. Discrete velocity c_i can be divided into three categories based on the rate and speed direction of D2Q9 model:

$$c_i = \begin{cases} c(0,0) & i = 0 \\ c(\cos\frac{i-1}{2}\pi, \sin\frac{i-1}{2}\pi) & i = 1,2,3,4 \\ \sqrt{2}c(\cos\frac{i-5}{2}\pi+\frac{\pi}{4}, \sin\frac{i-5}{2}\pi+\frac{\pi}{4}) & i = 5,6,7,8 \end{cases} \quad \ldots (7)$$

Where, $c = \Delta x/\Delta t$, Δx, Äx is lattice spacing, Ät is time step.

Discrete velocity f_i and the corresponding weight factor w_i are shown in Table 1. c_s is lattice velocity, can be calculated as follows:

$$c_s = c/\sqrt{3} \quad \ldots (8)$$

The force $F(x)$ which fluid particles are subjected to is given by:

$$\boldsymbol{F}\left(x\right) = -\varphi(x)\sum_i G\varphi\left(\boldsymbol{x}+\boldsymbol{c}_i\Delta t\right)\boldsymbol{c}_i \ \square \quad \ldots (9)$$

Where, $\varphi(x)$ is the effective density, and parameter G is inter-particle interaction intensity in each phase.

Pseudo-potential model assumes that there is a non-local interaction between fluid particles the corresponding density function has many forms. Only the effective density function, take a specific function, it can make consistent with thermodynamics theory. The effective density function is given by Shan-Chen:

$$\varphi(\rho) = \rho_0\left(1 - \exp\left(-\frac{\rho}{\rho_0}\right)\right) \quad \ldots (10)$$

State equation of Shan-chen pseudo-potential model is written by:

$$P = \rho RT + \frac{GRT}{2}\left(\varphi(\rho)\right)^2 \quad \ldots (11)$$

The effective density function $\varphi(\rho)$ is obtained by:

$$\frac{1}{2}\varphi^2\left(\rho\right) = \frac{\rho\frac{\partial g(\rho,T)}{\partial\rho} - g(\rho,T)}{RT} - \rho \quad \ldots (12)$$

$g(\rho,T)$ is free energy density function of thermodynamic, which can be expressed as [25]:

$$g(\rho, T) = \rho RT \ln\left(\frac{\rho}{1-0.3\rho}\right) - 1.018226\rho^2 \qquad \dots (13)$$
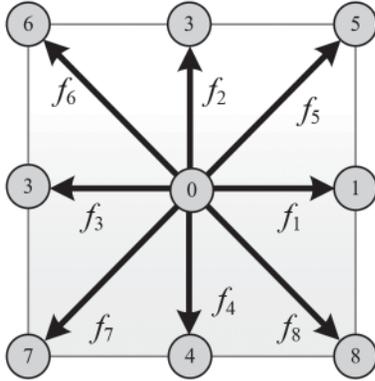
Fig.1: D2Q9 Lattice model

TABLE 1: PARAMETERS OF D2Q9 MODEL

Distribution function	Discrete velocity	Weight factor
f_0	(0,0)	4/9
$f_1 \sim f_4$	(1,0), (0,1), (-1,0), (0,-1)	1/9
$f_5 \sim f_8$	(1,-1), (-1,1), (-1,-1), (1,-1)	1/36

3.0 Gas-liquid phase transition simulation

The lattices of numerical calculation area are divided into 200×200, equilibrium state function adopts D2Q9 model, and relaxation time $\tau = 1$. Boundary conditions of upper and lower, left and right boundary are periodic boundary conditions [26].

The ambient temperature is 0.9, the critical temperature is 1.0, the critical density is 1.0, and initial density is 0.9. Add a random disturbance r_i in the range of –0.02 to 0.02 for system density to separate gas liquid phase. And gas-liquid phase change process is simulated. Fig.2 (a)-(f) shows the evolution process of gas-liquid phase separation at 50, 100, 150, 350, 550, and 950 time steps, respectively. As time evolves, gas-liquid two-phase system appears gas-liquid two-phase separation and phase transition. Where, yellow represents liquid phase, blue represents gas phase, liquid phase density is 1.523 and gas phase density is 0.45.There is a thin layer of mixed gas-liquid two-phase near the gas-liquid interface.

As can be seen from Fig.2, gas-liquid two-phase is separated with evolution time increases. In mixed fluid, liquid phase is gradually divided as gas phase increases, and the continuity is broken. Droplets gradually condense to form in gas phase. In the course of evolution, some small droplets undergo phase change. The small droplets disappear and become gas phase. As the phase transition go further, liquid phase which not have phase change has formed droplets. And these droplets coalesce into large droplets under the effect of surface tension.

The initial density is less than the critical density of the system in the simulations above, which show that gas accounts for a greater proportion relative to liquid phase. Therefore, phase separation is dominated by gas phase, liquid phase is split and form droplets under the effect of surface tension.

The initial density value is set to 1.1, other conditions are the same as above, phase transition process is simulated. The critical fluid density distribution under evolutionary time of 50, 100, 150, 350, 550, 950 time steps are shown in Fig.3 (a)-(f). Where, yellow represents liquid phase, blue represents gas phase, liquid phase density is 1.523 and gas phase density is 0.45. There is a thin layer of mixed gas-liquid two-phase near the gas-liquid interface.

As can be seen from Fig.3, gas-liquid two-phase is separated with evolution time increases. Gas phase is divided, gas bubbles gradually condense to form in liquid phase. In the course of evolution, some small bubbles undergo phase change. The small bubbles disappear and gradually become liquid phase. The small bubbles coalesce into larger bubbles under the effect of surface tension.

In the simulation of phase transition with 1.1 initial density the initial density is greater than the critical density, the proportion of liquid phase is greater than gas phase in critical fluid before phase transition. Therefore, when gas-liquid separation occurs, liquid phase are dominant, gas phase is divided into discontinuous fluid and forms gas bubbles under the effect of surface tension.

Through the above phase transition simulation, it is found that the initial density has a decisive effect on bubbles and droplets formation in gas-liquid two phase system. For wellbore annulus fluid mixed with supercritical fluid, when the initial density of mixed fluid is less than the critical density, which shows that gas accounts for a greater proportion relative to liquid phase. Once the gas-liquid separation occurs, gas phase occupies a large area. Due to the effect of surface tension, liquid phase forms droplets, and gas phase forms a continuous gas in phase transition position of the wellbore; when the initial density of the mixed fluid is greater, which means the proportion of liquid phase is relatively greater. Once the gas-liquid separation occurs, liquid phase occupies a large area. Due to the effect of surface tension, gas changes in phase behavior migrate along the wellbore in the form of bubbles. This reflects there is difference in gas-liquid distribution after phase transition, which has an important influence on flow pattern identification and wellbore pressure variation.

4.0 Validation of simulation results

Gas-liquid two-phase separation only occurs when the ambient temperature is below the critical temperature. One of the key to simulate gas-liquid two-phase flow with LBM is to perform Maxwell integration using P-V line of the corresponding state equations [27-28]. That is, given a related parameter T (less than the critical temperature T_c), determine
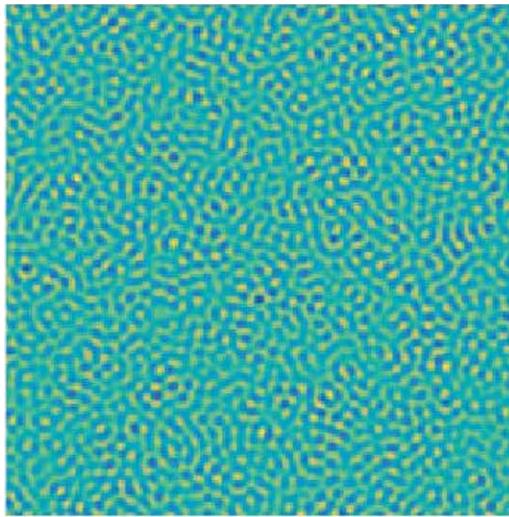
(a) evolution time steps is 50
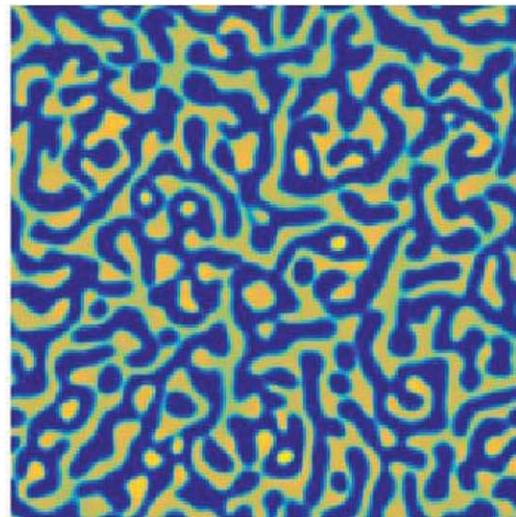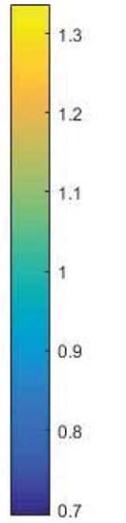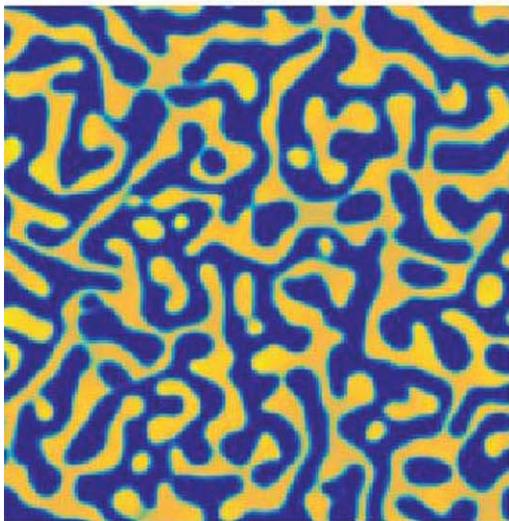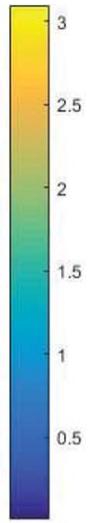
(b) evolution time steps is 100

(c) evolution time steps is 150
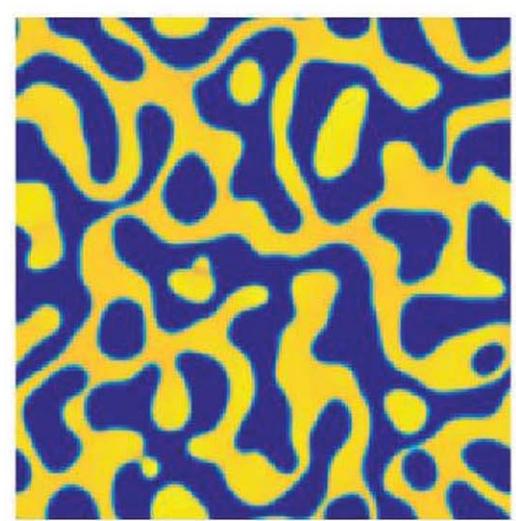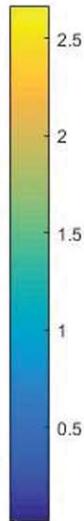
(d) evolution time steps is 350
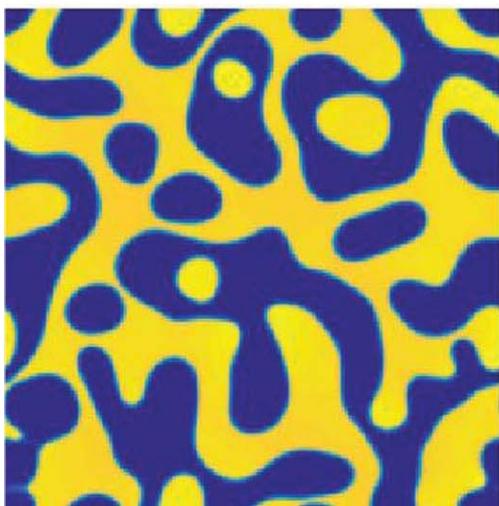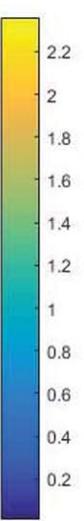
(e) evolution time steps is 550
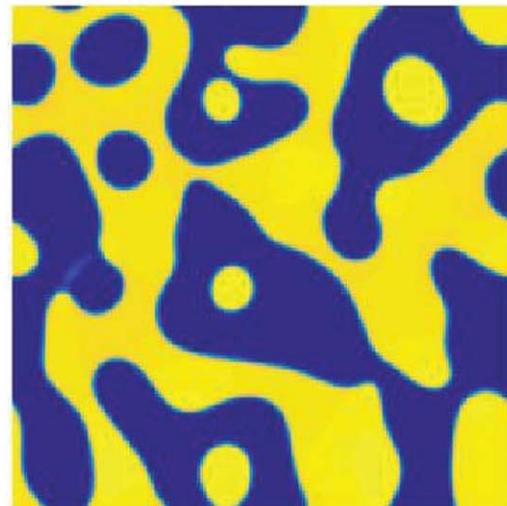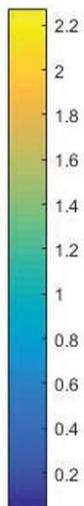
(f) evolution time steps is 950
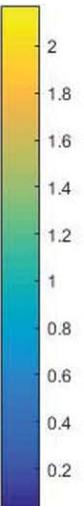
Fig.2: Evolution diagram of droplets during gas-liquid phase transition
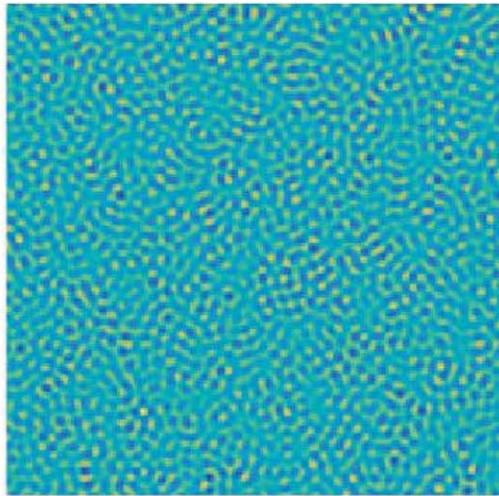
(a) evolution time steps is 50
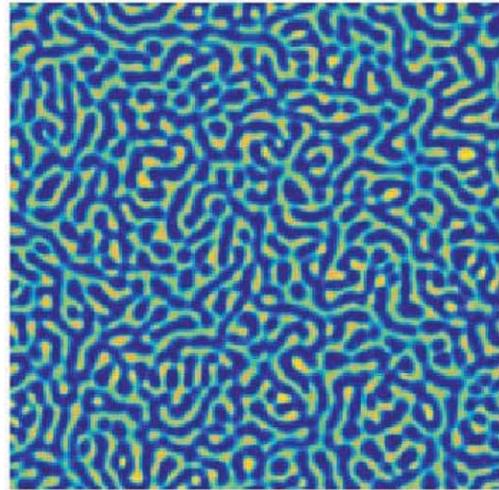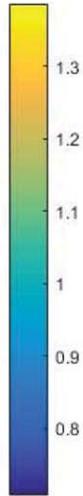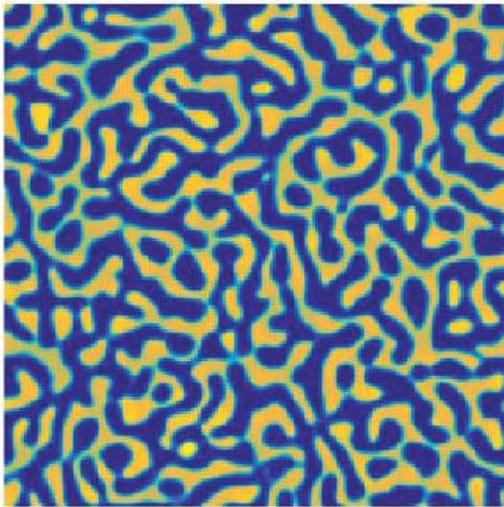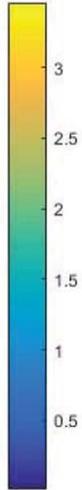
(b) evolution time steps is 100

(c) evolution time steps is 150
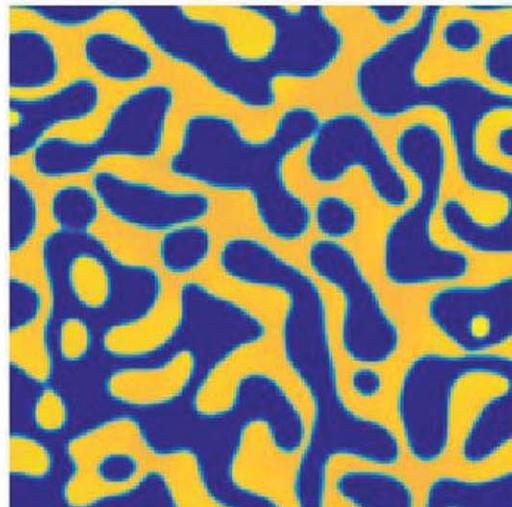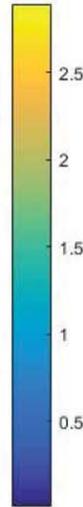
(d) evolution time steps is 350
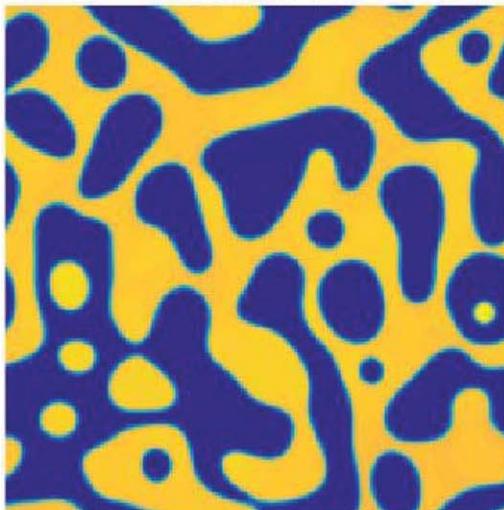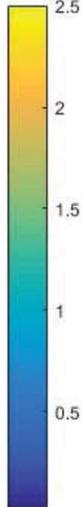
(e) evolution time steps is 550
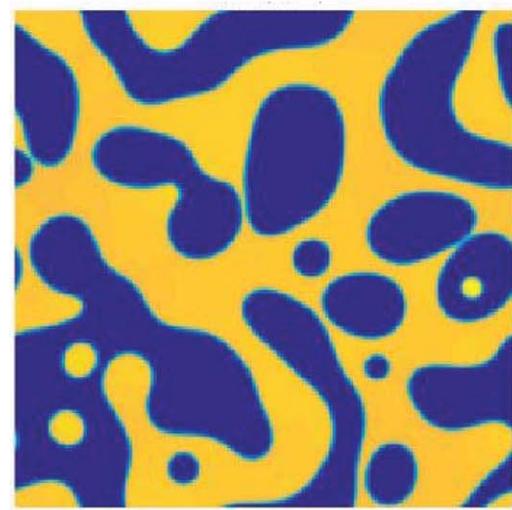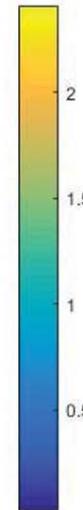
(f) evolution time steps is 950
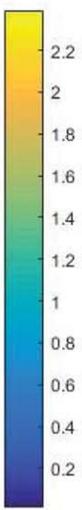
Fig.3: Evolution diagram of gas bubble during gas-liquid transition

pressure and volume theoretical integral values of gas and liquid phases. Maxwell's integral equation can be described as follows:

$$\int_{V_g}^{V_1} PdV = P(V_1 - V_g) \tag{14}$$

Where, V_1 and V_g are liquid and gas volume, respectively. P is the pressure of gas-liquid two-phase system.

The simulated density of gas and liquid phases using LBM can be compared to theoretical integral values obtained from Maxwell's equation. As is shown in Fig.4, the solid line represents theoretically calculated values obtained by Maxwell's integral equation. The discrete solid point represents simulation values by lattice Boltzmann method. As can be seen from Fig.4, theoretical calculation values of gas-liquid two-phase density is in good agreement with numerical simulation results. The feasibility and accuracy of the simulation of gas-liquid phase change using LBM are verified.
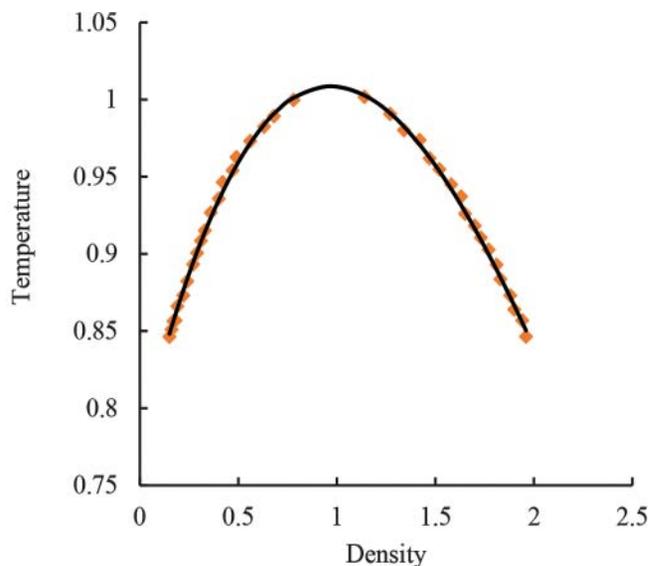
Fig.4: Density distribution lines of gas-liquid two-phase at different ambient temperatures

5.0 Conclusions

• Based on LBM method, single-component pseudo-potential model is used to study gas-liquid phase transition. The simulation results show that different initial densities have a significant effect on gas-liquid distribution after phase transition.

• When the initial density of mixed fluid is less than the critical density, liquid phase forms droplets, and gas phase forms a continuous gas in phase transition position of the wellbore under the effect of surface tension; when the initial density of the mixed fluid is greater, gas changes in phase behavior migrates along the wellbore in the form of bubbles. The difference in gas-liquid distribution after phase transition has an important influence on flow pattern identification and wellbore pressure variation.

• The application of LBM to simulate wellbore gas-liquid phase transition behavior provides a new method and idea for wellbore gas-liquid two-phase flow.

7. References

[1] Assael M. J.,TruslerJ. P. M.,Tsolakis T. F. (196): An introduction to their prediction thermophysical properties of fluids [M]. U. K. Imperial College Press.

[2] Prausnitz J.M., Lichtenthaler R. N., Azevedo E.G. (1999): Molecular thermodynamics of fluid-phase equilibria 3rd [M]. Nopardazan.

[3] Tilman Knorr, Eberhard Aust, Karl-Heinz Jacob (2004): Calculation of vapor-liquid equilibrium data of binary mixtures using vapor pressure data [J]. *Chemical Engineering & Technology*, 2010, 33(12): 2089-2094.

[4] Gossett J. M. (198): Measurement of Henry's law constants for C_1 and C_2 chlorinated hydrocarbons [J]. E*nvironmental Science & Technology,* 21(2): 202-208.

[5] Meylan W. M, Howard P. H. (1991): Bond contribution method for estimating Henry's law constants [J]. *Environmental toxicology and chemistry,* 10(10): 1283-1293.

[6] Duan Z, Mao S. (2006): A thermodynamic model for calculating methane solubility, density and gas phase composition of methane-bearing aqueous fluids from 273 to 523 K and from 1 to 2000 bar[C]. *Geochimica Et Cosmochimica Acta*, 70(13): 3369-3386.

[7] O'Brien, T. B. ('1981): Handling gas in an oil mud takes special precautions [J]. *World Oil.* 192(1): 83-86.

[8] Silva C T, Mariolani J R L, Bonet E J, et al. (2004): Gas solubility in synthetic fluids: A well control issue[C]. Society of Petroleum Engineers.

[9] Berthezene N, Hemptinne J C D, Audibert A, et al. (1999): Methane solubility in synthetic oil-based drilling muds[J]. *Journal of Petroleum Science & Engineering*, 23(2):71-81.

[10] FU Jianhong, XU Chao, ZHANG Zhi.(2012): Gas solubility calculation in oil-based drilling fluid during deepwater drilling[J]. Drilling & Production Technology, 35(4):85-87.

[11] Monteiro E N, Ribeiro P R, Lomba R F T. (2010): Study of the PVT properties of gas-synthetic-drilling-fluid mixtures applied to well control [J]. SPE Drilling & Completion, 25(1): 45-52.