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Vibration and buckling behaviour of delaminated composite plates subjected to localised edge loads

Laminated composite plates are usually a part of complicated structures. Most of the cases these components may undergo delamination due to the transferring of various complicated loads. In this investigation the effect of extent of delamination on the vibration and buckling characteristics of laminated composite plate is studied by using FE method. Towards this, an eight-noded plate element has been used, by incorporating the effect of shear deformation. In this work two types of boundary conditions are used for calculating pre buckling in-plane stresses and buckling loads respectively. Matlab code has been developed to carry out various parameters like extent of delamination, ply orientation, plate thickness, width and position of load and boundary conditions.

Keywords: Buckling, vibration, composite plates, delamination, edge load

1.0 Introduction

The sectors like aerospace, automobile, marine, civil structures etc., uses composites extensively due to its high-stiffness and strength to weight ratios, low thermal expansion and other excellent properties compared to general structural materials. The usage of such materials subjected to in-plane edge loads during its service life resulting loss of stability at lower stress levels, especially delaminated composite laminates. Delamination in general is nothing but the separation of one layer from other it may be mainly due to the poor fabrication techniques or by external damage. (Reddy, 1984) and (Reddy and Phan, 1985) have developed a mathematical model and analysed the buckling and vibration attributes of laminated composite panels by considering numbers of layers, ply orientation, plate aspect ratio, plate thickness ratio and boundary conditions. Noor, 1975 validated the classical and shear deformable theories for large number of multi-layered composite plates. A method to study free vibration of rectangular mindline plate having mixed boundary condition was proposed by Sakiyama and Matsuda, 1987. Ravi Kumar et al., 2003a considered tensile non-uniform in-plane edge loading to analyse the vibration and tension buckling behaviour of composite curved panels. In his study he considered various loading conditions like patch loads, point load and distributive loads. Earlier (Prabhakara and Datta, 1993), (Prabhakara and Datta, 1996) and (Prabhakara and Datta, 1997) studied the effects of internal cracks, cut outs on buckling and vibration of isotropic plates under in-plane compressive and tensile loading. Simitses et al., 1985 considered an axially loaded one dimensional homogeneous laminated plate having a delamination along the entire width of the plate was considered. The buckling loads were computed for various positions of delamination along the plate thickness. Finn and Springer, 1993 developed a model using finite element method to calculate the shape, size and location of delamination occurred in composite plate due to static transverse or impact loading. Venkatesh B K et al., 2020 compared the results of experimental investigation of mechanical properties of hybrid composites with simulation values. Oh et al., 2005 used higher-order zig zag theory to estimate the effects of multiple delamination in different layers on dynamic behaviour of the composite plates. Rajanna et al., 2016 considered different types of partial loads on the edges of the stiffened composite plate to evaluate its effect on buckling and vibration characteristics. The bending, vibration and transient responses of delaminated composite plates for different location, sizes of delamination with different plate aspect ratios and ratios of thickness were analysed by (Sahoo et al., 2016).

From the brief overview of the above literatures, it can be inferred that abundant literature are available on vibration and buckling behaviour of composite laminates under UDL, without considering the effect of delamination. By considering the effect of delamination under uniformly distributed in plane load is very much scanty in the literature. To the best of author's knowledge, no literature is available by considering the effect of various positioned delamination under the action of localized edge loads.

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2.0 Theory and mathematical formulation

The geometrical configuration of a composite plate of dimensions $a \times b$ with a square delamination of size $b_1 \times b_1$ along x-y direction is shown in Fig.1. The mid-plane displacements and mid-plane rotations are represented as u, v, w, q_y and q_x along x, y and z-axis respectively.

2.1 Strain-Displacement Relationship

Throughout proposed study for the formulation of the problem a Lagrange's strain displacement equations are used. The strain energies in a composite plate are the combination of linear and non-linear energies.

The linear strain relation expressions by considering shear deformation are written as:

$$\left.\begin{aligned} \varepsilon_{lx} &= \frac{\partial u}{\partial x} + zk_x \\ \varepsilon_{ly} &= \frac{\partial v}{\partial y} + zk_y \\ \gamma_{lxy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + zk_{xy} \\ \gamma_{txz} &= \frac{\partial w}{\partial x} + \theta_x \\ \gamma_{tyz} &= \frac{\partial w}{\partial x} + \theta_y \end{aligned}\right\} \quad \text{... (1)}$$

In the above equations,

- ε_{lx}, ε_{ly} and γ_{lxy} represents linear in-plane normal and shear strains;
- γ_{txz} and γ_{tyz} represents the transverse shear strains;
- The distance of any layer from the middle plane is denoted by z;
- The curvature of the plate is denoted by k.

The non-linear strain displacement relations of the plate are as follows (Bathe, 1996).

$$\left.\begin{aligned} \varepsilon_{nlx} &= \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta_x}{\partial x}\right)^2 + \left(\frac{\partial \theta_y}{\partial x}\right)^2\right] \\ \varepsilon_{nly} &= \frac{1}{2}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 + \frac{1}{2}z^2\left[\left(\frac{\partial \theta_x}{\partial y}\right)^2 + \left(\frac{\partial \theta_y}{\partial y}\right)^2\right] \\ \gamma_{nlxy} &= \frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\frac{\partial v}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} + z^2\left[\frac{\partial \theta_x}{\partial x}\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x}\frac{\partial \theta_y}{\partial y}\right] \end{aligned}\right\} \quad \text{...2}$$

Where, ε_{nlx}, ε_{nly} and γ_{nlxy} are non-linear normal and shear strains.

The stress-strain relation for a lamina with reference to the fibre direction shown in Fig.1 is given by

$$\{\sigma\} = \left[\overline{Q}_{ij}\right]_k \{\varepsilon_l\} \quad \text{... (3)}$$

Using Eq.1 and Eq.3, the constitutive relation for the laminated plate can be given by

$$\begin{Bmatrix} \{N_i^P\} \\ \{M_i^P\} \\ \{Q_i^P\} \end{Bmatrix} = \begin{bmatrix} [A_{ij}^P] & [B_{ij}^P] & 0 \\ [B_{ij}^P] & [D_{ij}^P] & 0 \\ 0 & 0 & [S_{ij}^P] \end{bmatrix}_k \begin{Bmatrix} \{\varepsilon_j^P\} \\ \{\varepsilon_j^P\} \\ \{\varepsilon_j^P\} \end{Bmatrix} \quad \text{... (4)}$$

Abbreviated as $\{N^P\} = [D^P]\{\varepsilon^P\}$

Where, $\{N_i^P\} = [N_x^P \quad N_y^P \quad N_{xy}^P]^T$ are stress resultants, $\{M_i^P\} = [M_x^P \quad M_y^P \quad M_{xy}^P]^T$ are moment resultants, $\{Q_i^P\} = [Q_{xz}^P \quad Q_{yz}^P]^T$ are transverse shear stress resultants; similarly, $\{\varepsilon_i^P\}, \{k_i^P\}$ and $\{\gamma_i^P\}$ are mid-plane strains, mid-plane curvatures, and transverse shear strains respectively. $[A_{ij}^P], [B_{ij}^P]$ and $[D_{ij}^P]$ are extension-extension, extension-bending and bending-bending components of a stiffness matrix, these can be expressed as

$$\left(A_{ij}^P, B_{ij}^P, D_{ij}^P\right) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \left(\overline{Q}_{ij}\right)_k \left(1, z, z^2\right) dz, \quad {\scriptstyle i,j=1,2,6} \quad \text{... (5)}$$

and $$S_{ij}^P = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \alpha \left(\overline{Q}_{ij}\right)_k dz, \quad {\scriptstyle i,\, j=4,5} \quad \text{... (6)}$$

In the above Eq.5 and Eq.6 n is the number of layers, α is the shear correction factor, which is taken as 5/6 (Kolli and Chandrashekhara, 1996), (Reddy, 2003) and $\left[\overline{Q}_{ij}\right]_k$ is the stiffness matrix of the lamina at k^{th} layer with respect to the plate axis (Reddy, 2003).

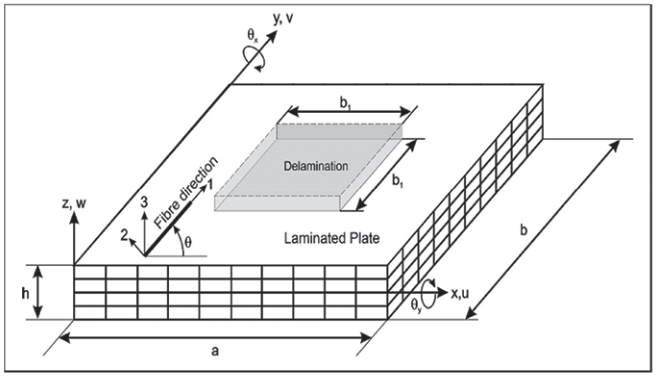

Fig.1: Geometrical configuration of delaminated composite plate

2.2 Finite Element Method

In the proposed investigation an eight noded iso-parametric plate element is considered. The element is assumed to have five degrees of freedom i.e. u, v, w, θ_y and θ_x respectively. The node numbering for the element is as represented in the Fig.2.

The various matrices of the plate element required to perform analysis are as follows:

$$[K_e] = \int_{-1}^{1}\int [B]^T [D][B] \,|\, J \,|\, d\xi \, d\eta \quad \text{... (7)}$$

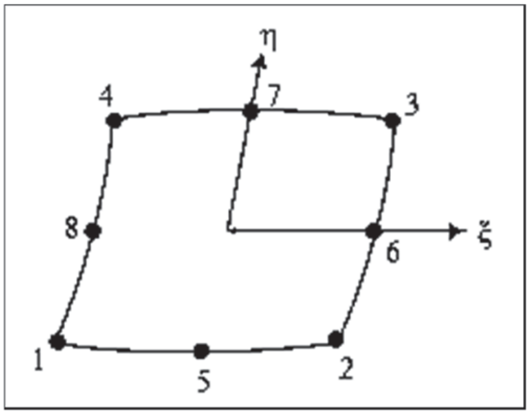

Fig.2: Eight noded quadrilateral plate element

$$[M] = \int_{-1}^{1}\int_{-1}^{1} [N]^T [\rho][N] \,|J|\, d\xi \, d\eta \quad \text{... (8)}$$

$$[K_g] = \int_{-1}^{1}\int_{-1}^{1} [G]^T [S][G] \,|J|\, d\xi \, d\eta \quad \text{... (9)}$$

The Eqs.7 to 9 represent element elastic stiffness, element mass and element geometric stiffness matrices due to applied in-plane loads.

2.3 Governing Equations

The distribution of stress within an element is highly non-uniform in nature for a given boundary and loading conditions. Therefore, dynamic approach has been used to calculate buckling loads. The generalized differential equation of motion for solving buckling problems by using dynamic approach can be written by using extended Hamilton principle as follows,

$$[M]\{\ddot{q}\} + \left([K] - P_{cr}[K_G]\right)\{q\} = 0 \quad \text{... (10)}$$

The governing equation to solve vibration problem can be derived by substituting, P=0 in Eq.10 as,

$$-\omega^2 [M]\{q\} - [K_e]\{q\} = 0 \quad \text{... (11)}$$

The governing equation to solve buckling problem in static approach can be obtained by substituting, $\{\ddot{q}\}$ in Eq. (10) as,

$$[K_e]\{q\} - P_{cr}[K_g]\{q\} = 0 \quad (12)$$

The applied edge load is denoted as P and critical buckling load is represented as P_{cr}.

3.0 Results and discussions

It is evident from Eqs.11 and 12 that the buckling and vibration problems can be solved by extracting the Eigen values and Eigen vectors. In the present study an independent code was developed to solve the Eigen value problems using Matlab. With the use of shear deformable theories, the problem is valid for both thin and thick plates. Unless otherwise specified, the material properties considered in this study are as follows: E_{11}/E_{22} = 40.0, υ_{12} = 0.23, G_{12}/E_{22} = G_{12}/E_{22} = 0.60, G_{23} = 0.50.

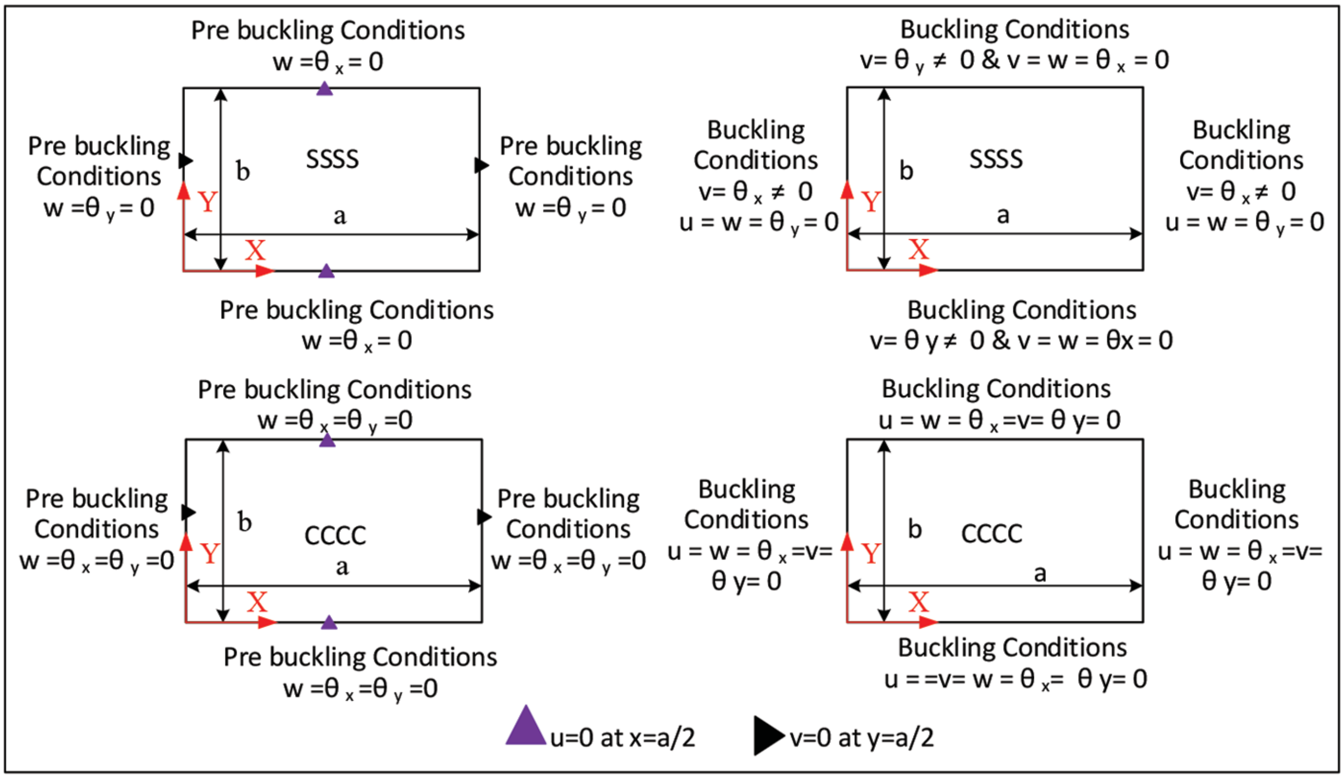

Fig.3: Boundary conditions used in the present work

3.1 Boundary Conditions, Material Properties

The notations S and C are indicated as simply support and clamped conditions respectively in the string of any support conditions. The displacement based boundary conditions considered in this work is shown in Fig.3.

The natural frequencies and critical buckling loads are presented in non-dimensional form as follows (Kumar et al., 2003b)

Non-dimensional frequency, $\bar{\omega} = \omega b^2 \sqrt{\left(\frac{\rho}{E_{22} t^2}\right)}$

Non-dimensional critical load, $\lambda_{cr} = \frac{P_{cr} b^2}{E_{22} t^3}$

3.2 Comparative studies

It is very important to validate the formulation in order to ensure the correctness of different matrices participating in the vibration and buckling problems. The validation of all these matrices is elaborated in the following sub sections.

3.2.1 Vibration analysis of composite plate

The numerical results of non-dimensional fundamental vibration frequencies of square plate with cross-ply and angle-ply layup sequences are studied in section and numerical results are shown in the following Table 1 along with the results available in the literature. The results shown in Table 1 is making good agreement with the literatures available which proves the correctness of consistent mass matrix.

Table 1: Comparative study of non-dimensional fundamental frequencies

b/h	(0/90)		(0/90)s	
	Present	(Reddy & Phan, 1985)	Present	(Reddy & Phan, 1985)
5	8.834	8.757	10.854	10.820
10	10.473	10.355	15.143	15.083
20	11.078	10.941	17.660	17.583
25	11.159	11.020	18.071	17.991
50	11.271	11.127	18.675	18.590
100	11.299	11.155	18.837	18.751

3.2.2 Buckling analysis of composite plate

The results of non-dimensional buckling loads for composite plates subjected to concentrated load and uniformly distributed load with simply supported boundary conditions are compared to the available literatures.

Table 2 shows non-dimensional buckling load for a simply supported 2, 3 and 4 layered cross ply composite plates for different thickness ratios. The results are matching with literatures mentioned in the table. From the Tables 1 and 2 it is evident that the results are in line with the literature and same formulation can be used to study on the further intended parameters. From the above comparisons the geometric stiffness matrix and also the code written to calculate the buckling load are validated.

3.3 Effects of Delamination on Buckling and Vibration characteristics

3.3.1 Effects of delamination on buckling characteristics

In this section the buckling characteristics of a plate under in-plane load will be studied for different extent of delamination, layer of delamination by varying aspect ratio, thickness ratio, ply orientation, boundary condition under point loading, patch loading and distributed loading conditions. The loading conditions considered in the study is shown in Fig.4(a), (b) and (c).

Fig 4 (a) shows the in-plane point loading condition, the plate is loaded under in-plane point load at one end. The position of load (c/b ratio) is given by e which varies from 0.0 to 1.0. Fig.4(b) depicts the patch load applied similar to the point load but, the ratio of load (e) changes from 0.1 to 1.0; 0.1 is a patch load increase and becomes UDL when e becomes 1.0. Fig.4(c) shows the distributive load which is applied at the centre of the plate and gradually increases from the centre symmetrically on either side along the edge and become UDL. In all the cases an equal and opposite load is applied on other edge of the plate.

Before presenting the results, the authors would like to make some generalized assumptions which will be applied in entire paper unless specified.

- The plate is having an aspect ratio (α) of 1.0;

Table 2: Non-dimensional buckling load for laminated composite panel for simply supported for different thickness ratios

b/h	(0/90)		(0/90/0)		(0/90/90/0)	
	Present	(Reddy & Phan, 1985) FSDPT	Present	(Reddy & Phan, 1985) FSDPT	Present	(Reddy & Phan, 1985) FSDPT
5	7.901	8.142	10.720	10.525	11.407	11.533
10	11.105	11.099	22.071	21.643	23.212	23.270
20	12.423	12.208	31.055	30.664	31.565	31.432
25	12.605	12.356	32.696	32.515	33.052	32.872
50	12.858	12.559	35.194	34.936	35.295	35.037
100	12.924	12.611	35.883	35.589	35.909	35.629

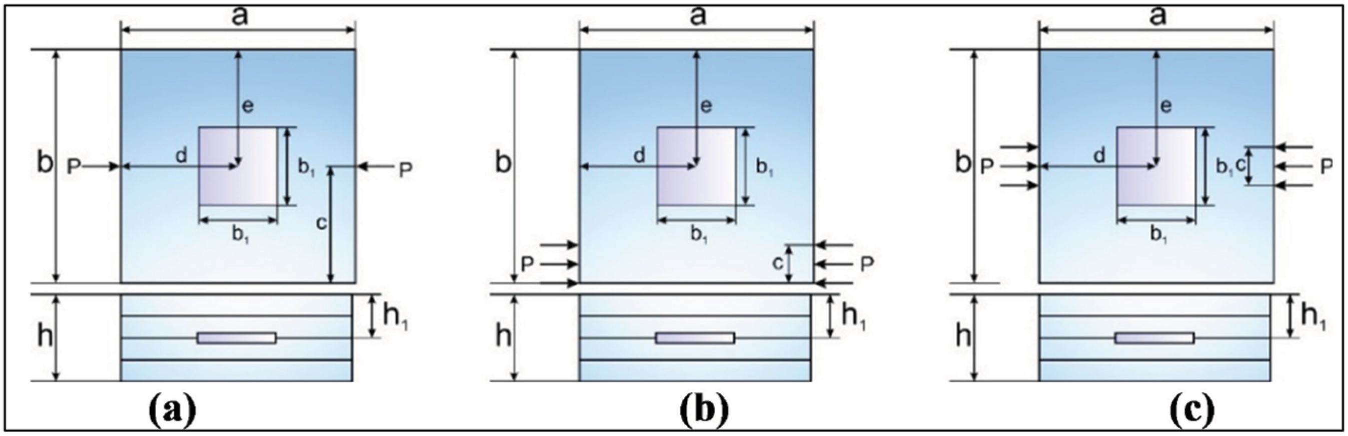

Fig.4: Delaminated composite plate with in-plane loading conditions:
(a) Point Load (b) Patch load and (c) Distributed load

- The plate is having simply supported (S-S-S-S) boundary condition;
- The plate is having an eight layer with symmetric cross ply orientation $(0/90)_{2s}$;
- Delamination is at ζ=0.5 and η=0.5 in *x-y* plane.
- Extent of delamination is taken as ζ=0.4;

3.3.2 Study of buckling behaviour on a delaminated composite plate under in-plane point load

The results are shown for the values of a varying from 0.5 to 3.0 and ε moves from 0 to 1.0 in the Fig.5. From the figure it can be noticed that as α increases the critical buckling load decreases i.e. plate become less stable. When an in-plane point load position moves from 0 i.e. b=0; it can be observed that critical buckling load gradually reduces till the load approaches the centre of the plate along the its loading edges and increases further showing exact symmetry. For any given α and ζ the movement of point load towards the centre reduces the buckling load.

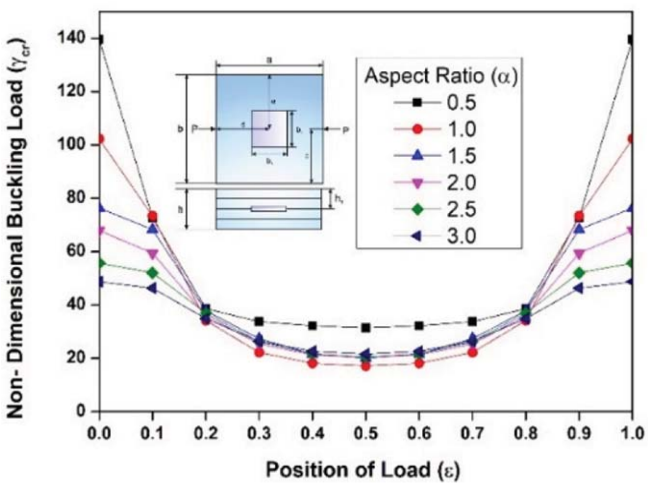

Fig.5: Variation of non-dimensional buckling load under different position of in-plane point load with different aspect ratios

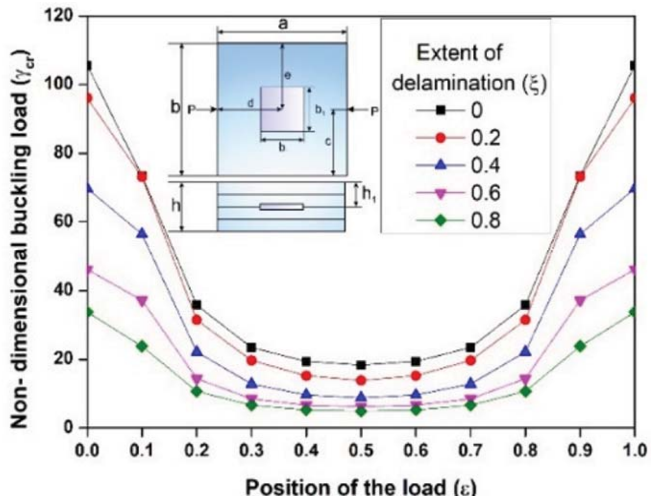

Fig.6: Variation of non-dimensional buckling load under different position of in-plane point load with different extent of delamination

The Fig.6 shows the effect of ξ on composite plate under in-plane point loading conditions. The plate was examined for ξ = 0, 0.2, 0.4, 0.6 and 0.8; with an in-plane moving load. For a plate having ξ = 0.8 and ε = 0.5 will reduce the buckling load by 27% when compared to a plate without delamination (ξ = 0.0) which is very critical.

A composite plate with cross ply, angle ply and quasi-isotropic ply orientations were considered to study the effects on buckling load and the results are as shown in the Fig 7. From the figure it can be seen that for any given condition the cross ply have greater stability when the in-plane load is applied at the corners and angle ply has the least stability. As the in-plane load moves towards the centre the plates behaves indifferent angle ply showing highest stability and cross ply the least. The quasi isotropic ply orientation shows less stability than cross ply and more stability than angle ply when the loads are at the corners and vice versa for central in-plane loads.

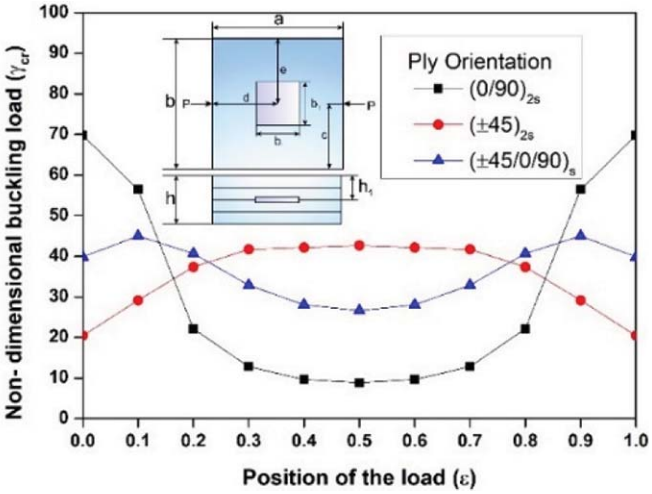

Fig.7: Effects of ply orientation on the non-dimensional buckling load under in-plane point load

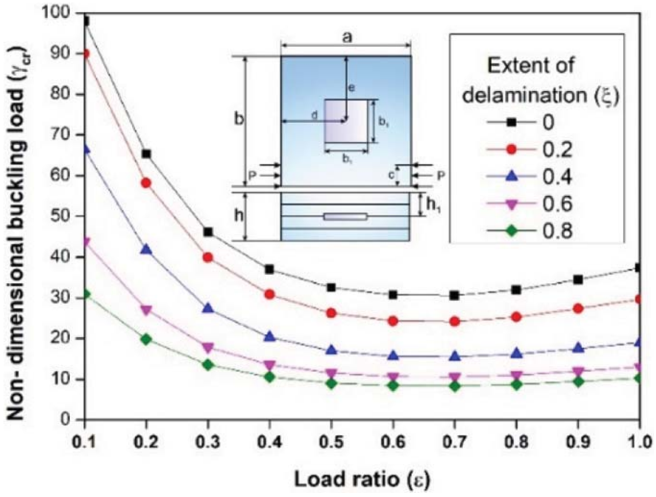

Fig.9: Variation of non-dimensional buckling load under in-plane patch load with different extent of delamination

3.3.3 Study of buckling behaviour on a delaminated composite plate under in-plane patch load

The buckling characteristics of delaminated composite plates under a patch load which is extending gradually over the entire span of the plate from one end for different values of α is shown in Fig.8. It is observed that as the value of α increases the plate tends to become unstable. It can also be seen that when the patch load is in the lower extreme position the buckling load will be maximum for any particular case of α, as the patch load increases the buckling load reduces however, for ε greater than 0.6 the plate become slightly more stable.

Fig.9 shows the buckling behaviour under patch load for different ξ. From the figure it can be seen that as load extends from one end to other the stability of the plate decreases till it extends 60% of the plate edge and later the slight increase in the stability.

As the delamination size in the plate increases the plate becomes less stable it is observed from the Fig. 9 a plate with ξ=0.8 shows reduction of buckling load by 28% in comparison with a plate without delamination.

The plate was examined under three ply orientations i.e. angle ply, cross ply and quasi-isotropic plate loaded under patch load and the results are presented in Fig.10. It can be seen that for a lower load ratio the plate shows higher stability for the cross-ply and quasi-isotropic plate orientation, for an angle ply plate orientation it shows low stability. Venkatesha B K et al. 2020 investigated the influence of stacking sequence of multi layered of hybrid composites and [0°/90°] lay-up showed better result. As the load ratio increases the angle ply gains stability and other plates becomes less stable up to the load ratio is 60% of the entire load. however, it is observed that the plate cross ply orientation shows larger deviation in the buckling loads for the variation in the load ratio.

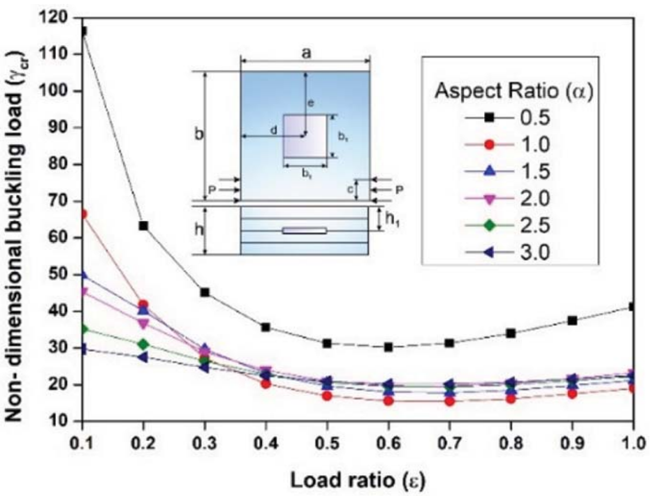

Fig.8: Effects of aspect ratio on the non-dimensional buckling load under in-plane patch load

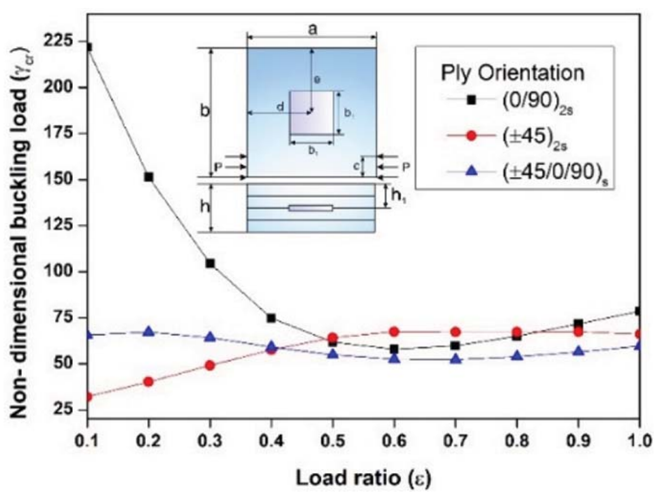

Fig.10: Effects of ply orientation on the non-dimensional buckling load under in-plane patch load

3.3.4 Study of buckling behaviour on a delaminated composite plate under in-plane distributed load.

In this section the plate is examined under distributive load by varying various parameters and obtained results are discussed as follows. A composite plate with a cross ply orientation is considered to know the effects of ξ on buckling load and the results are shown in Fig.11.

The delamination was assumed on the mid-plane of the plate. It can be seen from the figure that as ε increases over the width of the plate, the stability of the plate will improve. However, the presence of delamination reduces the stability of the plate and by increasing ξ the plate stability will be reduced further.

3.3.5 Effects of delamination on vibrational characteristics

The study has been done to understand the effect of delamination on natural frequency of a composite plate without considering the initial stress condition. Table 3 shows the

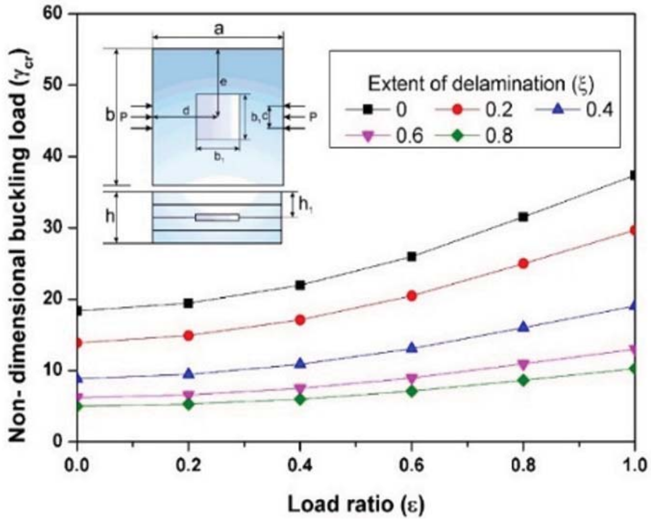

Fig.11: Variation of non-dimensional buckling load under in-plane distributed load with different extent of delamination

TABLE 3: NON-DIMENSIONAL VIBRATION FREQUENCIES FOR LAMINATED COMPOSITE PANEL FOR DIFFERENT ASPECT RATIOS

Extent of delamination (ξ)	Aspect ratio (α)					
	0.5	1.0	1.5	2.0	2.5	3.0
0	15.294	18.838	28.616	45.047	67.436	95.372
0.2	13.145	16.871	26.056	41.167	61.543	86.718
0.4	9.9455	13.550	21.603	34.429	51.402	72.002
0.6	7.905	11.140	18.399	29.686	44.560	62.732
0.8	6.934	9.880	16.781	27.441	41.530	58.899

TABLE 4: NON-DIMENSIONAL VIBRATION FREQUENCIES FOR LAMINATED COMPOSITE PANEL FOR DIFFERENT THICKNESS RATIOS FOR DIFFERENT EXTENT OF DELAMINATION HAVING DIFFERENT PLY ORIENTATIONS

Extent of delamination (ξ)	Plate thickness ratio (α)					
	100	50	25	20	10	5
(0/90)2s						
0	18.838	18.712	18.234	17.901	15.736	11.526
0.2	16.871	16.731	16.310	16.037	14.324	10.881
0.4	13.550	13.475	13.228	13.063	11.987	9.644
0.6	11.140	11.100	10.964	10.869	10.226	8.646
0.8	9.880	9.855	9.768	9.708	9.282	8.100
(±45)2s						
0	25.040	24.767	23.773	23.108	19.166	12.842
0.2	23.489	23.235	22.376	21.805	18.375	12.587
0.4	20.875	20.679	20.041	19.617	16.981	12.101
0.6	18.585	18.403	17.893	17.573	15.575	11.547
0.8	15.875	15.667	15.204	14.941	13.452	10.487
(±45/0/90)s						
0	24.569	24.315	23.380	22.750	18.967	12.778
0.2	22.996	22.760	21.961	21.425	18.152	12.511
0.4	20.226	20.028	19.423	19.030	16.582	11.944
0.6	17.686	17.473	16.939	16.625	14.765	11.116
0.8	14.753	14.465	13.865	13.549	12.029	9.523

effects of extent of delamination for various α. Table 4 shows the effect of ply orientation for various ξ and κ.

It can be seen from the Tables 3 and 4 that the plate without delamination i.e. (ξ=0) has highest vibrating frequencies for any given condition, and as the ξ increases the vibrating frequency reduces. From Table 3 it can be understood that as α increases the vibrating frequency also increases.

It can be seen from Table 4 that for any given condition the plate having angle ply orientation i.e. (±45)2s has highest vibrating frequencies, the plate with cross ply configuration is having least vibrating frequencies. However, it can be notices that the quasi-isotropic plate exhibit close values near to the angle ply configuration.

4.0 Conclusions

The present investigation shows the results for vibration and buckling characteristics of a delaminated plate considering various parameters which can be summarized as follows.

i. As the extent of delamination increases the buckling load decreases for any given ply orientation.
ii. For a given extent of delamination, the buckling load increases with the increase in plate aspect ratios.
iii. The reduction in buckling load due to the extent of delamination is not that much predominant in the case of thick plates as compared to that of thin plates.
iv. It is observed from the results that the cross ply orientation always yields the maximum buckling as well as natural frequencies as compared to that of angle-ply orientation.
v. The buckling load keep on reduces with the increase in the load width from both edges, but this is found to be reversed when the load width increases from the center of plate.
vi. For any given ply orientation, extent of delamination, loading and boundary condition as α increases the vibrating frequency increases.

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