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Effect of time-dependent sinusoidal boundary temperatures on the onset of ferroconvection in a porous medium

The effect of temperature modulation on the onset of Darcy ferroconvection in a horizontal porous layer heated from below is investigated. The analysis is based on the assumption that the amplitude of the temperature modulation is small enough compared with the imposed steady temperature difference. The effect of the oscillating temperature field is treated by a perturbation expansion in powers of the amplitude of the applied field. The effect of magnetic parameters, Vadasz number and temperature modulation in the cases of symmetric, asymmetric and bottom wall modulation, were discussed. The study divulges that subcritical motion exists for symmetric temperature modulation for low frequency. In the case of asymmetric and bottom wall modulation only supercritical motion exists.

Keywords: *Ferrofluid, porous media, magnetic field, temperature modulation*

1.0 Introduction

Thermo-mechanical interactions in fluids make the possible onset of convection induced by externally applied temperature gradients. The theory of thermal instability in a horizontal fluid layer heated from below was investigated by Lord Rayleigh (Rayleigh, 1916) and termed that phenomenon of buoyancy-induced instability as Rayleigh Bénard convection. The Rayleigh Bénard convection that occur in fluids with magnetic particles is named as ferroconvection. Ferrofluids are colloidal suspensions of surfactant-coated magnetic particles in a liquid medium, where the sizes of the particles are of several nanometers. They exhibit a variety of unusual properties, for instance, these fluids exhibit increased viscosity and apparent density in magnetic field gradients. Due to its wide range of applications in aerospace, industrial equipment designs, loudspeaker audio, biomedicals etc. (Papell, 1964) and (Scherer and Neto, 2005) a great effort has been devoted to the study of ferroconvection during the past three decades.

The convective instability of a ferromagnetic fluid for a fluid layer heated from below in the presence of uniform vertical magnetic field has been first investigated by Finlayson (Finlayson, 1970). Gupta and Gupta (Gupta, 1979), Gotoh and Yamada (Gotoh and Yamada, 1982), Stiles and Kagan (Stiles and Kagan, 1990) Russell et al (Russell et al, 1995) extended the pioneering work of Finalyson (Finlayson, 1970) to deal with the influence of a strong magnetic field and large wave number convection. Maruthamanikandan (2003) employed the Rayleigh-Ritz technique to examine the problem of onset of Bénard convection in a horizontal layer of a radiating ferromagnetic fluid. The effect of viscosity variation on non-Darcy ferroconvection was paid attention by Soya Mathew and Maruthamanikandan (2018) and Maruthamanikandan et al. (2018).

The motivation for the study of convection in a fluid saturated a porous medium has rich technological applications in chemical engineering, geothermal activities, oil recovery techniques and biological processes. Darcy (Darcy, 1856), Muskat (Muskat, 1937), Hubbert (Hubbert, 1956), Whitaker (Whitaker, 1966) has developed the flow of fluids through porous media and the equations pertaining to it.

In many systems, such as charges in electrostatic field and ferromagnetic resonance, modulation of a suitable parameter can have marked effects on the motion and can result in increased stability of the system. Venezian (Venezian, 1969) investigated the stability of a horizontal layer of fluid heated from below when, in addition a steady temperature difference between the walls of the layer, a time-dependent sinusoidal perturbation is applied to the wall temperatures. He showed that at low frequencies the equilibrium state becomes unstable because at that frequency the disturbances grow to a sufficient size so that the inertia effect becomes important. Malashetty and Wadi (Malashetty and Wadi, 1999) investigated the stability of a Boussinesq fluid saturated horizontal porous layer with time-dependent wall temperatures. It is shown that the system is most stable when the boundary temperature is modulated out of phase. Nisha Mary and Maruthamanikandan (Nisha and Maruthamanikandan, 2018) used regular perturbation

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technique to address the non-Darcy ferroconvection problem with gravity modulation. It is made clear that gravity modulation and magnetic mechanisms have opposing influence. Maruthamanikandan et al (2021) explored the combined effect of centrifugal acceleration and time-varying boundary temperatures on the onset of convective instability of a rotating magnetic fluid layer. It is established that, for bottom wall modulation, rotation tends to stabilize the system at low frequencies and the opposite is true for moderate and large frequencies.

The critical Rayleigh number in thermal modulation problems relies on the frequency of modulation and it proves to be possible to hasten or delay the onset of instability by tuning the frequency of modulation. In the present study we aim at investigating the problem of convective instability on a Darcy ferroconvection subject to time-periodic boundary temperatures with the intention of exploring the possibility of subcritical or supercritical motions.

2.0 Mathematical formulation

We consider a ferromagnetic fluid layer confined between two infinite horizontal surfaces with height "d". A vertical downward gravity force acts on the fluid together with a uniform, vertical magnetic field H_o. A cartesian frame of reference is chosen with the origin in the lower boundary and the z-axis vertically upwards.

The Boussinesq approximation is applied to account for the effect of density variation. The governing equations describing flow in an incompressible, non-conducting magnetic fluid saturated porous layer are

$$\nabla \cdot \vec{q} = 0 \qquad \text{... (1)}$$

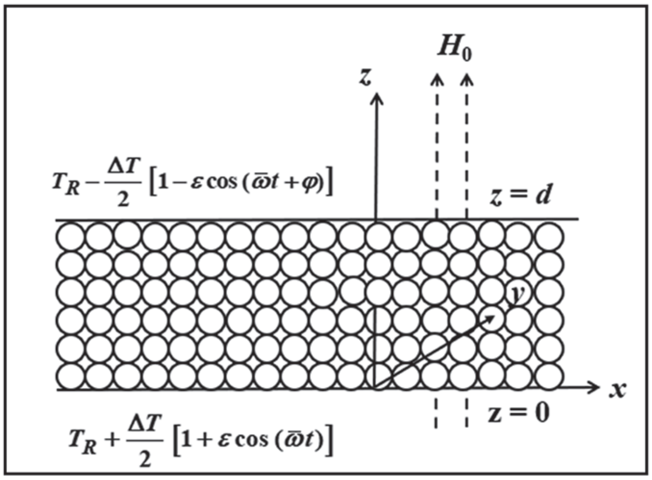
$$\rho_o\left[\frac{1}{\varepsilon_p}\frac{\partial \vec{q}}{\partial t} + \frac{1}{\varepsilon_p^2}(\vec{q}\cdot\nabla)\vec{q}\right] = -\nabla p + \rho\vec{g} - \frac{\mu_f}{k}\vec{q} + \nabla\cdot\left(\vec{H}\vec{B}\right) \qquad \text{... (2)}$$

$$\varepsilon_p\left[\rho_o\bar{C}_{V,H} - \mu_o\vec{H}\cdot\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\right]\left[\frac{\partial T}{\partial t} + (\vec{q}\cdot\nabla)T\right] + \mu_0 T\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\cdot\left[\frac{\partial\vec{H}}{\partial t} + (\vec{q}\cdot\nabla)\vec{H}\right] + \left(1-\varepsilon_p\right)\left(\rho_0 C\right)_s\frac{\partial T}{\partial t} = K_1\nabla^2 T \qquad \text{... (3)}$$

$$\rho = \rho_0\left[1 - \alpha\left(T - T_R\right)\right] \qquad \text{... (4)}$$

$$\vec{M} = \frac{\vec{H}}{H}M(H\,,\,T) \qquad \text{... (5)}$$

$$M = M_0 + \chi_m\left(H - H_0\right) - K_m\left(T - T_R\right) \qquad \text{... (6)}$$



Schematic of the problem

where $\vec{q}=(u,v,w)$ is the fluid velocity, ρ_0 is the reference density, ε_p is the porosity, t is the time, p is the pressure, $\vec{g}$ is the acceleration due to gravity, ρ is the fluid density, μ_f is the dynamic viscosity, k is the permeability of the porous medium, $\vec{H}$ is the magnetic field, $\vec{B}$ is the magnetic induction, T is the temperature, μ_0 is the magnetic permeability, $\vec{M}$ is the magnetization, K_1 is the thermal conductivity, α is the thermal expansion coefficient, $C_{V,H}$ is the specific heat at constant volume and magnetic field, χ_m is the magnetic susceptibility, K_m is the pyromagnetic coefficient and T_R denotes the reference temperature.

The relevant Maxwell equations are

$$\nabla\cdot\vec{B} = 0, \quad \nabla\times\vec{H} = 0, \quad \vec{B} = \mu_0\left(\vec{H} + \vec{M}\right) \qquad \text{... (7)}$$

The surface temperatures are

$$T_R + \frac{\Delta T}{2}\left[1 + \varepsilon\cos(\bar{\omega}t)\right],\ at\ z = 0$$
$$T_R - \frac{\Delta T}{2}\left[1 - \varepsilon\cos(\bar{\omega}t + \varphi)\right], at\ z = d \qquad \text{... (8)}$$

where ΔT is the temperature difference between the two surfaces in the unmodulated case, ε the amplitude of the thermal modulation, $\bar{\omega}$ the frequency and φ the phase angle.

3. Stability analysis

On applying an infinitesimal thermal perturbation and introducing the magnetic potential Φ, we obtain the following stability equations

$$\left(\frac{1}{Va}\frac{\partial}{\partial t} + 1\right)\nabla^2 w = R\left[1 - M_1(\varepsilon f - 1)\right]\nabla_1^2 T + RM_1(\varepsilon f - 1)\frac{\partial}{\partial z}\left(\nabla_1^2\Phi\right) \qquad \text{... (9)}$$

$$\frac{\partial T}{\partial t} + (\varepsilon f - 1)w = \nabla^2 T \qquad \text{... (10)}$$

$$\left(\frac{\partial^2}{\partial z^2} + M_3 \nabla_1^2\right)\Phi = \frac{\partial T}{\partial z} \qquad \text{... (11)}$$

where, $f = \mathrm{Re}\left[\left\{A(\lambda)e^{\lambda z} + A(-\lambda)e^{-\lambda z}\right\}e^{-i\omega t}\right]$,

$A(\lambda) = \frac{\lambda}{2}\left[\frac{e^{-i\phi} - e^{-\lambda}}{e^{\lambda} - e^{-\lambda}}\right]$, $\lambda = (1-i)\left(\frac{\omega}{2}\right)^{1/2}$, ω is the dimensionless frequency of modulation, $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and $\nabla^2 = \nabla_1^2 + \frac{\partial^2}{\partial z^2}$.

The dimensionless parameters are *Va* the Vadasz number, R the Darcy-Rayleigh number, M_1 the buoyancy magnetization parameter, M_3 the non-buoyancy magnetization parameter.

The boundary conditions are taken to be

$$w = T = \frac{\partial \Phi}{\partial z} = 0,\ at\ z = 0, 1. \qquad \text{... (12)}$$

At this point, we would like to indicate that it is convenient to express the entire system of equations in terms of the vertical component w of the fluid velocity. On combining Eqns. (9) through (11), we obtain we obtain an equation for the vertical component of the velocity w in the form

$$\left(\frac{1}{Va}\frac{\partial}{\partial t} + 1\right)\left(\frac{\partial}{\partial t} - \nabla^2\right)\left(\frac{\partial^2}{\partial z^2} + M_3\nabla_1^2\right)\nabla^2 w$$
$$= -R\left(\frac{\partial^2}{\partial z^2} + M_3\nabla_1^2\right)(\varepsilon f - 1)\nabla_1^2 w \qquad \text{... (13)}$$
$$+ RM_1M_3(1 - 2\varepsilon f)\nabla_1^4 w$$

4. Method of solution

The eigen functions and eigen values of the current study differ from the classical Rayleigh-Bénard problem by quantities of order ε. We therefore assume the solution of Eqn. (13) in the form

$$w = w_o + \varepsilon w_1 + \varepsilon^2 w_2 + \cdots \qquad \text{... (14)}$$

$$R = R_o + \varepsilon R_1 + \varepsilon^2 R_2 + \cdots \qquad \text{... (15)}$$

where R_o is the critical Rayleigh number for the corresponding unmodulated problem. The zero-order problem is equivalent to the problem of Rayleigh-Benard ferroconvection in the absence of thermal modulation. The expression for the Rayleigh number is given by

$$R_o = \frac{\left(\pi^2 + M_3\alpha^2\right)\left(\pi^2 + \alpha^2\right)^2}{\alpha^2\left[\pi^2 + M_3(1 + M_1)\alpha^2\right]} \qquad \text{... (16)}$$

Following the analysis of Venezian (Venezian, 1969), we obtain the following expression for.

$$R_2 = K_3 \sum_{n=1}^{\infty}\left[n^2\pi^2 + M_3(1 + 2M_1)\alpha^2\right]\frac{|B_n(\lambda)|^2 C_n}{D_n} \qquad \text{... (17)}$$

where $K_3 = \frac{R_o^2\alpha^2\left[\pi^2 + M_3(1 + 2M_1)\alpha^2\right]}{2\left[\pi^2 + M_3(1 + M_1)\alpha^2\right]}$

$$|B_n|^2 = \frac{16\pi^4 n^2\omega^2}{\left[\omega^2 + (n+1)^4\pi^4\right]\left[\omega^2 + (n-1)^4\pi^4\right]}$$

$$C_n = \frac{\omega^2}{Va}\left(n^2\pi^2 + \alpha^2\right)\left(n^2\pi^2 + M_3\alpha^2\right)$$
$$-\left(n^2\pi^2 + \alpha^2\right)^2\left(n^2\pi^2 + M_3\alpha^2\right)$$
$$+ R_o\alpha^2\left[n^2\pi^2 + M_3(1 + M_1)\alpha^2\right]$$

$$D_n = A_1^2 + A_2^2$$

$$A_1 = \frac{\omega^2}{Va}\left(n^2\pi^2 + \alpha^2\right)\left(n^2\pi^2 + M_3\alpha^2\right)$$
$$-\left(n^2\pi^2 + \alpha^2\right)^2\left(n^2\pi^2 + M_3\alpha^2\right)$$
$$+ R_o\alpha^2\left[n^2\pi^2 + M_3(1 + M_1)\alpha^2\right]$$

$$A_2 = \frac{\omega}{Va}\left(n^2\pi^2 + \alpha^2\right)^2\left(n^2\pi^2 + M_3\alpha^2\right)$$
$$+ \omega\left(n^2\pi^2 + \alpha^2\right)\left(n^2\pi^2 + M_3\alpha^2\right)$$

It should be remarked that supercritical instability exists if R_{2C} is positive and subcritical instability occurs when R_{2C} becomes negative. We evaluate R_{2C} for the following cases:

Case (i): the oscillating temperature field is symmetric so that the wall temperatures are modulated in phase (φ=0).

Case (ii): the oscillating temperature field is asymmetric corresponding to an out-of-phase modulation ($\varphi=\pi$).

Case (iii): when only the temperature of the bottom wall is modulated ($\varphi=-i\infty$).

5.0 Results and discussion

The problem considered is that to obtain the criteria for the effect of thermal modulation on the onset of convection in a horizontal porous layer of a ferromagnetic fluid heated from below. The effects of the oscillating temperature field are treated by a perturbation expansion in powers of the amplitude of the applied field. The shift in the critical Rayleigh number is calculated as a function of the frequency of temperature modulation, magnetic parameters and Vadasz number. The analysis presented is based on the assumption that the amplitude of the temperature modulation is small enough compared to the imposed steady temperature difference. It should be remarked that the validity of the results obtained depends on the range of the frequency of modulation ω. When ω is small, the period of modulation becomes large so that the disturbances may grow to such an extent that the finite amplitudes become significant. On the other hand, in the limit as $\omega \to \infty$, the effect of modulation is confined to a narrow boundary layer and outside this boundary layer the basic temperature field has essentially a linear gradient varying in time. Thus the effect of temperature modulation is perceptible for moderate values of the frequency of modulation ω (Venezian, 1969). Three different thermal excitations, viz., symmetric temperature modulation, asymmetric temperature modulation and the bottom wall temperature modulation are examined. The results of the present study are illustrated with the help of Figs.1 through 9. The results pertaining to the bounding wall temperatures

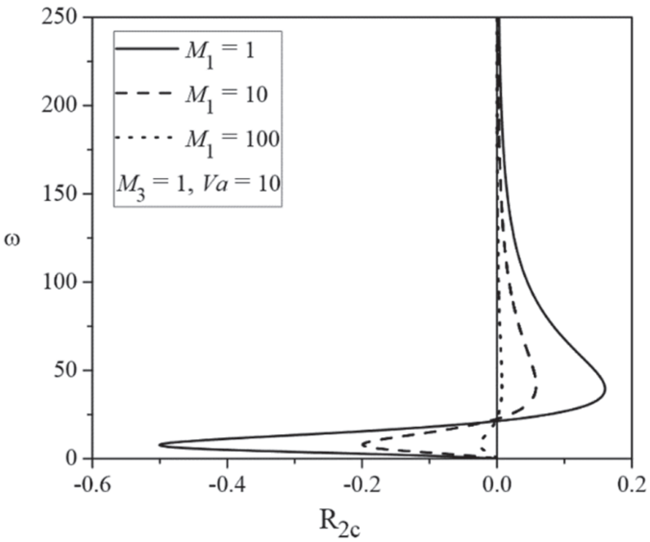


Fig.1: Plot ω of verses R_{2C} with variations in relating to in M_1 phase modulation

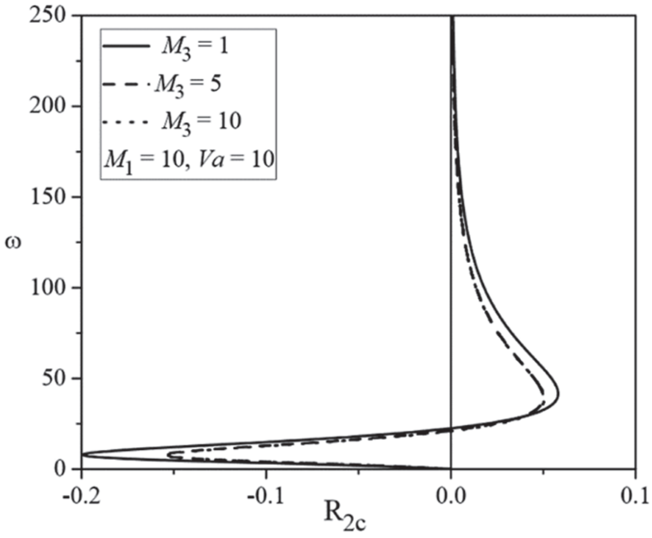


Fig.2: Plot ω of verses R_{2C} with variations in M_3 relating to in-phase modulation

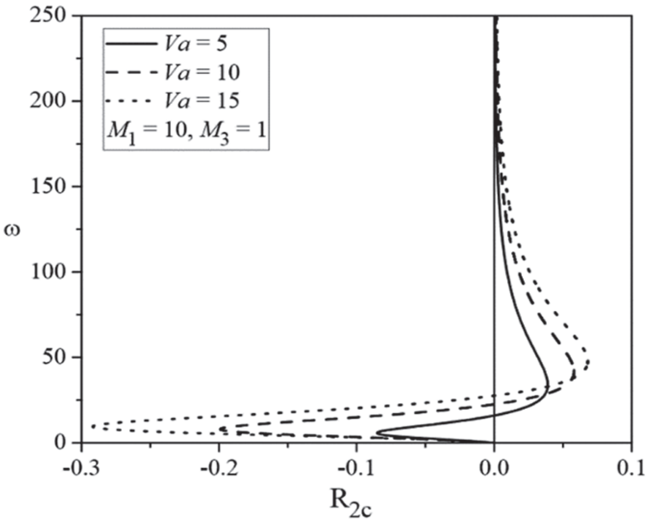


Fig.3: Plot ω of verses R_{2C} with variations in Va relating to in-phase modulation

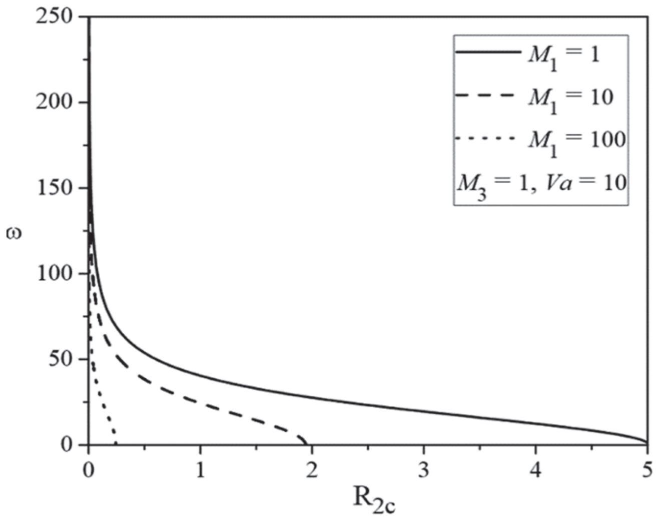


Fig.4: Plot ω of verses R_{2C} with variations in M_1 relating to out-of-phase modulation

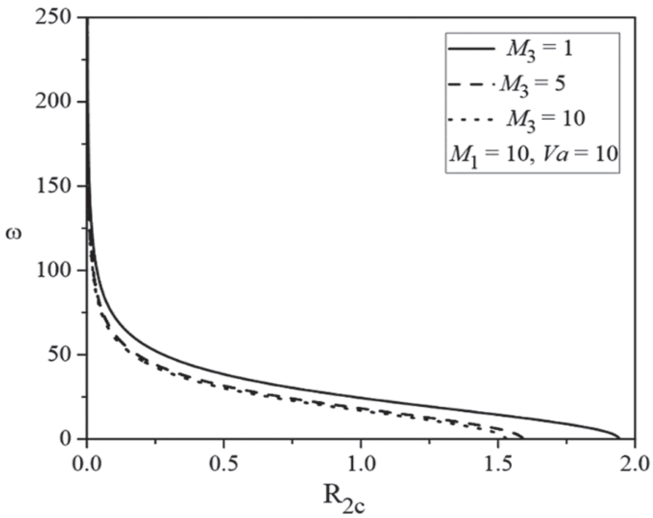


Fig.5: Plot ω of verses R_{2C} with variations in M_3 relating to out-of-phase modulation

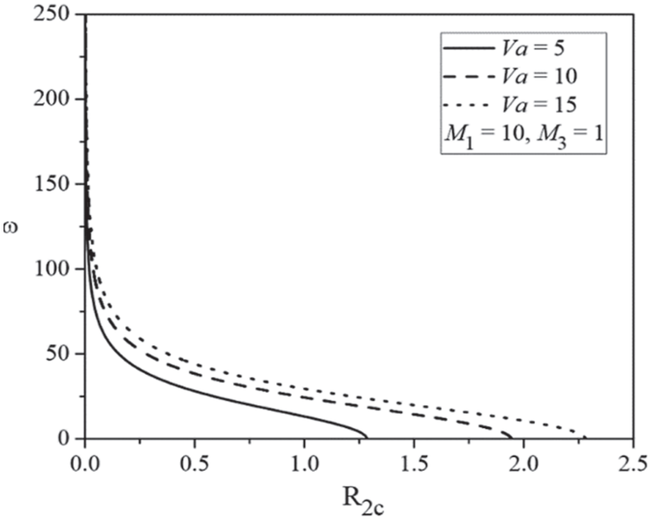


Fig.6: Plot ω of verses R_{2C} with variations in Va relating to out-of-phase modulation

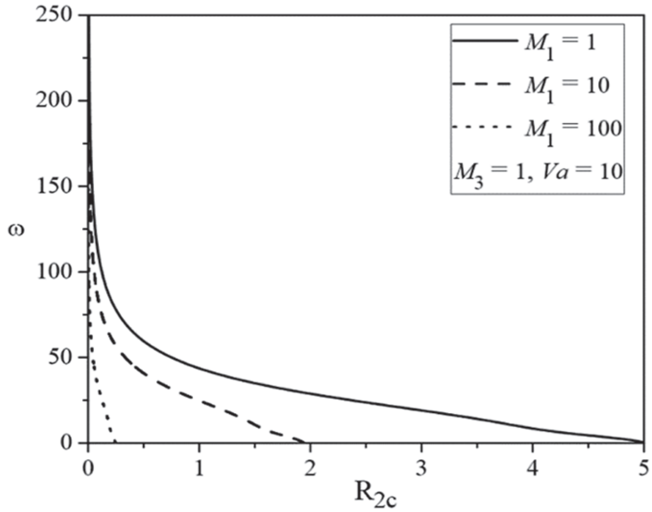


Fig.7: Plot ω of verses R_{2C} with variations in M_1 relating to bottom wall modulation

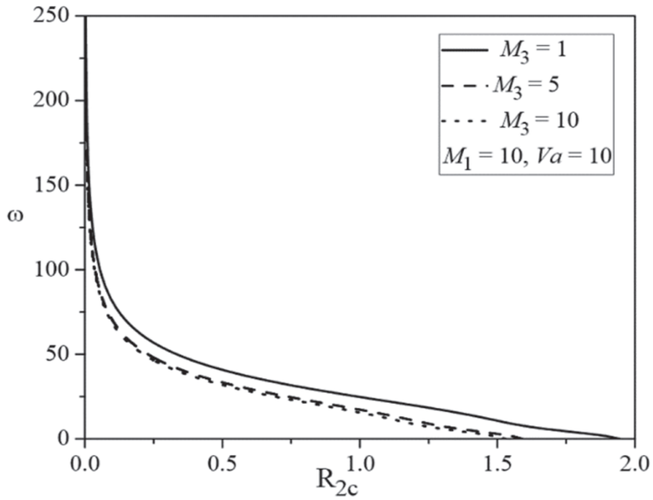


Fig.8: Plot ω of verses R_{2C} with variations in M_3 relating to bottom wall modulation

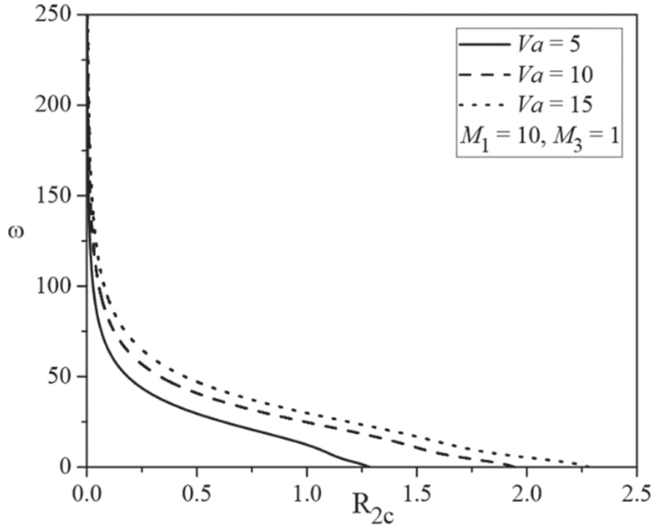


Fig.9: Plot ω of verses R_{2C} with variations in relating to bottom wall modulation

are modulated in phase are exhibited in Figs.1 through 3, thermal excitation is asymmetric in Figs.4 through 6, only the bottom wall temperature is modulated in Figs.7 through 9. The parameter M_1 is the ratio of magnetic force to gravitational force. The parameter M_3 measures the departure of linearity in the magnetic equation of state. The Vadasz number Va administrates the effect of porosity on flow in a porous media.

Since R_{2C} is negative only for symmetric modulation, as seen in Figs.1 through 9, it follows that subcritical motion is non-existent for asymmetric and bottom wall modulation. This could be a consequence of the symmetric modulation resulting in a nonlinear imposed temperature gradient.

It is proved in Figs.1 through 3 that the parameters M_1 and M_3 incline to stabilize the system and the opposite behaviour applies to Vadasz number Va. It is interesting to spot that symmetric modulation can also lead to supercritical instability for low Vadasz number ferrofluids provided the frequency of modulation ω is moderate in value.

As for the asymmetric modulation, as seen in Fig.4 through 6, the parameters M_1 and M_3 have the tendency to destabilize the system and the opposite is true for Vadasz number Va. As pointed out earlier, only supercritical instability exists for out-of-phase modulation.

Figs.7 through 9 link to the bottom wall thermal modulation. It is understood that the influences of the parameters M_1, M_3 and Va are analogous to that of asymmetric modulation.

Furthermore, moderate and large values of ω happen to scale down the influences of both magnetic and porous mechanisms. Moreover, the effect of magnetic forces, porous medium and temperature modulation disappear for sufficiently large values of the frequency of the temperature modulation. The problem throws light on external means of controlling convection in ferromagnetic fluid applications.

6.0 Conclusions

The combined effect of thermal modulation and magnetic parameters on the onset of convection in a densely packed ferromagnetic fluid layer is investigated and the following conclusions are drawn:

1. Subcritical motion exists for symmetric temperature modulation provided the frequency is low and only supercritical motions exist in the case of asymmetric and bottom wall modulation.
2. The effect of symmetric temperature modulation is destabilizing for small values of frequency and stabilizing for moderate values of frequency.
3. The increase in magnetic forces is to reduce the symmetric temperature modulation effect. The opposite is true for the effect of the Vadasz number.
4. In the case of asymmetric and bottom wall temperature modulation, the effect of temperature modulation is stabilizing for small and moderate values of frequency.
5. In the case of asymmetric and bottom wall temperature modulation, increase in magnetic forces is to hasten the onset of ferroconvection and the opposite is true for the effect of the Vadasz number.
6. In all the three cases, the magnetic, modulation and porous effects disappear altogether provided the frequency of temperature modulation is sufficiently large.

7.0 References

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