

# Thermal Management for Natural Convection within Porous Enclosures

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## Abstract

Various simulations and numerical studies have been carried out for free convection heat transfer of a square enclosure filled with porous matrix for sinusoidal and uniform heating conditions also varying Rayleigh values from 1 to 500 by using a mathematical finite element simulation. The porous media chosen for this research is saturated water medium (H<sub>2</sub>O). The enclosure properties have been predefined as follows: The top and base walls of the enclosure are made adiabatic, whereas the right wall is kept at a steady cold temperature. The hot left wall is treated at varying temperatures. The Nusselt values as well as mean Nusselt values have been quantified for the aforementioned values. It has been observed that the mean Nusselt values rise with the surge in Rayleigh values. For the Rayleigh value of 500, we found that uniform heating condition showed larger values of streamline function when compared to sinusoidal heating conditions. These simulations were performed by considering a solar fresnel as a real-world example. We hope that scholars and engineers collectively working on enhancing or aspiring to numerically model the thermal exchange through a square enclosure filled porous matrix will benefit from the information provided in this research article.

**Keywords:** Square Trapezoidal cavity, buoyant heat transfer, sinusoidal heating, aspect ratio, and Nusselt number.

## 1. Introduction

There has been a wide array of researches performed regarding thermal exchange which could also be called as heat transfer and its engineering applications. It is obvious from the literature that a good amount of work has been carried out by many researchers to understand various aspects and phenomenon in porous medium, but still there are wide open areas of heat and mass exchange in porous medium which need to be addressed. It is important to understand the different methods of heat transfer i.e. convection and conduction methods to apply on real world engineering applications such as packed beds and fluidized beds in chemical reactors, solar absorbers, boilers and furnaces, heat

exchangers, building design, HVAC, solar fresnel etc. Hence it is essential to use the right techniques for the enhancement of heat transfer for the optimization of the aforementioned applications. Another interesting and important study has also been carried out regarding environmental engineering.

The porous matrix chosen for the thermal exchange application also plays a crucial role in analyzing various factors with respect to heat transfer. Free convection heat transfer of a porous matrix in an enclosure has been the subject of study and investigations by scholars and engineers in the recent years.

Adrian Bejan is a pioneer of heat transfer and thermodynamics aptly defines porous media as a material made up of a solid matrix which contains a void that is

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interconnected [1]. A term associated with porous matrix is Porosity ( $\delta$ ) and can also be defined as the fraction of the total volume of the porous media matrix that is occupied by void space. Generally, for natural porous matrix, the porosity usually does not exceed 6/10. There have been various researches regarding the geometry by considering various such as aspect ratio, angle of inclination and shape of the geometry.

Akeel Mohammed et al. [2] studied the free convection heat transfer for various enhancement strategies and geometries including circular, triangular, square, and so forth. It was concluded that numerous factors play a vital role within the heat transfer techniques inclusive of type of fluid, supply of heat, the shape of enclosure geometry, angle of inclination of the enclosure tailored, presence of porous and buoyancy strength.

K Aparna et al. [3] examined the impact of uniform and non-uniform temperature on fluid flow and heat transfer through the walls of a trapezoidal space packed with porous matrix. In this paper, numerous numerical analyses and simulations were computed and presented for the free convection of porous-filled trapezoidal enclosure. The following conclusions were derived from the research. The mean Nusselt values ( $N_{ui}$ ) grow with the rise in Rayleigh values ( $R_a$ ) for the lower and top walls of the cavity enclosure. The mean Nusselt values ( $N_{ui}$ ) is reduced with the rise in aspect ratio (AR). The mean Nusselt range was observed going up with the inclination angle of the cavity enclosure walls.

Tanmay Basak et al. [4] carried out various simulations using Penalty FEA with quadratic polynomials having four degrees to inspect the flow of free heat convection in a trapezoidal hollow space packed with a porous matrix at distinct inclination angles. The study was also done by changing various frameworks such as Rayleigh ( $R_a$ ) values, Prandtl ( $P_r$ ) values and Darcy ( $D_a$ ) values varying those values to obtain various results. The analysis has been performed for several substances such as molten metal, saltwater, and olive oil while preceding works of literature focused mainly on water ( $H_2O$ ) and air. It was seen that regardless of Rayleigh ( $R_a$ ) and Prandtl ( $P_r$ ) numbers, the heat transfer took place through conduction at low Darcy ( $D_a$ ) values. For higher values of  $R_a$  and  $P_r$ , the circulation intensity and temperature gradient greatly increased when angle of inclination was increased from  $0^\circ$  to  $45^\circ$ . It was determined that for steady flow at the bottom wall, the Nusselt values ( $N_u$ ) have been the identical for inclination angles  $30^\circ$  and  $45^\circ$ . It was observed that the mean Nusselt numbers ( $N_{ui}$ ) almost remained unchanged with respect to inclination angles for higher  $R_a$  values during non-uniform heating. The  $N_{ui}$  displayed that after the inclination attitude is going up, the heat transfer rate will lower in most instances. It was also concluded that for convection heat transfer, the mean Nusselt

values ( $N_{ui}$ ) pass up with Rayleigh numbers ( $R_a$ ). Though Tanmay Basak focused on different inclination angles, subsequent workings for higher angles were not performed.

M. J Voon et al. [5] investigated and studied the thermal exchange, temperature fields, and flow of a square cavity consisting of four-layered porous media which is located layer by layer in the hollow space. The hollow space filled with layered porous media is heated from underneath to perform a numerical evaluation. In addition, they analyzed authenticity through the use of Lumped-System method for heat transfer prediction. They studied for mainly two types of porous media position i.e. four-layered horizontal and four-layered vertical media. Since the wall underneath is heated we know the heat go with the flow course is perpendicular to horizontal sub layers and parallel to vertical sub layers. This paper additionally discovered that for horizontally layered porous media, if the powerful penetrability (KA) is based on the Harmonic mean, the records are graphed into a curve and if KA is primarily based on the mathematics mean, then the information is dispersed.

Tanmay Basak et al. [6] studied the phenomena of free convective flow in a trapezoidal enclosure full of porous matrix. A penalty FEA with bi-quadratic elements is carried out to research the effect of uniform and non-uniform heating of the bottom wall at the same time as vertical partitions are maintained at consistent cold temperature and the top wall is well insulated, inclination angles ( $u$ ), Prandtl quantity ( $P_r$ ), Darcy quantity ( $D_a$ ), Nusselt values ( $N_u$ ), Rayleigh values ( $R_a$ ). The penalty FEA technique helps in obtaining smooth solutions in terms of stream function and isotherm contours for uniform and non-uniform heating of the base wall with wide ranges of  $P_r$ ,  $R_a$ , and  $D_a$ .

Yasin Varol et al.[7] investigated the up thrust influenced convective and flow of fluid in cavities saturated with fluid porous matrix which can be observed in many engineering applications such as solar electricity collector, nuclear reactor, double-wall insulations, geothermal utility, electric powered equipment, the cooling systems of electronic devices etc. Poulidakos analyzed a porous layer filled with  $H_2O$  (water) for stable convection flow at the largest denseness and heating done differentially within the horizontal direction. He proved the problem analytically and calculated numerically at low Rayleigh values ( $R_a$ ) with a limited volume scheme. It was discovered that the bi-cellular flow field, that is a direct result of the presence of the max density ( $\rho_m$ ) analogous with the highest temperature ( $T_m$ ) of  $3.98^\circ C$ , is an attribute of the numerical solution. Pop and Saeid performed numerical analysis on a similar study of the free convective flow in a two-dimensional (2D) partially heated rectangular empty space saturated with a porous matrix i.e. water which has a max density in the vicinity area of the max temperature of  $3.98^\circ C$ . A numerical modelling of the stable convective flow in a trapezoidal enclosure full of cool water at round  $3.98^\circ C$

has been experimented with the use of a finite-difference analysis.

A.C Baytas et al. [8] worked out a numerical study of the steady free convective heat transfer flow with distinct inclinations of the trapezoidal enclosure with a top cylinder surface packed with a fluid-saturated permeable medium. In this study, the top surface of the cylinder enclosure is cooled down and the base surface is heated and the other two non-parallel plane sides walls do not take part in the exchange of heat and mass with the environment. Results are got numerically for the trapezoidal hollow space of  $K=30$ , Aspect Ratio  $AR=2$ , and various tilt angles ranging from  $15^\circ$  to  $165^\circ$ . The Rayleigh values ( $R_a$ ) ranges from 100 to 900. The mean Nusselt value ( $N_{ui}$ ) will increase with the rise of  $R_a$  till it reaches max value at  $60^\circ$ .

## 2.0 Governing Equations

In order to achieve the mathematical governing equations few presumptions were considered. The presuppositions were as follows: criteria for the porous matrix and the fluid were made to be a constant value, the boundary conditions were considered to be hermetic in nature, Boussinesq approximation has been used to compute density, inertia and viscous drag quantities in the momentum mathematical equation are assumed to be insignificant. Based on these presumptions the dimensional governing mathematical equations (continuity, momentum, energy) are as follows,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \dots (1)$$

Darcy formula (Momentum equation)

$$u = \frac{-K}{\mu} \frac{\partial p}{\partial x} \quad \dots (2)$$

Darcy's law velocity in the horizontal direction can be used to explain the velocity in the x-direction.

$$u = \frac{-K}{\mu} \frac{\partial p}{\partial x} \quad \dots (3)$$

Velocity in x-direction is shown below,

$$v = \frac{-K}{\mu} \left( \frac{\partial p}{\partial y} + \rho g \right) \quad \dots (4)$$

The permeability  $K$  of permeable medium can be expressed as

$$K = \frac{D_p^2 \phi^3}{180(1-\phi)^2} \quad \dots (5)$$

The Boussinesq approximation as

$$\rho = \rho_\infty [1 - \beta_T (T - T_\infty)] \quad \dots (6)$$

Momentum equation

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{Kg \beta_T}{\gamma} \frac{\partial T}{\partial x} \quad \dots (7)$$

Energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad \dots (8)$$

Revised Rayleigh (Ra) number

$$Ra = \frac{g \beta_T \nabla T K L}{\nu \alpha} \quad \dots (9)$$

The Boundary conditions set for the model have been shown in Figure 1.

On the bottom wall (Isothermal heating) :

$$0 < x < L, y = 0, \theta = (x), u = 0, v = 0.$$

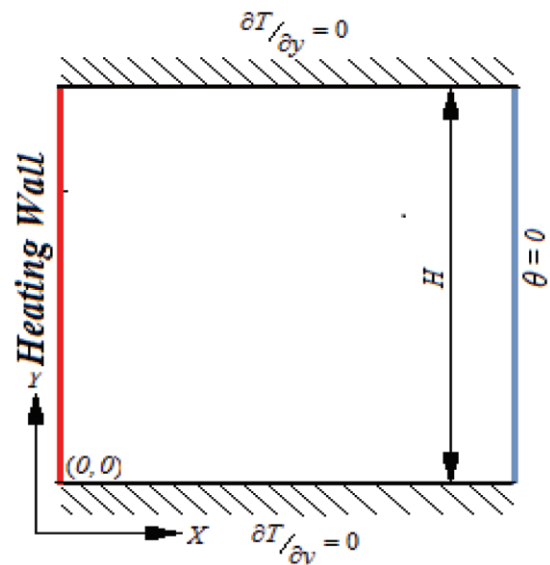


Figure 1: Physical Domain

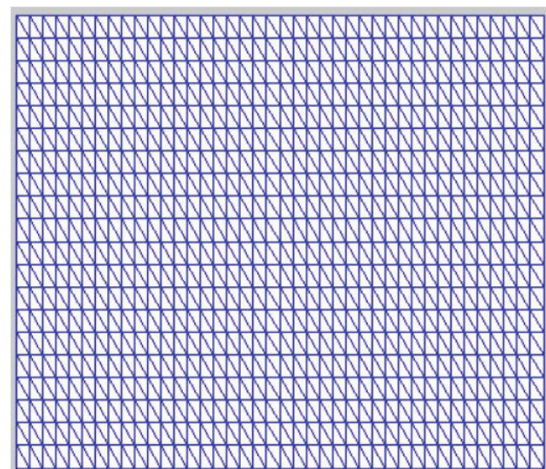


Figure 2: Triangular Mesh for porous cavity

The finite element method was used to analyse and solve the aforementioned governing mathematical equations. In order to confirm the predictive similarity and accurateness of the current methodology, the numerical computations on free convective thermal removal in a square trapezoidal cavity are chosen.

The different sizes of grid ranging from  $11 \times 11$ ,  $21 \times 21$ ,  $31 \times 31$  and  $41 \times 41$ , are used for the computation. The overall heat transfer rates are tabulated (Table 2) for sinusoidal heating condition. We have chosen up to grid of  $41 \times 41$  for computations because for further grid values i.e  $51 \times 51$ ,  $61 \times 61$ ,  $71 \times 71$  etc. Show the same mean Nusselt value and also the computational time and power required for grid values more than  $41 \times 41$  is a lot.

**Table 1: Grid convergence at Rayleigh number (Ra) = 500**

Grid independence Test	
Grid Dimension	<i>NU</i>
$11 \times 11$	0.4235
$21 \times 21$	0.6213
$31 \times 31$	1.1852
$41 \times 41$	1.3234
$51 \times 51$	1.3256
$61 \times 61$	1.3325

### 3.0 Code Validation

To validate the methodology we used, we recreated the set up done by Al-Rashed. Figure 3 shows the comparison of results obtained in their study and our study.

In Figure 3, the graphs on the left show the current study and the graphs on the right are graphs obtained by Al-Rahed et al. We can see that outcomes derived in our study are almost analogous to the results obtained in Al-Rashed et al. study. This acts as an evidence for the validity of our code used for this research.

### 4.0 Results and Discussion

The thermal exchange rate surges with the rise of Rayleigh value. The temperature contours become more dense with the

increase of Ra and pushes towards the top right of cavity. The local Nusselt value as well as the mean Nusselt value increases with the surge of Rayleigh value from 1 to 500. The heating conditions chosen for the present study have been uniform and sinusoidal heating conditions. The Rayleigh values chosen for this research are 1, 10, 100, 250 and 500. The local Nusselt values have also been computed for both of the heating conditions and Table 2 shows the variation of local Nusselt values for the Rayleigh values mentioned above. Figures 5, 6 and 7 show the isotherm contours and stream functions for the aforementioned heating conditions. Various graphs have also been derived and plotted with respect to the different heating conditions by considering the variation of mean Nusselt values and dimensionless length for the left wall as well as the right wall of the porous square cavity.

In Figure 4, as expected, due to sinusoidal heating and cooling of the two sidewalls, the isotherm contours tend to narrow down at the top right side of the cavity with the rise of Rayleigh values. It can be observed that the convective flow of heat at the centre seems to be in the counter clockwise direction. The bulb shape is observed with rise of Ra in the isotherm silhouettes. The increase in Ra leads to shifting of the isotherms towards edges of the cavity. The streamline function value for Ra=1 is 0.055 in counter clockwise direction with a gradual increase to 1.2 in the same direction for Ra=500. The change in shape of the streamline contour can be observed as well. In Figure 5, we can easily observe that for uniform heating, the stream contours gradually expand with the rise of Ra and the temperature contours expand rapidly towards the right side of the wall with the rise of Ra as well. With the rise of Ra the isotherm silhouettes gain the bulb structure here as well. The maximum local Nusselt value was observed for Ra=500 at the left side of the wall. By comparing uniform heating with sinusoidal we can observe that streamline functions were larger for uniform than sinusoidal heating conditions. We can also observe the increase of  $\phi$  with the surge in Ra.

Figs. 6 and 7 show variation of local Nusselt values for Left and Right wall of square porous cavity with uniform and sinusoidal heating respectively for Ra values ranging from 1 to 500.

Fig. 8 shows the variation of the mean Nusselt values with respect to Rayleigh values for different heating conditions. Uniform heating condition has the mean Nusselt value for the Rayleigh values considered for this study. Sinusoidal heating condition showed the lowest mean Nusselt value when compared to uniform heating conditions.

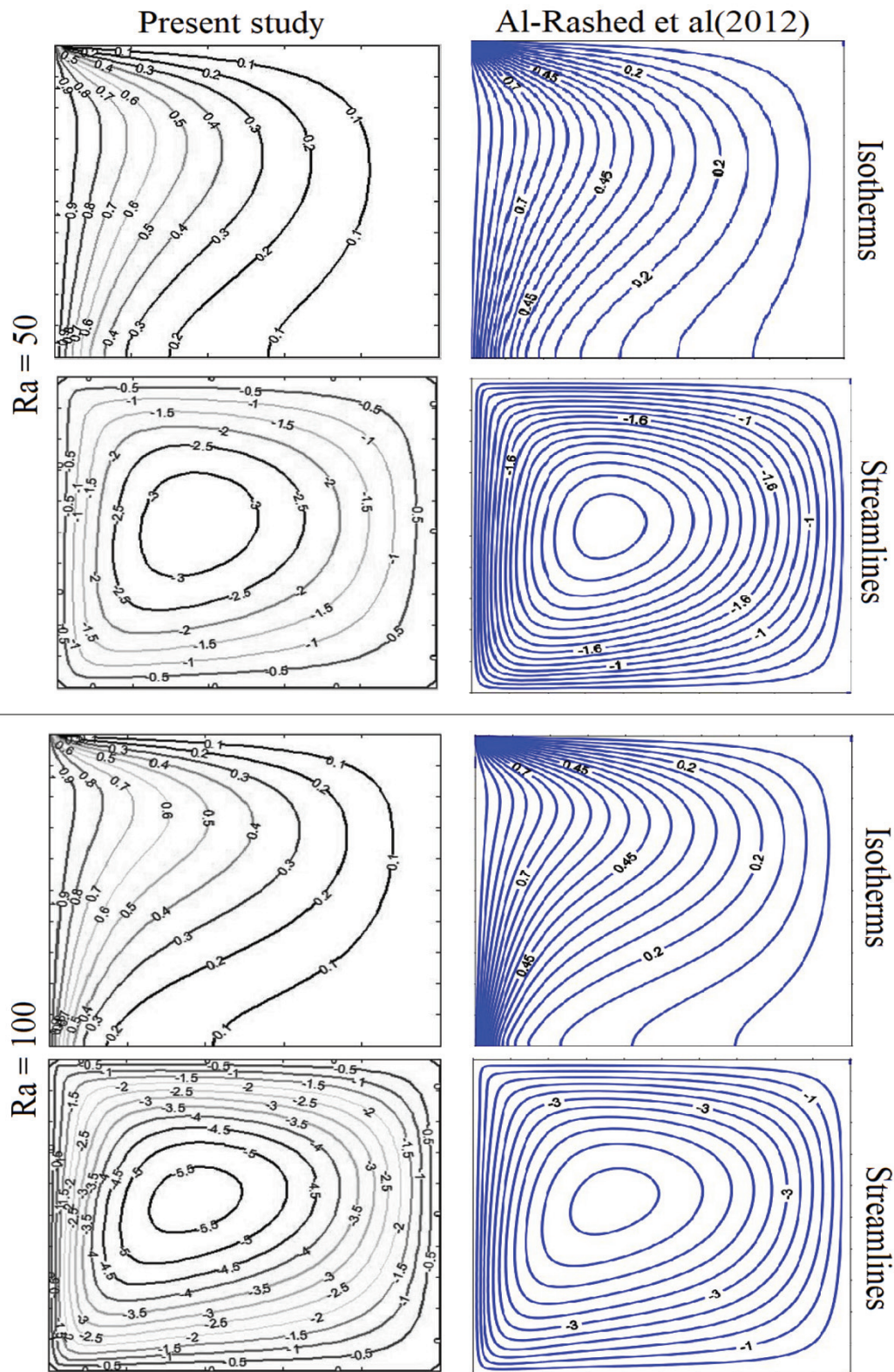


Figure 3: Comparison of Streamlines and Isotherms at Ra =50 and Ra=100 for square cavity heated left sidewall. (Present Studyv/s Al-Rashed et al.)

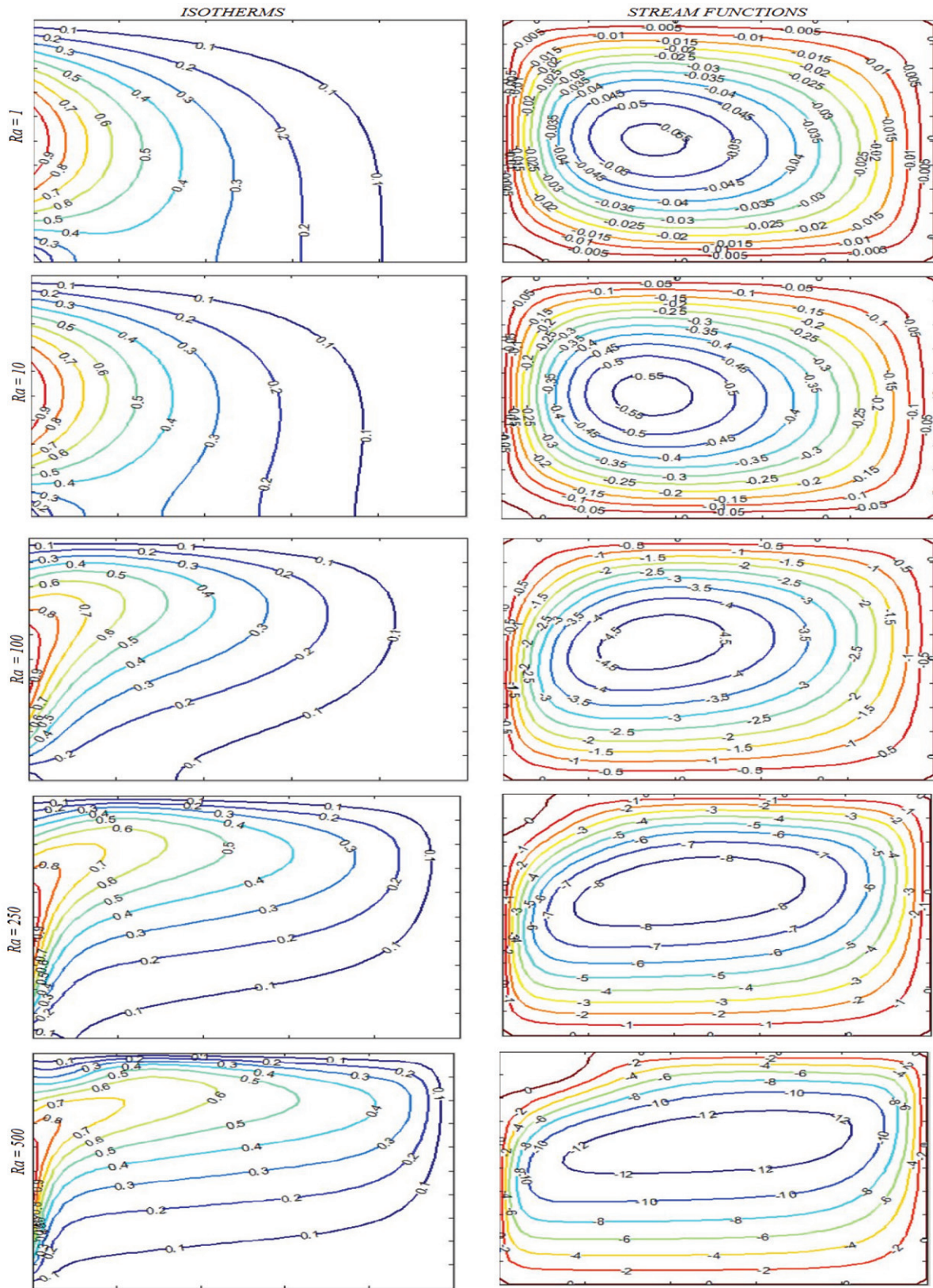


Figure 4: Isotherms and Streamlines of square cavity Sinusoidal Heating with Different Rayleigh numbers

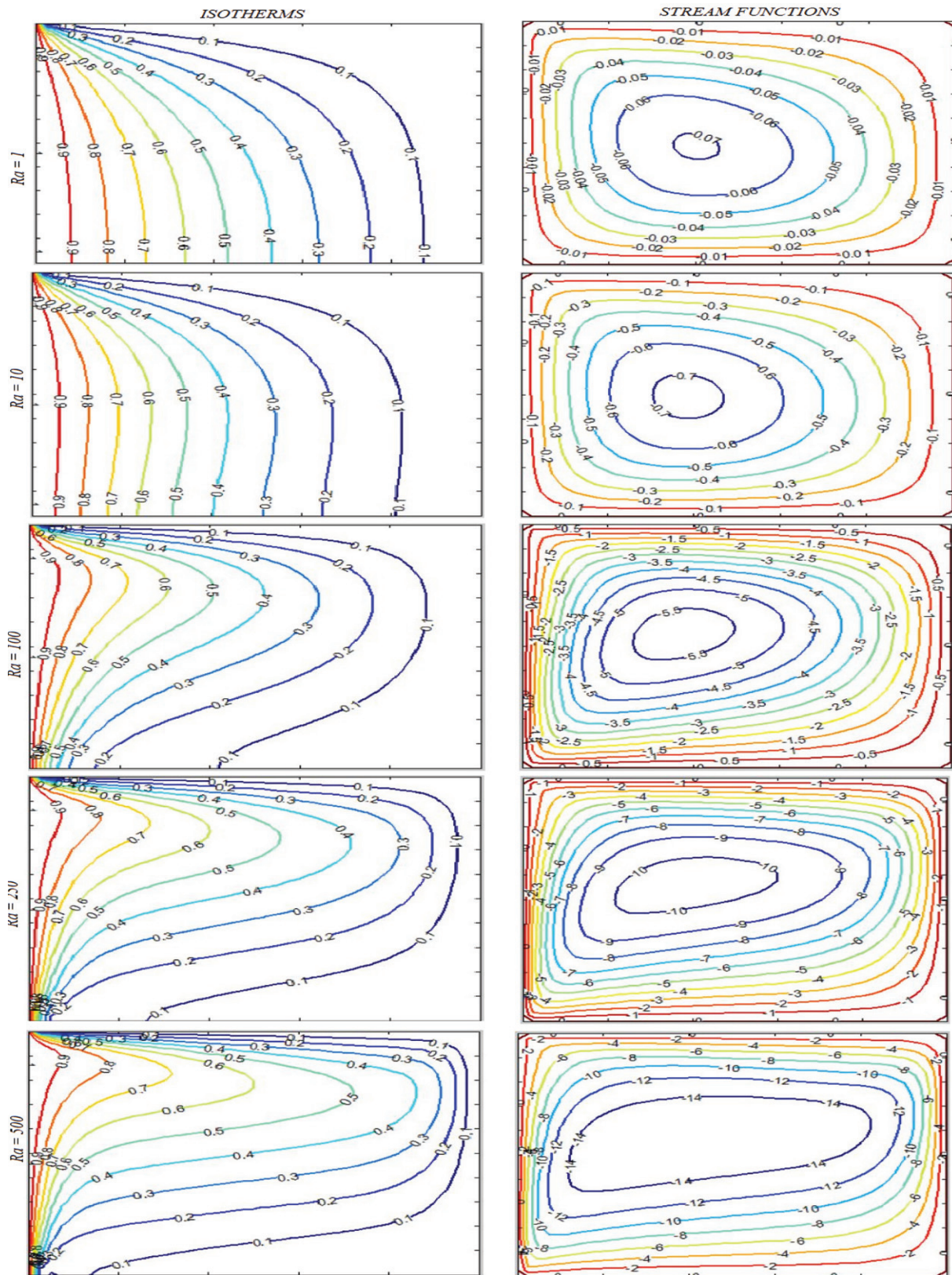


Figure 5: Isotherms and Streamlines of square cavity for Uniform Heating with Different Rayleigh numbers

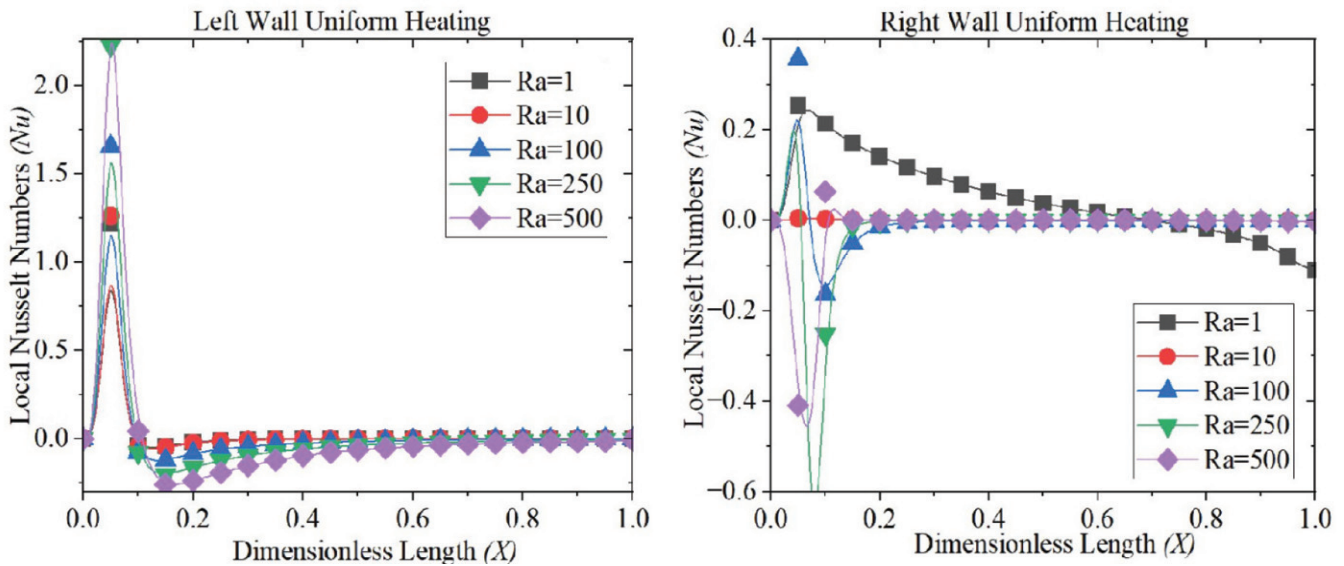


Figure 6: Variation of Local Nu with respect to uniform heating

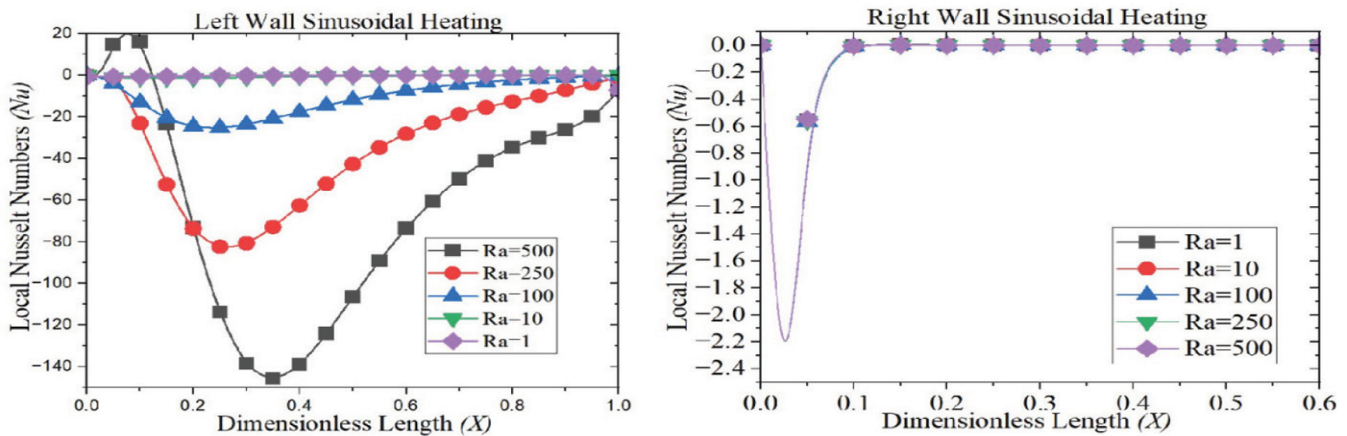


Figure 7

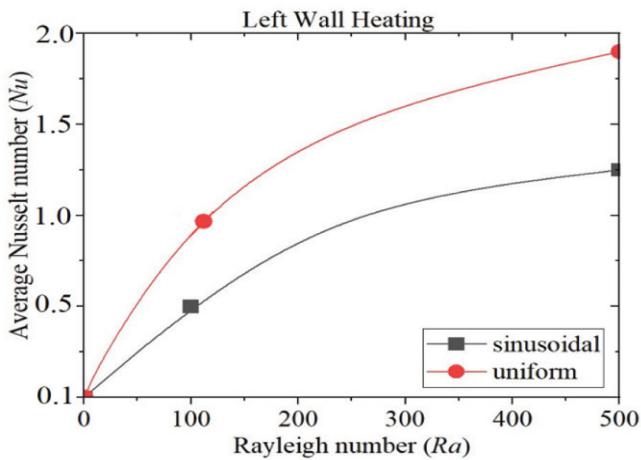


Figure 8: Variation of MeanNu for different heating conditions at the left wall

## 5.0 Conclusions

The main investigation of the present study is to investigate the effect of Ra with respect to various heating boundary conditions for the left wall of the square cavity on free convection heat transfer. The following conclusions have been understood from the present computations:

- (a) The mean Nusselt value rises with the rise of Rayleigh values.
- (b) The mean Nusselt values were higher for uniform heating when compared to sinusoidal heating conditions.
- (c) Sinusoidal heating showed the lowest mean Nusselt values for the rise Ra when compared to uniform heating conditions.



- (d) The maximum local Nusselt values observed were generally large Rayleigh values i.e.  $Ra=250$  and  $Ra=500$ .
- (e) The isotherms  $\theta$  tends to gain a bulb shape with rise of  $Ra$  for all the heating conditions.
- (f) The streamline function  $\psi$  can be easily observed to incrementally rise with the surge of Rayleigh values.
- (g) The uniform heating condition shows the highest value of  $\psi$  for  $Ra=500$ .
- (h) The shape of streamline silhouette can be seen to change from a circular shape to an elliptical shape at the centre of the porous cavity with the increase of  $Ra$ .
- (i) By observing these patterns we can aptly conclude that the mode of thermal exchange moves from thermal conduction to thermal convection for all the heating conditions with the increase of  $Ra$  from 1 to 500.

## 6.0 References

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