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Mathematical modelling of a hydro-pneumatic type gas accumulator

In recent days, in the field of advanced technologies fluid power applications are playing a pivotal role in every aspect of engineering design and manufacturing. Every mechanical system is getting replaced with the hydraulic system. One of the main components of hydraulic system is accumulator. Accumulators play the same role as played by the flywheel in mechanical systems only difference is accumulator stores energy in the form of pressure energy while the latter one stores energy in the form of kinetic energy.

There are several types of accumulators being used based on different applications and hydraulic system requirements. In this paper bladder type accumulator, applications, its energy transfer and thermal characteristic equations and sizing of accumulators are clearly explained with the help of mathematical equations.

Keywords: *Fluid power; hydraulic system; accumulators; thermal characteristics.*

I. Introduction

The hydraulic accumulator is one of the hydraulic elements of the system that will be used to reduce pressure and speed pulsation inside it; thus, the selection and workflow modelling have a significant influence on the stability of the entire system. Hydraulic accumulators store the source of energy during loading and return it to the system when there is high load demand.

Hydraulic accumulators permit to reduce pump power and pressure pulsation which causes the pump, valves and other devices work and protect the system against possible hydraulic jerks, kinetic energy absorption during the large inertia load of the engine and to compensate for pressure changes in the fluctuations of temperature[1].

A gas hydraulic accumulator is made up of steel metal shell and comprising a bladder inside it which is made up of an elastic diaphragm or a synthetic polymer rubber like chloroprene, nitrile, etc. [2].

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The main advantage of a bladder type accumulator is that it responds quickly for receiving and expelling flow of oil and bladder type accumulators are generally preferred than piston accumulators due to lighter weight, lower cost, and compactness [3].

Usually bladder is filled with the nitrogen gas due to its non poisonous, not combustible and inert nature during empty of hydraulic fluid and it is known as pre-charge pressure generally less than nominal system pressure.

As the fluid enters the accumulator the bladder gets compressed and pressure inside the accumulator reaches the system maximum pressure and it represents the maximum amount of energy stored by the accumulator[4]. While compressing the gas it follows isothermal process and in expansion, it follows adiabatic process. A bladder type accumulator is schematically shown in Fig.1.

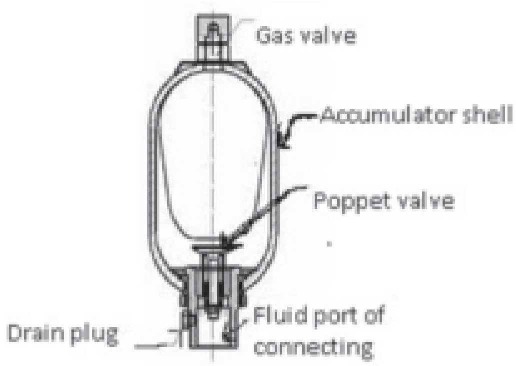


Fig.1 Schematic diagram of a bladder accumulator

These accumulators are made like a sphere used in the applicable mobile devices, especially in mobile equipment [5]. Other applications of gas accumulators involve leakage compensation, auxiliary power source and as energy conservative device in hydraulic systems and it is used in many hydraulic operations like steering system, bucket lifting operation so on employed in mining equipment to improve response time and for noise reduction.

Some of the cases of the scientific reviews based on the usage of accumulators are: modelling and control of open accumulator compressed air energy storage (CAES) system for wind turbines [6], usage of hydro-pneumatic accumulators for kinetic energy storage and thermal storage losses, and mathematical modelling and simulation of an industrial accumulator for elastic webs [7].

In this paper, mathematical modelling of the hydro-pneumatic type accumulator has been done to analyse the energy saving and thermal behaviour of the accumulator. Mathematical modelling explains the energy and thermal characteristics of the bladder type accumulator with the help of mathematical equations.

II. Mathematical modelling

A. Energy equations

A bladder type accumulator is shown in Fig.2

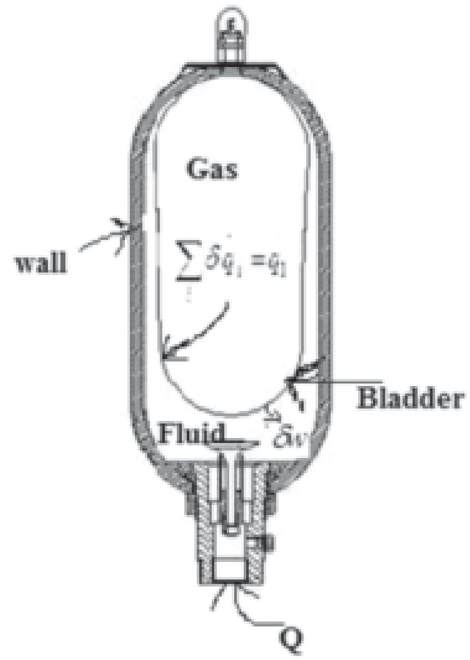


Fig.2 A schematic diagram of bladder accumulator

A schematic diagram of bladder accumulator showing energy transformations consists of a cylindrical shell in carbon steel, closed at both ends and diaphragm that acts as a separator between the gas and the liquid to determine the thermodynamic conditions assumed by the gas during a transformation of compression or expansion, whether it is regarded as the real or ideal type, the equation of energy for closed systems is applied [8] :

$$m_g \cdot \frac{du}{dt} = \sum_i \delta \dot{q}_i - \delta \dot{w} \qquad \text{... ... (1)}$$

$\delta \dot{w} =$ Exchange of work with the outside is determined by both the change in volume undergone by the gas as a result of the oil flow through the accumulator Q' and by internal viscous losses. In the figure are shown thermal exchanges between gas, the accumulator body and the oil in the hydraulic circuit. Thermal exchanges between gas and hydraulic oil are neglected in the model. Bearing in mind that:

$$du = c_v.dT_g + \left[T_g \left(\frac{\partial p_g}{\partial T_g} \right) - P_g \right].dv_g \qquad \text{... ... (2)}$$

$$\partial \dot{w} = P_g \frac{dv_g}{dt} - \delta \dot{L}_p \qquad \text{... ... (3)}$$

One dimensional Fourier heat equation is given by:

$$\sum_i \delta \dot{q}_i = \dot{q}_1 = \left(\frac{T_w - T_g}{R_{g-w}} \right)$$

$$R_{g-w} = \frac{1}{hA} \qquad \text{... ... (4)}$$

By substituting all the above equations in equation (1)

$$.m_g . \frac{\left(c_v .dT_g + \left[T_g \left(\frac{\partial p_g}{\partial T_g} \right) - P_g \right] .dv_g \right)}{dt} = \frac{\left(T_w - T_g \right)}{R_{g-w}} - P_g \frac{dv_g}{dt} + \delta \dot{L}_p$$

$$m_g c_v \frac{dT_g}{dt} + \left[T_g \frac{\partial P_g}{\partial T_g} - P_g \right] \frac{dV_g}{dt} = \frac{T_w - T_g}{R_{w-g}} - P_g \frac{dV_g}{dt} + \delta \dot{L}_p$$

$$\frac{dT_g}{dt} = \frac{T_w - T_g}{m_g c_v R_{w-g}} - \frac{P_g \frac{dV_g}{dt}}{c_v} + \frac{\delta \dot{L}_p}{m_g c_v} - \frac{T_g \frac{\partial P_g}{\partial T_g} \frac{dV_g}{dt}}{c_v} + \frac{P_g \frac{dV_g}{dt}}{c_v}$$

$$\frac{dT_g}{dt} = \frac{T_w - T_g}{m_g c_v R_{w-g}} + \frac{\delta \dot{L}_p}{m_g c_v} - \frac{T_g \frac{\partial P_g}{\partial T_g} \frac{dV_g}{dt}}{c_v}$$

$$\tau = m_g c_v R_{w-g}$$

$$\frac{dT_g}{dt} = \frac{T_w - T_g}{\tau} + \frac{\delta \dot{L}_p}{m_g c_v} - \frac{T_g \frac{\partial P_g}{\partial T_g} \frac{dV_g}{dt}}{m_g c_v} \qquad \text{... ... (5)}$$

Where,

τ = thermal time constant of the system and it is different for different size of accumulators and its working operations[8].

The derivative of the gas pressure with respect to its temperature at constant volume present on above equation of state is explained by the BWR model equation. The behaviour of gases in a wide of range of pressure and temperature with good accuracy is expressed by Benedict-Webb Rubin equation (BWR). The BWR equation for calculating the compressibility co-efficient 'z' is given below:

$$z = 1 + \frac{1}{v}\left(B_o - \frac{A_o}{RT} - \frac{C_o}{RT^3}\right) + \frac{1}{v^2}\left(b - \frac{a}{RT}\right) +$$

$$\frac{a * \alpha}{RT * v^5} + \frac{c}{v^2 * RT^3} \times \left[\left(1 + \frac{\gamma}{v^2}\right)\exp\left(\frac{-\gamma}{v^2}\right)\right]$$

$$z = \frac{P_g v}{RT_g}$$

$$P_g = \frac{RT_g}{v} + \frac{1}{v^2}\left(B_o RT - A_o - \frac{C_o}{T_g^2}\right) + \frac{1}{v^3}\left(bRT_g - a\right) +$$

$$\frac{a * \alpha}{v^6} + \frac{c}{T_g^2 v^3}\left(\left[\left[\left(1 + \frac{\gamma}{v^2}\right)\exp\left(\frac{-\gamma}{v^2}\right)\right]\right]\right)$$

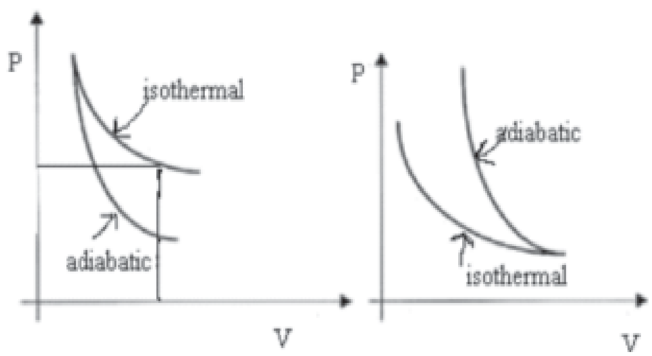
$$\frac{dP_g}{dT_g} = \frac{R}{v}\left(1 + \frac{b}{v^2}\right) + \frac{1}{v^2}\left(B_o R + \frac{2C_o}{T_g^3}\right)$$

$$- \frac{2c}{v^3 T_g^3}\left(1 + \frac{\gamma}{v^2}\right)\exp\left(\frac{-\gamma}{v^2}\right) \qquad \text{... ... (6)}$$

B. Thermal Characteristics

Thermal characteristics involve charging and discharging of the accumulator. As earlier said the hydraulic fluid is pumped into the accumulator, thereby compressing the gas and increasing the pressure in the accumulator.

This compression in the accumulator chamber or charging of the accumulator follows isothermal process because work required is minimum in isothermal process and sudden release of pressure by the accumulator or discharging of the accumulator follows adiabatic process because as there is no heat transfer losses will be reduced and it reaches the required point much faster than isothermal.These two facts are very well explained with the help of Fig.3 as is shown below:



(a) Discharging (b) Charging

Fig.3 Charging and discharging characteristics of an accumulator

In accumulators bladder is filled with nitrogen gas and it takes the shape of the accumulator shell this is known as pre-charge position as shown in Fig.3.(a), by pumping the hydraulic fluid into the accumulator the gas contained in the bladder will be compressed isothermally.

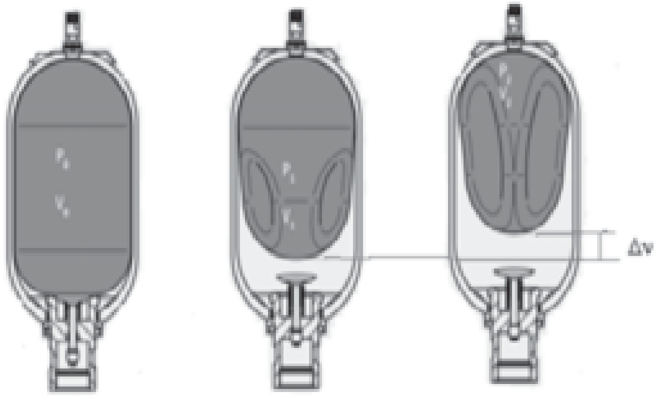


Fig.4 Schematic diagram showing working of an accumulator in different stages

The gas volume is reduced to v_2 and pressure increases and it reaches the maximum pressure p_2. Hydraulic fluid accumulates in the unit as shown in Fig.3.(b). Work involved in this process is given by the equation (9).

$$W = p_2 v_2 \ln \frac{p_2}{p_0} \qquad \text{... ... (7)}$$

Pressure is released by the accumulator to the system from p_2 to p_1 as shown in Fig.3(b).

When the system pressure is less than the gas pressure this discharging of pressure follows adiabatic path to minimize the losses and work equations are given by:

$$E = \frac{p_1 v_1}{n-1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right] \qquad \text{... ... (8)}$$

$$v_1 = v_o \left[\frac{p_o}{p_1}\right]^{\frac{1}{n}} \qquad \text{... ... (9)}$$

Furthermore considering the pre-charge condition, the gas fills the volume v_o of the reservoir at the pressure p_o the equation becomes:

$$\frac{E}{p_2} = \frac{\frac{p_1}{p_2}\left(\frac{p_o}{p_1}\right)^{\frac{1}{n}} . v_o \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]}{n-1}$$

$$\frac{E}{p_2 v_o} = \frac{\frac{p_1}{p_2}\left(\frac{p_o}{p_1}\right)^{\frac{1}{n}} . \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]}{n-1} \qquad \text{... ... (10)}$$

C. Sizing of an Accumulator

In an accumulator at any instant we either compress a pre-charged gas or allow it to expand. This compression or expansion brings change in state of a gas which is governed by ideal gas equation (11):

$$pv = mRT \qquad \text{... ... (11)}$$

P Pressure in bar, *v* volume in   , *T* means temperature and *mR* for a particular gas (nitrogen) is constant .Then the equation turns out to be as:

$$\frac{pv}{T} = \text{constant} \qquad \text{... ... (12)}$$

In isothermal condition i.e. in compression

$$p_0 v_o = p_1 v_1 = p_2 v_2 \qquad \text{... ... (13)}$$

While in case of adiabatic condition

... ... (14)

1) Sizing accumulators for isothermal condition

For isothermal condition,

$$v_1 = v_o \frac{p_o}{p_1} \qquad \text{... ... (15)}$$

$$v_2 = v_o \frac{p_0}{p_2} \qquad \text{... ... (16)}$$

The difference between v_1 at the minimum operating pressure p_1 and v_2 at the maximum operating pressure p_2 gives the amount of the stored fluid Dv = =

$$\text{Dn} = v_0\left(\frac{p_0}{p_1} - \frac{p_0}{p_2}\right)$$

$$v_o = \frac{\Delta v}{\left(\frac{p_0}{p_1}\right) - \left(\frac{p_0}{p_2}\right)} \qquad \text{... ... (17)}$$

υ_1, υ_2 are nitrogen volumes at the pressures and v_0 is the nitrogen precharge volume at the pressure p_o in litres . It is the maximum volume of the gas that can be stored in the accumulator. While conforming to standard sizes a tolerance level of 5-10% higher than volume v_o .

2) Sizing of accumulators under adiabatic condition

Starting from the basic formula $p_0 v_0^n = p_1 v_1^n = p_2 v_2^n$, values of maximum value v_0 at the precharge pressure p_0 and the stored volume of the given oil is given by the equation:

$$\Delta v = v_0\left[\left(\frac{p_0}{p_1}\right)^{0.7143} - \left(\frac{p_0}{p_2}\right)^{0.7143}\right] \qquad \text{... ... (18)}$$

III. Conclusion

This paper involves energy and heat transfer equations taking place inside accumulatorand thermal characteristics of bladder accumulators while undergoing compression and expansion in terms of gas properties and sizing of accumulators under isothermal and adiabatic conditions are explained with the help of mathematical equations. This a general approach to understand the sizing of accumulators. Practical values may differ with the theoretical values obtained with the help of above equations.

List of symbols			
$A_o, B_o, C_o, a, b, c, \alpha, \gamma$	Coefficients of BWR	v	Specific volume of the gas
C_v	Specific heat at constant volume	z	Compressibility coefficient
E	Energy of compression	p_o	Pre-charge pressure
A	Convective heat transfer area	p_2	Maximum operating pressure
h	Coefficient of convective heat transfer	p_1	Minimum operating pressure
K	Thermal conductivity of the accumulator wall	v_o	Maximum accumulator capacity
	Flux of work	v_1	Gas volume at p_1
	Flux of viscous losses	v_2	Gas volume at p_2
	Mass of the gas	Δv	Delivered and/ or absorbed recovery volume between and p_2
n	Adiabatic exponent		
p	Pressure	**Subscripts' Used**	
q	Flux of heat	g	Gas
R	Thermal resistance gas constant	exp	Exponential
T	Temperature	i	Internal
t	Time	*w*	Hydraulic fluid
u	Internal energy		

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various losses occurring in the form of flow loss and torque loss during the operation of the pump. Numerical expressions in the form of mathematical equations have been provided to analyze the characteristics of the pump. Expressions for volumetric as well as mechanical efficiencies have been generated to calculate the overall efficiency, which has been presented in a graphical form. Ultimately the optimum condition pertaining to maximum efficiency has been laid out.

Nomenclature

b	=	gear width
B_v	=	viscous friction coefficient
d	=	piston diameter
D_p	=	pump displacement
l	=	stroke length
P_1	=	outlet pressure
P_2	=	inlet pressure
$\overline{P_l}$	=	load pressure
P_f	=	friction pressure loss term
P_q	=	leakage pressure loss term
Q	=	pump flow rate
Q_c	=	fluid compressibility flow loss
Q_{cp}	=	cross-port leakage loss
Q_{ext}	=	external flow loss
r_p	=	pitch circle radius
R_{cp}	=	cross-port resistance
R_{ext}	=	external resistance
R_{tim}	=	timing groove resistance
T_c	=	coulomb friction running torque
T_{loss}	=	torque loss
T_{sc}	=	striction-coulomb friction torque loss function
α	=	torque loss constant
β	=	fluid effective bulk modulus
η_m	=	mechanical efficiency
η_v	=	volumetric efficiency
ψ	=	pressure angle
ω	=	pump speed

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