K. PRIYA AJIT
ABHINAV GAUTAM
and
P. K. SARKAR

Fatigue damage estimation through continuum damage mechanics revisited

This paper presents an overview of estimation methodologies for predicting fatigue damage using continuum damage mechanics (CDM). It is comparatively a new tool for fatigue damage estimation. It is still progressing from inception in last three decades. During this period, many models are developed and discussed by several authors. The present paper is a review of some of well accepted models for estimation of fatigue damage to predict safe life of a structure.

Keywords: *Fatigue damage; CDM.*

Introduction

Damage mechanics is currently one of the major area of concern for solving many engineering problems. Damage, in its mechanical sense in solid materials, is the creation and growth of micro voids or micro cracks which are discontinuities in a medium treated to be continuous at a larger scale [1]. It involves heterogeneous micro processes during straining the materials and structure at macroscale. Damage process correspond to localizations and accumulations of strains those are irreversible in nature. Kachanov in 1958 first proposed a continuous variable in terms of density of defects [2]. This variable has constitutive equation for evolution, which can be written in terms of stress or strain and may be used in calculation in order to predict the initiation of micro cracks. This concept has been further improved by incorporation of framework of thermodynamics of irreversible process by Lemaitre [1] and initiated a new concept of continuum damage mechanics. This concept opens the door for complementary possibilities to fracture mechanics. This model is further enriched by many researchers for describing various types of material damage related to fatigue damage, ductile damage and creep damage.

Fatigue damage in metals is caused by the micro crack formation in accumulated slip bands due to repeated loading. This concept of damage analysis play a key role in predicting the life of component subjected to field load history. More than 90 years ago Palmgren [3] introduces the damage accumulation concepts, which is known as linear damage rule. After 20 year, in 1945 miner [4] got success to expresses this concept in terms of mathematical expression. Since then this concept gets more attention and a lot of researcher extend their work in this field. In the course of time a lot of models have been developed but none of them is accepted universally. On the basis of published result fatigue damage prediction method can be classified into six groups [5]. These can be stated as (a) linear damage evolution and linear summation, (b) nonlinear damage curve and two-stage linearization approaches, (c) life curve modifications to account for load interactions, (d) approaches based on crack growth concept, (e) models based on CDM and (f) energy-based methods. This paper present an overview of CDM approach to fatigue damage prediction..

Messrs. K. Priya Ajit, Mining Machinery Engineering, Abhinav Gautam, and P. K. Sarkar, Mechanical Engineering, Indian School of Mines, Dhanbad, India. E-mail: ajit160984@gmail.com, abhinav980026898@gmail.com, sarkarpk1955@yahoo.co.in

CDM apporach to estimate fatigue damage

CDM approach is not only applicable to metallic materials but also to composites and concrete. Wide applicability of this approach encouraged many researchers to apply and develop the CDM model in different form of fatigue damage. The first fatigue damage model based on CDM is delivered by Chaboche [6,7]. This one-dimensional nonlinear continuous damage model is presented as a function of the load condition and damage state. The nonlinear damage model is formulated as:

$$D = 1 - \left[1 - r^{1(1-\alpha)}\right]^{1/(1+\beta)} \qquad \text{... ... (1)}$$

Here β is a material constant, α is a function of the stress state and r is isotropic hardening state variable. Parameters are evaluated on the findings of completely reversed strain controlled fatigue test and tensile test. Effective stress concept is used for finding the parameters. This model have some advantages over the previous exiting model. First is the damage growth can be absorbed below the fatigue limit, second, the model can easily consider the influence of initial hardening effect which consider the large plastic strain range in the prior loading history and third the mean stress effect is directly incorporated in the model. It has some limitations [8], since the model is stated for uniaxial form with scalar damage variable so it is difficult to application of the model for multiaxial case of loading.

After this model many authors developed fatigue damage model based on CDM concept for specific purpose. All these models are almost similar to chaboche model, differences are on the basis of parameters used in specific model. Lemaitre and Plumtree [9] developed the unified model for high temperature failure (eqn 2). Where *p* is accumulated plastic strain. LCF (low cycle fatigue), HCF (high cycle fatigue) and monotonic creep tests were carried out on OFHC copper at 540°C and Sanicro 31 (alloy 800) at 600°C. Damage model shows good agreement with the experimental results. This model is further used by Socie et.al [10] for the evaluation of fatigue damage of cast iron. It gives better result than the miner's linear damage rule. Hua and Socie [11] present a comparison of chaboche and Lemaitre-Plumgreen model for biaxial fatigue study. They reported chaboche model gives better result. Other uniaxial fatigue model base on effective stress concept were developed in course of time by Lemaitre and Chaboche [12], Wang [13], Wang and Lou [14]. These model is similar to above discussed model of chaboche. The models differs by the variable used in the equations.

$$D = 1 - (1 - r)^{1/(1+p)} \qquad \text{... ... (2)}$$

Chow and Wie [15] proposed a generalized fatigue damage model by introducing damage tensor. This is the first model which can be used for the multiaxial loading condition. The fatigue evolution based on overall fatigue damage which includes both the elastic and plastic damage. Damage evolution equation is formulated using an appropriate damage dissipative potential and fatigue damage criterion. Constitutive equation of elasticity and plasticity coupled with damage is verified by comparing the experimental result. Tensile test of Al 2024 T3 conducted to find out effective elasticity and Poisson's ratio which is further used to find out the other material parameters. Model is used to predict the number of cycles to failure of aluminum 2024-T3; it shows excellent agreement.

A model for low cycle fatigue is presented by June Wang [16]. Variation of damage parameter with cycle is considered for the damage model (eqn 3). Where N_f is the number cycle to failure and is material parameter which depends upon stress ratio R. Three different stress ratios, namely R = -0.2, 0, and 0.2, were chosen for the experimental investigation. Rupture time of hot rolled steel (RS 50) is predicted by the damage model and compared to experimental result. The proposed damage model is found to be more physically reasonable as the stress amplitude approaches zero.

$$D = 1 - \left(1 - \frac{N}{N_f}\right)^{1/n(R)+1} \qquad \text{... ... (3)}$$

Low cycle fatigue damage evolution studied by Yang et.al [17] and proposed a model, based on the model developed by Chaboche and Lemiatre [12]. The model almost similar to LCF model of June Wang. Damage variable (*D*) is calculated based on the concept of effective stress (eqn 4), where $\Delta\sigma_0$ is stress of initial cycle and $\Delta\sigma$ is the stress as cycle grow. Derived isotropic CDM model is used to analyze the strain-controlled LCF damage evolution of steam turbine blade material 2Cr13 steel. The evolution of microstructure is studied using TEM (transmission electron microscopy). The microscopy analysis shows that the initial increment of damage value during fatigue is caused by the evolution of dislocation substructure. After the formation of stable dislocation cell structure, the damage increases very slowly until 80% N_f, then it increases quickly because of the nucleation of fatigue micro cracks. At the end of cycling the damage value is:

$$D = 1 - \frac{\Delta\sigma}{\Delta\sigma_0} \qquad \text{... ... (4)}$$

Xiao et.al [18] proposed a CDM model of high cycle fatigue with brittle damage mechanism. It is based on thermodynamic framework developed by Lemaitre and Choboche. This model can be used, where elastic strain is dominating factor of fatigue damage. Model is verified experimentally, with help of Woehler's curve for aluminum alloy 7075-T6. This model is derived from a brittle damage mechanism, so it really cannot be used to cope with strain control problems such as low cycle fatigue and creep which have to describe using ductile damage model.

Bhattacharya and Elinwood [19] describe the fatigue life of defect free component consists of crack initiation and fatigue crack propagation period. Significant portion of total life of a component is associated to the crack initiation period. Three different model for fatigue damage growth is discussed by author (a) Isotropic fatigue damage growth under uniaxial loading (b) Fatigue damage growth under constant amplitude loading (c) Fatigue damage growth under variable loading. Principal of thermodynamics and soild mechanics is used to derive the model with consideration of isotropic damage. Crack initiation behaviour is predicted with help of CDM damage variable. Fatigue damage is computed recursively as a function of fatigue cycle for four different materials with the help of their material properties. Both stress and strain controlled fatigue life is compute and compared well to experimental findings. This model is further used by Upadhyaya and Sridhara [20] to predict the fatigue life of ferrous alloy E19. All basic mechanical testing is conducted for the study which includes Monotonic test, cyclic test, low cycle fatigue test, fracture toughness test and fatigue crack growth test.

Nonlinear damage CDM model for fracture mechanics proposed by Bonora, N. [21] is further extended to the case of cyclic loading. Taking into account that total strain as a combination of elastic and plastic strain and total fatigue life include crack initiation as well as propagation. Three possible formulations are proposed and discussed that take into account in different ways the accumulation of damage, plastic strain and the material cyclic properties change. Fully coupled life model is used to predict low cycle fatigue life in AI 2024 T3 alloy and HYS0 low carbon steel. Comparison with a large fatigue experimental data set is also presented. The model, that has been shown to be particularly well suited to describe the evolution of ductile damage as a function of strain in metals, is able to predict the effect of cycling loading in the low cycle fatigue range. The fully coupled life model seems to be more consistent from the conceptual point of view and found a better agreement with the experimental data available in the literature.

According to varying behaviour of material ductility in process of fatigue a nonlinear fatigue damage model based on CDM is proposed by Sangh and Yao [22]. Recurrence formula of fatigue damage model is derived under multilevel loading. Proposed model is verified by experimental data. For multilevel loading, two categories experimental data of smooth and notched specimen of both the material are used to verify the case of two level loading. The tests were performed on PQ 1-6 rotating bending testing machine with the stress controlling and fully reversed loading condition and good result is obtained, though the results are not verified for different materials but it suits well for ductile material and further study is needed for brittle material.

A similar fatigue model based on continuum damage mechanics (CDM) approach is formulated in terms of accumulation of plastic strain with a number of cycles by Abilio et.al [23]. It predicts a nonlinear damage evolution, with the number of cycles. The fatigue model is implemented in a nonlinear finite element code in conjunction with a continuum plasticity model with multiple nonlinear kinematic hardening variables. Result are verified with both wohler and damage evolution curve of P355NL1 steel. Further this model is applied to fatigue analysis of nozzle to plate attachment of the P355NL1 steel. Predicted results are verified by published experimental data.

A modified nonlinear CDM model based on the framework of Lemaitre and Chaboche is proposed by Dattoma et.al [24]. Formulated model taking into account the material damage evolution at different sequence of load application and considering damage initiation are lower than fatigue limit. Fatigue tests data for hardened and tempered steel have been used to verify the proposed model.

Conclusions

A short but effective overview of fatigue damage estimation through CDM is presented. More than 20 fatigue models have been proposed since the Chaboche's nonlinear continuous damage model (NLCD). Most of the models are based on the initial work of Lemaitre and chaboche. Four categories of fatigue damage model based on CDM is discussed:

1. Model based on nonlinear uniaxial damage model
2. Low cycle fatigue damage model
3. High cycle fatigue damage model
4. Fatigue life estimation based on two stage, crack initiation and crack propagation

No clear boundary exists among these approaches. Approaches based on the nonlinear continuum damage has wide degree of acceptance and most of the models developed are based on this concept. Most of models are verified experimentally and some of the recent works are verified numerically as well as using the finite element approach.

A number of fatigue damage models are developed but unfortunately not a single model has universal acceptance. Each of model accounts for one or several phenomenological factors like load dependence, load sequence, nonlinear damage, small amplitude cycle below fatigue limit, influence of hardening exponent, mean stress etc. Some models are applicable to specific materials. Due to complicity none of the fatigue damage models incorporates the factors all together. Nonlinear continuous damage model developed by Lemaitre and Chaboche is still dominantly used for analysis. More effort in the study of fatigue damage is needed to predict reliable life estimation and developing a unified damage model.

References

1. Lemaitre and Desmorat (2005): Engineering Damage Mechanics, Springer-Verlag Berlin Heidelberg 2005.
2. Kachanov, L. M. (1984): "Time to the rupture process under creep conditions," *Izvestiia AN SSSR, OTN(8)*, pp.26-31,1984.
3. Palmgren, A. (1924): Die Lebensdauer von Kugellagern. Veifahrenstechinik, Berlin, vol.68, 339-341.1924.
4. Miner, M. A. (1945): "Cumulative damage in fatigue," *Journal of Applied Mechanics*, vol.67, pp. AI59-AI64,1945.
5. Fatemi and Vang, L. (1998): "Cumulative fatigue damage and life prediction theories: a survey of the state of the art for homogeneous materials," *Int. J. Fatigue* Vol. 20, No. I, pp. 9-34, 1998.
6. Chaboche, J. L. (1987): "Continuum damage mechanics and its application to structural lifetime predictions," Recherchea 'erospatiale, English edition, vol.4, pp.37-54, 1987.
7. Chaboche, J. L. (1974): "A differential law for nonlinear cumulative fatigue damage," In Materials and Building Research, Paris Institut Technique Du Batiment Et Des TravausPublies, Annales de l'ITBTP, HS No. 39, pp. 117-124, 1974.

8. Chaboche, J. L. and Lesne, P. M. (1988): "A non-linear continuous fatigue damage model," *Fatigue and Fracture of Engineering Materials and Structures*, vol.11(1), pp.1-7, 1988.

9. Lemaitre, J. and Plumtree, A. (1979): "Application of damage concepts to predict creep-fatigue failures," *ASME Journal of Engineering,Materials and Technology*, vol.101, pp.284-292, 1979.

10. Socie, D. F., Fash, J. W. and Leckie, F. A. (1983): "A continuum damage model for fatigue analysis of cast iron," In Advances in Life Prediction Methods, ed, D. A. Woodford and J, R. Whitehead. The American Society of Mechanical Engineers, New York, pp. 59-64, 1983.

11. Hua, C. T. and Socie, D. F. (1984): "Fatigue damage in 1045 steel under constant amplitude biaxial loading," *Fatigue of Engineering Materials and Structures*, vol. 7(3), pp.165-179, 1984.

12. Lemaitre, J. and Chaboche, J. L. (1978): "Aspect phenomenologique de la ruptutre par endommagement," *Journal Mecanique Appliquee*, vol. 2(3), pp.317-365, 1978.

13. Wang, J. (1992): "A continuum damage mechanics model for low-cycle fatigue failure of metals," *Engineering Fracture Mechanics*, vol. 41(3), pp.437-441, 1992.

14. Wang, T. and Lou, Z. (1990): "A continuum damage model for weld heat affected zone under low cycle fatigue loading," *Engineering Fracture Mechanics*, vol.37 (4), pp.825-829, 1990.

15. Chow, C. L. and Wei, Y. (1991): "A model of continuum damage mechanics for fatigue failure," *International Journal of Fracture*, vol.50, pp.301-316, 1991.

16. Wang, June (1992): "A Continuum Damge Mechanics Model For Low-Cycle Fatigue Failure Of Metals," *Engineers Fracture Mechanics*, vol. 41(3), pp. 437-441, 1992.

17. Yang, Xiaohua, Li, Nian, Jin, Zhihao and Wang, Tiejun (1997): "A continuous low cycle fatigue damage model and its application in engineering materials," *Int. J. Fatigue*, Vol. 19(3) pp. 687-692, 1997.

18. Xiao, Y.-C., Li, S. and Gao, Z. (1998): "A continuum damage mechanics model for high cycle fatigue," *Int. J. Fatigue*, vol. 20(7), pp. 503-508, 1998.

19. Bhattacharya, Baidurya and Ellingwood, Bruce (1998): "Continuum damage mechanics analysis of fatigue crack initiation," *Int. J. Fatigue*, vol. 20(9), pp. 631-639, 1998.

20. Upadhyaya, Y. S. and Sridhara, B. K. (2012): "Fatigue life prediction: A continuum damage mechanics and fracture mechanics approach," *Materials and Design*, vol.35, pp.220-224, 2012.

21. Bonora, N. and Newaz, G. M. (1998): "Low cycle fatigue life estimation for ductile metals using a nonlinear continuum damage mechanics model," *Int. J. Solids Structures*, Vol. 35(16), pp. 1881-1894, 1998.

22. Shang, De-Guang and Yao, Wei-Xing (1999): "A nonlinear damage cumulative model for uniaxial fatigue," *International Journal of Fatigue*, vol. 21, pp.187-194, 1999.

23. Jesus, Abilio M. P. De, Riberio, Alfredo S. and Fernandes, Antonio A. (2005): "Finite element modeling of fatigue damage using continuum damage mechanics," *Journal of Pressure Vessel Technology*, vol.127, pp.157-164, 2005.

24. Dattoma, V., Giancane, S., Nobile, R. and Panella, F. W. (2006): "Fatigue life prediction under variable loading based on new non-linear continuum damage mechanics model," *International Journal of Fatigue*, vol. 28, pp.89-95, 2006.

FATIGUE DAMAGE ESTIMATION THROUGH CONTINUUM DAMAGE MECHANICS

(Continued from page 213)

- For a combined torsional and pressure loading the angle ply-orientation (Gr/E [+45/-45]2) is most preferable.

References

1. Zou, G. P. and Taheri, F. (2006): "Stress analysis of adhesively bonded sandwich pipe joints subjected to torsional loading" *Int. J. Solids Struct.* 43, 5953-5968 (2006).

2. Das, R. R. and Pradhan, B. (2011): "Finite Element Based Design and Adhesion Failure Analysis of Bonded Tubular Socket Joints Made with Laminated FRP Composites," *J. Adhes. Sci. Technol.* 25, 41-67 (2011).

3. Lees, J. M. (2004): Combined pressure/tension behaviour of adhesive-bonded GFRP pipe joints. In: Hollaway LC, Chryssanthopoulos MK, Moy SSJ, editors. Advanced polymer composites for structural applications in construction, ACIC. Woodhead Publishing Ltd; p. 377-383, (2004)..

4. Panigrahi, S. K. and Pradhan, B. (2007): "Delamination damage analyses of adhesively bonded lap shear joints in laminated FRP composites," *International Journal of Fracture*, 148, pp. 373-385 (2007).

5. Esmaeel, R. A. and Taheri, F. (2009): "Infuence of adherend's delamination on the response of single lap and socket tubular adhesively bonded joints subjected to torsion" *J Adhes Sci Technol*, 23 p. 1827-1844 (2009).