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Strength criterion effect on the mechanical response of elasto-brittle plastic rock mass considering different post-peak elastic strain forms

The strength criterion is an extremely important basis for evaluating the stability of surrounding rock and optimizing the support pressure design. In this paper, nine different strength criterions are summarized and simplified based on the reasonable assumption. And then, a new unified criterion equation is established which includes all of the strength theories proposed by this paper. Meanwhile, a new unified closed-form solution for circular opening based on the newly proposed unified criterion equation is deduced with the non-associative flow rule under plane strain conditions. In the plastic zone, four different elastic strain assumptions are applied to solve the plastic zone deformation. The validity of the solution is also verified by comparing with the traditional solution. Finally, the influences of strength criterions, dilation coefficient, elastic strain form of plastic zone and rock mass damage on the mechanical response of surrounding rock are discussed in detail. The research result shows that Tresca(TR) and Von Mises (VM) criterions give a largest plastic zone radius, followed by Inscribe Drucker-Prager (IDP), Mohr-Coulomb (MC) and Middle Circumscribe Drucker-Prager (MDP) criterions and seem to underestimate the self-strength of rock mass. Circumscribe Drucker-Prager (CDP) criterion gives a smallest plastic zone radius and may be overestimated the self-strength of rock mass; Unified Strength Theory (UST0.5), Generalized SMP criterion (GSMP), Mogi-Coulomb (GMC) and Generalized Lade-Duncan (GLD) criterions which reasonably consider the effect of internal principal stresses give an intermediate range and can be strongly recommended for evaluating the mechanics and deformation behaviour of surrounding rock. As the dilation coefficient gradually increases, the dimensionless surface displacement presents the nonlinear increase characteristics; The deformation of plastic zone which are closely related to the strength criterions, are also greatly influenced by the elastic strain assumption in the plastic zone and rock mass damage degree. The assumption that the elastic strain satisfies Hook's law (Case 3) may be more reasonable compared to the continuous elastic strain (Case 1) and thick-walled cylinders (Case 2) assumptions. In addition, the Young's modulus power function damage model seems to give more reasonable solution for the deformation of plastic zone and is suggested to be a preferred method for solving plastic displacement. The research results can provide very important theoretical basis for evaluating the tunnel stability and support design reliability for different lithology rock mass in underground engineering.

Keywords: *Unified criterion equation; circular tunnel; post-peak elastic strain; Young's modulus attenuation*

1. Introduction

Accurate prediction for stresses and displacement distribution of circular opening play a crucial role in evaluating the mechanics and deformation behaviour of rock mass in civil, mine, oil engineering and natural gas development engineering. The circular opening may include tunnel, vertical shaft and boreholes. As for most of the analyses reported in the past, the solution are given considering different yield criterion, like linear Mohr-Coulomb (MC), Tresca (TR) and nonlinear Hoek-Brown (HB) criterion [1-9]. Nevertheless, above studies didn't consider the effect of intermediate principal stress on the mechanical response of surrounding rock. Many research results have shown that the intermediate principal stress has significant influence on the failure behaviour of rock mass [12-14, 17, 18]. In addition, rock mass as a natural geological material, the yield failure criterion is more complicated under the influence of internal crack and joint. Therefore, it is extremely difficult to reconcile the calculation results with the field measured results if only one or two yield criterion is used to predict the stresses and displacement behaviour of surrounding rock. In this paper, as shown in Table.1, different yield criterion, such as Mogi-Coulomb (GMC) [10-11], Drucker-Prager (CDP, MDP, IDP) [12-15] , Generalized Lade-Duncan (GLD) [16], Generalized SMP (GSMP) [17-19] and Von Mises (VM) [3, 20], unified strength (UST) criterions [26-28] et al, will be summarized and simplified, then used to

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study the mechanical response of rock mass.

For an elastic-brittle plastic rock mass, the post-peak deformation is closely related to the assumed form of elastic strain in the plastic zone. Brown and Braw et al. researched the ground response curve for the rock tunnel by assuming that the elastic strain in the plastic region was equal to that on the elastic-plastic interface [21]. Sharan presented a series of new closed-form solution for the prediction of displacements around circular openings in a brittle rock mass with nonlinear Hoek-Brown criterion by regarding the elastic strain of plastic zone as the thick-walled cylinder [22]. Yu, Zhang and Reed et al hold that the elastic strain in the plastic zone satisfies the generalized Hooke's theorem, and then derived the deformation of plastic zone with non-associative flow laws [23-25]. In addition, Park summarized above three different definitions for elastic strains in the plastic zone and analyzed the deformation law of plastic zone with Mohr-Coulomb and Hoke-Brown criterion under three different cases conditions [7]. However, the post-peak Young's modulus attenuation along the radii direction is ignored in this study.

In this paper, different yield criterion of rock mass is firstly summarized, and then a unified yield criterion forms are derived by simplifying above criterions. Next, a new closed-form solution for stresses and deformation distribution around a circular opening subjected to the hydrostatic pressure is also obtained with the new proposed unified yield criterion. In the plastic zone, five different definitions for elastic strains in the plastic zone and non-associated flow rule are adopted to establish the radii displacement solution. The correctness of the solution is also verified by comparing with a series of traditional solution. Finally, the influences of strength theory and elastic strain definitions of plastic zone on the mechanical response of surrounding rock are discussed in detail.

2. Brief description of main yield criterion

2.1 Mohr-Coulomb criterion

The linear Mohr-Coulomb (MC) criterion ignored the influence of intermediate principal stress had been widely used in geotechnical engineering. The governing equation for MC criterion based on the cohesion (*c*) and internal friction angle (φ) can be expressed as following [1-2].

$$\sigma_1 = N\sigma_3 + \sigma_c \quad \text{... (1)}$$

Where *N* is a constant which is a function of the internal friction angle and σ_c is the uniaxial compressive strength (USC). They can be expressed as following:

$$N = \frac{1+\sin\varphi}{1-\sin\varphi} \quad \text{... (2)}$$

$$\sigma_c = \frac{2c\cos\varphi}{1-\sin\varphi} \quad \text{... (3)}$$

2.2 Tresca criterion

Tresca (TR) criterion is a simplified form of MC criterion and assumes failure occur if the maximum shear stress (τ_{max}) inside any plane of rock mass reaches a critical value. The expression form is

$$\frac{\sigma_1 - \sigma_3}{2} = \tau_{\max} = c \quad \text{... (4)}$$

2.3 Generalized Lade-Duncan criterion

In the early 1973, Lade-Duncan criterion has been firstly proposed by considering the intermediate principal stresses based on the triaxial compression test for non-cohesive soil. Then, since 1999, Lade-Duncan criterion was modified by Ewy so that it can reasonably describe the strength characteristics of cohesive soil by introducing bound stress (σ_0). Its generalized expression is also given as [16].

$$\sigma_1 + \sigma_0 = \frac{1}{4}[\sqrt[3]{K_{\text{Lade}}} - 1 + \sqrt{(\sqrt[3]{K_{\text{Lade}}} - 1)^2 - 4}]^2(\sigma_3 + \sigma_0) = \eta_{\text{Lade}}(\sigma_3 + \sigma_0) \quad \text{... (5)}$$

Where, $\sigma_0 = c\cot\varphi$; $K_{\text{Lade}} = \dfrac{(3-\sin\varphi)^3}{(1+\sin\varphi)(1-\sin\varphi)^2}$;

$$\eta_{\text{Lade}} = \frac{1}{4}[\sqrt[3]{K_{\text{Lade}}} - 1 + \sqrt{(\sqrt[3]{K_{\text{Lade}}} - 1)^2 - 4}]^2$$

2.4 Generalized SMP criterion

Considering the effect of internal principal stress, H. Matsuoka and T. Nakai proposed SMP criterion which can be expressed by three principal stress invariants [19]. Then, according the coordinate translation method, the generalized SMP criterion is also deduced as following [18].

$$\begin{aligned}\sigma_1 + \sigma_0 &= \frac{1}{4}[\sqrt{K_{\text{SMP}}} - 1 + \sqrt{K_{\text{SMP}} - 3 - 2\sqrt{K_{\text{SMP}}}}\,]^2(\sigma_3 + \sigma_0) \\ &= \eta_{\text{GSMP}}(\sigma_3 + \sigma_0)\end{aligned} \quad (6)$$

Where,

$$K_{\text{SMP}} = 8\tan^2\varphi + 9\,;\ \eta_{\text{GSMP}} = \frac{1}{4}[\sqrt{K_{\text{SMP}}} - 1 + \sqrt{K_{\text{SMP}} - 3 - 2\sqrt{K_{\text{SMP}}}}\,]^2$$

2.5 Mogi-Coulomb criterion

Based on the Mogi's theory [10], Al-Ajmi and Zimmerman found that the polyaxial test data can be fitted by linear relationships in $\tau_{act} - \sigma_{m,2}$ space.

$$\tau_{oct} = a\sigma_{m,2} + b \quad \text{... (7)}$$

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2} \quad \text{... (8)}$$

$$\sigma_{m,2} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \quad \text{... (9)}$$

Where, $\sigma_{m,2}$ is mean normal stress and τ_{oct} is octahedral shear stress. The parameters *a* and *b* can be related exactly to the Coulomb strength parameter.

$$a = \frac{2\sqrt{2}}{3}\sin\varphi, b = \frac{2\sqrt{2}}{3}c\cos\varphi \quad ... (10)$$

As the Von Mises criterion, we take $\sigma_2 = (\sigma_1 + \sigma_3)/2$. Then, the Mogi-Coulomb (GMC) criterion can be obtained by integrating Eqs.(7), (8) and (9).

$$f(\sigma_1, \sigma_3) = \sigma_1 - \frac{\sqrt{6}+3a}{\sqrt{6}-3a}\sigma_3 - \frac{6b}{\sqrt{6}-3a} = 0 \quad ... (11)$$

2.6 Unified Strength Theory

Based on the twin shear yield criterion, the unified strength theory (UST) is established by considering the influence of all the stress components on the material yield failure [26-28]. In geotechnical engineering, the cohesion and internal friction angle are usually used to represent this yield theory. The yield function can be expressed as follows:

If $\sigma_2 \le \frac{\sigma_1+\sigma_3}{2} - \frac{\sigma_1-\sigma_3}{2}\sin\varphi$

$$f(\sigma_1, \sigma_3) = \frac{1-\sin\varphi}{1+\sin\varphi}\sigma_1 - \frac{b\sigma_2+\sigma_3}{1+b} = \frac{2c\cos\varphi}{1+\sin\varphi} \quad ... (12)$$

If $\sigma_2 \ge \frac{\sigma_1+\sigma_3}{2} - \frac{\sigma_1-\sigma_3}{2}\sin\varphi$

$$f(\sigma_1, \sigma_3) = \frac{1-\sin\varphi}{(1+b)(1+\sin\varphi)}(\sigma_1 + b\sigma_2) - \sigma_3 = \frac{2c\cos\varphi}{1+\sin\varphi} \quad ... (13)$$

Where, b represents the yield parameter related to the intermediate principal stress, which can reflect the effect of the intermediate principal shear stress and the positive stress on the yield failure of the rock material, and $0<b<1$. As the Fig.3, if $b = 0$, UST will translate into the MC criterion; if $b=1$, UST is converted into general twin shear strength (GTSS) criterion; if $0<b<1$, UST is a series of other ordered new strength criteria.

Just like sections 2.5, if,can be judged by substituting into Eq. (12a) or (12b), Therefore, UST can be rewritten as:

$$f(\sigma_1, \sigma_3) = \sigma_1 - \frac{2+b+(2+3b)\sin\varphi}{(2+b)(1-\sin\varphi)}\sigma_3 - \frac{4(1+b)c\cos\varphi}{(2+b)(1-\sin\varphi)} = 0 \quad (14)$$

2.7 Drucker-Prager criterion

Drucker-Prager (DP) criterion is an extension of the Von Mises criterion, which takes into account the effect of intermediate principal stress and hydrostatic pressure on yield failure of the materials. It can be expressed as

$$f(I_1, J_2) = \sqrt{J_2} + \alpha I_1 - k = 0 \quad ... (15)$$

Where, I_1 and J_2 are respectively the first principal stress invariants and the second deviator stress tensor.

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 \quad ... (15)$$

$$J_2 = \frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_1-\sigma_3)^2}{6} \quad ... (16)$$

The parameters a and k are the material constants, which can be determined from the slope and the intercept of the failure envelope. The parameter k is related to the cohesion and internal friction angle of rock mass. The parameter a is only related to the friction angle. Therefore, the Mohr-Coulomb parameter can be used to descript the DP criterion. By comparing with the yield curve of MC criterion in the π-plane (Fig.1), the DP criteria can be divided into Circumscribe Drucker-Prager (CDP) criterion, Middle Circumscribe Drucker-Prager (MDP) criterion and Inscribe Drucker-Prager (IDP) criterion.

- When the yield curve of DP criterion is the circumcircle of MC criterion, the material parameter and for CDP criterion can be obtained by Deng and Zhang [12,14].

$$\alpha = \frac{2\sin\varphi}{\sqrt{3}(3-\sin\varphi)}, k = \frac{6c\cos\varphi}{\sqrt{3}(3-\sin\varphi)} \quad ... (17a)$$

- When the yield curve of DP criterion is the inscribed circle of MC criterion, the solution of a and k parameter for IDP criterion presented by Vekeens and Walters [15]

$$\alpha = \frac{\sin\varphi}{\sqrt{(9+3\sin^2\varphi)}}, k = \frac{\sqrt{3}c\cos\varphi}{\sqrt{(3+\sin^2\varphi)}} \quad ... (17b)$$

- When the yield curve of DP criterion is between CDP and IDP, the material parameter a and k for MDP criterion will be expressed as follows

$$\alpha = \frac{2\sin\varphi}{\sqrt{3}(3+\sin\varphi)}, k = \frac{6c\cos\varphi}{\sqrt{3}(3+\sin\varphi)} \quad ... (17c)$$

As shown in sections 2.6, the Drucker-Prager criterion can be deduced by taking $\sigma_2 = (\sigma_1 + \sigma_3)/2$ into Eq. (14).

$$f(\sigma_1, \sigma_3) = \sigma_1 - \frac{1+3\alpha}{1-3\alpha}\sigma_3 - \frac{2k}{1-3\alpha} = 0 \quad ... (18)$$

2.8 Establishment of unified criterion equation

As shown in Table.1, the unified equation of different yield criterion can be summarized based on the above analysis under plane strain conditions.

$$f(\sigma_1, \sigma_3) = \sigma_1 - \xi_j\sigma_3 - Y_j = 0 \quad ... (19)$$

Where, the subscript "j" represents different yield criterions. ξ_j and Y_j are the material constants of different criterions which can be divided into two different cases. When ξ_j=1, the Eq.(16) corresponds to the TR and VM criterions; when ξ_j≠1,the Eq.(16) represents the other criterions. Therefore, it can be regarded as a unified criterion equation to research the mechanical response of rock mass.

3. Problem description

3.1 Establishment of calculating model

Fig.2 shows that a circular opening excavated in a finite, isotropic, homogeneous elastic-brittle plastic rock mass

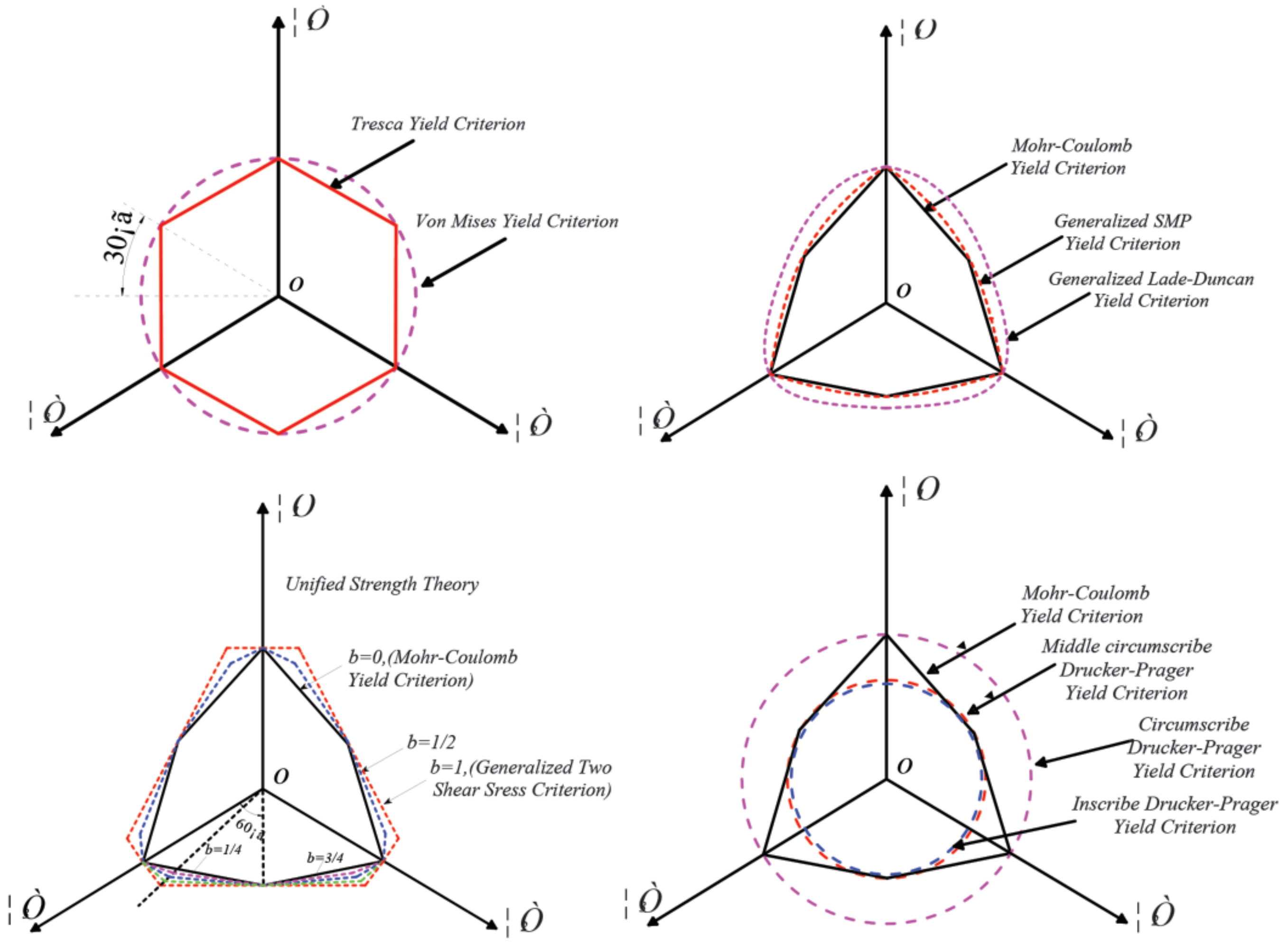


Fig.1 Different yield criterion curves in the π-plane under plane strain condition

subjected to an inner pressure σ_0 at the inner radii R_0 and a hydrostatic pressure p_0 at the external radius R_2. As σ_0 gradually decrease, the displacement will occur and the plastic zone with the radii R_1 firstly develops around the circular opening when the maximum principal stress and minimum principal stress satisfy the initial yield condition. The influence of the rock mass weight in the plastic zone on the radial displacement and inner pressure is ignored. In this paper, the brittle plastic rock mass is introduced to research the post-peak mechanical behavior of rock material. As shown in Fig.3, the strength of the rock mass suddenly drops after peak-load and the post-peak softening behavior of strength parameters will occur. In other words, the post-peak cohesion c_{res}, internal friction angle φ_{res}, Young's modulus E_{res} and Poisson's ratio v_{res} are used to solve the stress and displacement distributions in the plastic zone.

Under axisymmetric plane strain condition, when $p_0 > \sigma_0$, the hoop stress σ_0 and radial stress σ_r are respectively maximum principal stress and minimum principal stress; the tangential strain ε_θ and radial strain ε_r are respectively maximum and minimum principal strain. Accordingly, the unified criterion equation can be rewritten as follows:

$$\sigma_{\theta(i)} = \xi_j^{ini}\sigma_{r(i)} + Y_j^{ini} \quad \text{For peak stage} \qquad \text{... (20a)}$$

$$\sigma_{\theta(i)} = \xi_j^{res}\sigma_{r(i)} + Y_j^{res} \quad \text{For post-peak stage} \qquad \text{... (20b)}$$

Where, the subscript "i" represents different zone (i =1,2). ξ_j^{ini} and Y_j^{ini} are the initial strength parameters, respectively. ξ_j^{res} and Y_j^{res} are the residual strength parameters, respectively.

3.2 Based equation

For the axisymmetric plane strain problem, the equilibrium differential equation can be expressed as (ignoring the body force of rock masses) [22, 24].

$$\frac{d\sigma_{r(i)}}{dr} + \frac{\sigma_{r(i)} - \sigma_{\theta(i)}}{r} = 0 \qquad \text{... (21)}$$

The geometric equation, based on the small deformation assumption, can be denoted as

$$\varepsilon_{r(i)} = \frac{\partial u_{r(i)}}{\partial r}\,;\ \varepsilon_{\theta(i)} = \frac{u_{r(i)}}{r} \qquad \text{... (22)}$$

TABLE 1 A SUMMARY ON THE YIELD CRITERION UNDER PLANE STRAIN CONDITIONS

Failure criterion	Initial yield criterion	Simplified yield criterion ($\sigma_1 = \xi_j \sigma_3 + Y_j$)	Linearity	Intermediate principal stress
Mohr-Coulomb (MC) [1-2]	$\sigma_1 = \frac{1+\sin\varphi}{1-\sin\varphi}\sigma_3 + \frac{2c\cos\varphi}{1-\sin\varphi}$	$\xi_{\mathrm{MC}} = \frac{1+\sin\varphi}{1-\sin\varphi}$; $Y_{\mathrm{MC}} = \frac{2c\cos\varphi}{1-\sin\varphi}$	Linear	No
Tresca (TR)[3]	$\frac{\sigma_1-\sigma_3}{2} = \tau_{\max} = c$	$\xi_{\mathrm{TR}} = 1$; $Y_{\mathrm{TR}} = 2c$	Linear	No
Generalized Lade-Duncan (GLD) [18]	$\frac{(I_1^{'})^3}{I_3^{'}} = K_{\mathrm{Lade}}$; $K_{\mathrm{Lade}} = \frac{(3-\sin\varphi)^3}{(1+\sin\varphi)(1-\sin\varphi)^2}$	$\xi_{\mathrm{GLD}} = \eta_{\mathrm{GLD}}$; $Y_{\mathrm{GLD}} = (\eta_{\mathrm{GLD}}-1)\sigma_i$	Non linear	Yes
Mogi-Coulomb (GMC) [10-11]	$\frac{1}{3}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2} = q_1\frac{\sigma_1+\sigma_2+\sigma_3}{3} + q_2$; $q_1 = \frac{2\sqrt{2}}{3}\sin\varphi, q_2 = \frac{2\sqrt{2}}{3}c\cos\varphi$	$\xi_{\mathrm{MGC}} = \frac{\sqrt{6}+3q_1}{\sqrt{6}-3q_1}$; $Y_{\mathrm{MGC}} = \frac{6q_2}{\sqrt{6}-3q_1}$	Linear	Yes
Von Mises (VM)[3,20]	$\sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}{6}} = \sqrt{J_2} = \frac{2c}{3}$	$\xi_{\mathrm{VM}} = 1$; $Y_{\mathrm{VM}} = \frac{4c}{\sqrt{3}}$	Non linear	Yes
Generalized SMP criterion (GSMP) [17-18]	$\frac{I_1^1 I_2^1}{I_3^1} = K_{\mathrm{GSMP}}$; $K_{\mathrm{GSMP}} = 8\tan^2\varphi + 9$	$\xi_{\mathrm{GSMP}} = \eta_{\mathrm{GSMP}}$, $Y_{\mathrm{GSMP}} = (\eta_{\mathrm{GSMP}}-1)\sigma_i$	Non linear	Yes
Circumscribe Drucker-Prager (CDP) [12-14]	$\sqrt{J_2} = k - \alpha I_1$ $k = \frac{6c\cos\varphi}{\sqrt{3}(3-\sin\varphi)}$, $\alpha = \frac{2\sin\varphi}{\sqrt{3}(3-\sin\varphi)}$			Yes
Middle Circumscribe Drucker-Prager (MDP) [12]	The expression form is same as CDP. $k = \frac{6c\cos\varphi}{\sqrt{3}(3+\sin\varphi)}$, $\alpha = \frac{2\sin\varphi}{\sqrt{3}(3+\sin\varphi)}$	$\xi_{\mathrm{CDP}} = \frac{1+3\alpha}{1-3\alpha}$, $Y_{\mathrm{CDP}} = \frac{2k}{1-3\alpha}$	Non linear	Yes
Inscribe Drucker-Prager (IDP)	The expression form is same as CDP. $k = \frac{\sqrt{3}c\cos\varphi}{\sqrt{(3+\sin^2\varphi)}}$, $\alpha = \frac{\sin\varphi}{\sqrt{(9+3\sin^2\varphi)}}$			Yes
Unified Strength Theory (UST) [26-27]	If $\sigma_2 \le (\sigma_1+\sigma_3)/2 - (\sigma_1-\sigma_3)\sin\varphi/2$ $F = \frac{1-\sin\varphi}{1+\sin\varphi}\sigma_1 - \frac{b\sigma_2+\sigma_3}{1+b} = \frac{2c\cos\varphi}{1+\sin\varphi}$; If $\sigma_2 \ge (\sigma_1+\sigma_3)/2 - (\sigma_1-\sigma_3)\sin\varphi/2$ $F^{'} = \frac{1-\sin\varphi}{(1+b)(1+\sin\varphi)}(\sigma_1+b\sigma_2) - \sigma_3 = \frac{2c\cos\varphi}{1+\sin\varphi}$	$\xi_{\mathrm{UST}} = \frac{2+b+(2+3b)\sin\varphi}{(2+b)(1-\sin\varphi)}$, $Y_{\mathrm{UST}} = \frac{4(1+b)c\cos\varphi}{(2+b)(1-\sin\varphi)}$	linear	Yes
Generalized Twin shear stress criterion (GTSS) [28]	When b=1, UST is equal to GTSS.	$\xi_{\mathrm{GTSS}} = \frac{3+5\sin\varphi}{3(1-\sin\varphi)}$, $Y_{\mathrm{GTSS}} = \frac{8c\cos\varphi}{3(1-\sin\varphi)}$	linear	Yes

Where, $U_{r(i)}$ is the radial displacement. Both the radial displacement u_{R1} and radial contact stress σ_{R1} should be continuous at the elastic-plastic interface, respectively. Therefore, the boundary conditions around the circular opening can be summarized as

$$\begin{cases} r = R_0, \sigma_{r(1)} = \sigma_0 \\ r = R_1, \sigma_{r(1)} = \sigma_{R_1}, u_{r(1)} = u_{R_1} \\ r = R_2, \sigma_{r(2)} = p_0 \end{cases} \quad \text{... (23)}$$

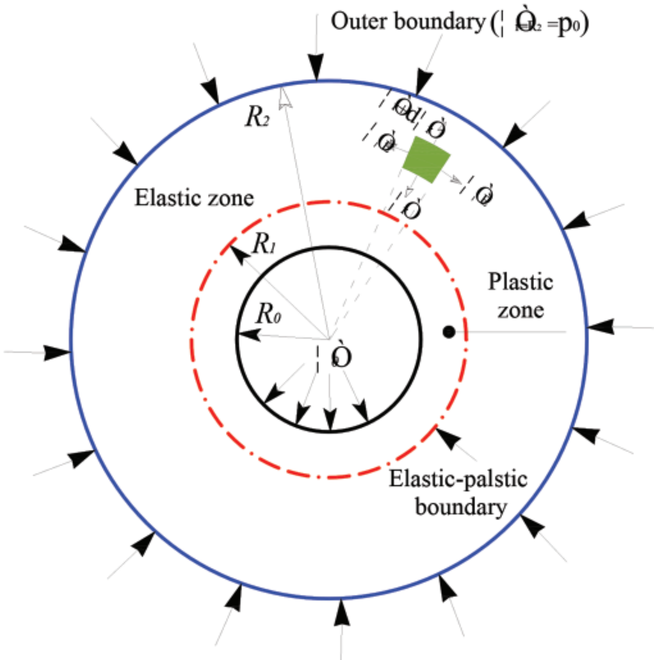


Fig.2 Calculation model of circular opening with infinite boundary ($R_2 \to \infty$).

4. New unified solution of circular opening

4.1 Stresses and deformation in elastic zone

For deep underground engineering, a circular opening subjected to an inner support pressure σ_0 at $r=R_o$ and hydrostatic pressure p_0 at infinite external boundary is a special case of the axisymmetric thick-wall cylinder. According to the elastic mechanics theory, the stresses and displacement solutions in the elastic zone can be easily deduced as following

$$\sigma_{r(2)} = p_0 - (p_0 - \sigma_{R_1})(\frac{R_1}{r})^2 \quad \text{... (24a)}$$

$$\sigma_{\theta(2)} = p_0 + (p_0 - \sigma_{R_1})(\frac{R_1}{r})^2 \quad \text{... (24b)}$$

$$u_{r(2)} = \frac{(1+\nu)(p_0 - \sigma_{R_1})}{E}\frac{R_1^2}{r} \quad \text{... (25)}$$

Where, E and ν are the initial Young's modulus and initial Poisson' ratio, respectively. The initial yield failure condition should be satisfied at the elastic-plastic interface. Therefore, the radial contact stress σ_{R1} can be derived by substituting Eq.(24a), (24b) into Eq.(20a) under different yield criterion.

$$\sigma_{R_1} = \frac{(2p_0 - Y_j^{ini})}{(1+\xi_j^{ini})} \quad \text{... (26)}$$

The radial displacement at the elastic-plastic interface ($r = R_1$) can be also determined by Eq.(25) as follows:

$$u_{R_1} = \frac{(1+\nu)(p_0 - \sigma_{R_1})}{E}R_1 \quad \text{... (27)}$$

4.2 Stresses and deformation in plastic zone

Obviously, the stresses in the plastic zone should satisfy the equilibrium differential equation and are easily deduced by submitting Eq.(20b) to Eq.(21) as well as combining with the boundary condition $\sigma_{r(1)} = \sigma_0$ at $r = R_0$. However, for $\xi_j^{res} = 1$ and $\xi_j^{res} \neq 1$, the stresses solution in the plastic zone are inconsistent.

If $\xi_j^{res} \neq 1$, the unified stresses solution in the plastic zone will be obtained as:

$$\sigma_{r(1)} = (\sigma_0 + \frac{Y_j^{res}}{\xi_j^{res}-1})(\frac{r}{R_0})^{\xi_j^{res}-1} - \frac{Y_j^{res}}{\xi_j^{res}-1} \quad \text{... (28a)}$$

$$\sigma_{\theta(1)} = \xi_j^{res}(\sigma_0 + \frac{Y_j^{res}}{\xi_j^{res}-1})(\frac{r}{R_0})^{\xi_j^{res}-1} - \frac{Y_j^{res}}{\xi_j^{res}-1} \quad \text{... (28b)}$$

Here, the subscript "j" represents MC, GMC, GLD, GSMP, CDP, MDP, IDP, UST and GTSS criterions.

Then, the radii of plastic zone can be determined by considering the boundary condition $\sigma_{r(1)} = \sigma_{R1}$ at the elastic-plastic interface. By integrating Eq.(24a) and (28a), the radius of plastic zone for deep circular opening can be easily deduced as follows:

$$R_1 = R_0 \sqrt[\xi_j^{res}-1]{\frac{(2p_0 - Y_j^{ini})/(1+\xi_j^{ini}) + Y_j^{res}/(\xi_j^{res}-1)}{\sigma_0 + Y_j^{res}/(\xi_j^{res}-1)}} \quad \text{... (29)}$$

If $\xi_j^{res} = 1$, the unified stresses solution of plastic zone based on the TR and VM criterions are

$$\sigma_{r(1)} = \sigma_0 + Y_j^{res}\ln(\frac{r}{R_0}) \quad \text{... (30a)}$$

$$\sigma_{\theta(1)} = \sigma_0 + Y_j^{res}[1 + \ln(\frac{r}{R_0})] \quad \text{... (30b)}$$

As the solution for Eq.(29), the unified radius solution of plastic zone based on TR and VM criterions are

$$R_1 = R_0 e^{\frac{(2p_0 - Y_j^{ini})/(1+\xi_j^{ini}) - \sigma_0}{Y_j^{res}}} \quad \text{... (31)}$$

In the plastic zone, the total hoop strain $\varepsilon_{\theta(1)}$ and $\varepsilon_{r(1)}$ radial strain are respectively composed of elastic strain and plastic strain. Therefore, the total strain can be expressed as:

$$\begin{cases} \varepsilon_{\theta(1)} = \varepsilon_{\theta(1)}^{e} + \varepsilon_{\theta(1)}^{p} \\ \varepsilon_{r(1)} = \varepsilon_{r(1)}^{e} + \varepsilon_{r(1)}^{p} \end{cases} \quad \text{... (32)}$$

Where, $\varepsilon^{e}_{\theta(1)}$ and $\varepsilon^{e}_{r(1)}$ and are respectively the hoop elastic strain and radial elastic strain in the plastic zone; $\varepsilon^{p}_{\theta(1)}$ and $\varepsilon^{p}_{r(1)}$ and are the hoop plastic strain and radial plastic strain of plastic zone, respectively.

For axisymmetric plane strain problem, the plastic strain relationships can be established by considering the small strain theory and non-associated flow rule [7, 24].

$$\varepsilon^p_{r(1)} + \beta\varepsilon^p_{\theta(1)} = 0 \quad ...(33)$$

Where, β is the dilation coefficient, $\beta = (1+\sin\psi)/(1-\sin\psi)$. ψ is the dilation angle.

By substituting Eqs.(22) and (33) into Eq.(32), the following differential equation for the radial displacement in the plastic zone can be derived as

$$\frac{\partial u_{r(1)}}{\partial r} + \beta\frac{u_{r(1)}}{r} = \varepsilon^e_{r(1)} + \beta\varepsilon^e_{\theta(1)} = f(r) \quad ...(34)$$

From Eq.(34), it can be seen that the radial displacement is closely related to the elastic strain form in the plastic zone. Then, the following function can be obtained by solving Eq.(34) as follows:

$$u_{r(1)} = \frac{1}{r^\beta}\int_{R_1}^{r} f(r)r^\beta dr + u_{R_1}(\frac{R_1}{r})^\beta \quad ...(35)$$

In order to obtain the radial displacement of plastic zone, the expression for elastic strain should be firstly determined. Generally, four different definitions for elastic strain can be used to research the deformation behavior of rock mass in the plastic zone.

(1) Case 1: It is assumed that the elastic strain in the plastic region is equal to that on the elastic-plastic interface. Then, the elastic strains can be expressed as [21]

$$\begin{cases} \varepsilon^e_{\theta(1)} = (\frac{u_{r(2)}}{r})_{r=R_1} = \frac{(1+\nu)(p_0-\sigma_{R_1})}{E} \\ \varepsilon^e_{r(1)} = (\frac{\partial u_{r(2)}}{\partial r})_{r=R_1} = -\frac{(1+\nu)(p_0-\sigma_{R_1})}{E} \end{cases} \quad ...(36)$$

Then, the function $f(r)$ is

$$f(r) = -\frac{(1+\nu)(1-\beta)}{E}(p_0-\sigma_{R_1}) = \delta_{case1} \quad ...(37)$$

By substituting Eq.(37) into Eq.(35), the radial displacement in the plastic zone can be derived as follows:

$$u^{case1}_{r(1)} = \frac{\delta_{case1}}{(1+\beta)}\frac{r^{\beta+1}-R_1^{\beta+1}}{r^\beta} + u_{R_1}(\frac{R_1}{r})^\beta \quad ...(38)$$

(2) Case 2: By regarding the plastic zone as the thick-wall cylinder subjected to the inner pressure σ_0 at $r = R_0$ and radial contact stress σ_{R1} at $r = R_0$, so, the elastic strain in the plastic zone can be written as [8,22].

$$\begin{cases} \varepsilon^e_{\theta(1)} = \frac{1+\nu_{res}}{E_{res}}[(1-2\nu_{res})C_1 - \frac{C_2}{r^2}] \\ \varepsilon^e_{r(1)} = \frac{1+\nu_{res}}{E_{res}}[(1-2\nu_{res})C_1 + \frac{C_2}{r^2}] \end{cases} \quad ...(39)$$

Where,

$$C_1 = \frac{(\sigma_{R_1}-p_0)R_1^2 - (\sigma_0-p_0)R_0^2}{R_1^2-R_0^2};\ C_2 = \frac{R_1^2R_0^2(\sigma_0-\sigma_{R_1})}{R_1^2-R_0^2}.$$

Therefore, the function $f(r)$ is

$$f(r) = \frac{1+\nu_{res}}{E_{res}}[(1+\beta)(1-2\nu_{res})C_1 + (1-\beta)\frac{C_2}{r^2}] = \delta_1^{Case2}(1+\beta) + \delta_2^{Case2}(1-\beta)r^{-2} \quad ...(40)$$

Where, $\delta_1^{Case2} = \frac{1+\nu_{res}}{E_{res}}(1-2\nu_{res})C_1$, $\delta_2^{Case2} = \frac{1+\nu_{res}}{E_{res}}C_2$.

By submitting Eq.(40) to Eq.(35), we can obtain the radial displacement of plastic zone as follows:

$$u^{case2}_{r(1)} = \frac{1}{r^\beta}[\delta_1^{Case2}(r^{\beta+1}-R_1^{\beta+1}) - \delta_2^{Case2}(r^{\beta-1}-R_1^{\beta-1})] + u_{R_1}(\frac{R_1}{r})^\beta \quad (41)$$

(3) Case 3: By adopting generalized Hooke's law for removing the effect of initial hydrostatic pressure p_0, the elastic strain in the plastic zone can be expressed as follows:

$$\begin{cases} \varepsilon^e_{\theta(1)} = \frac{(1+\nu_{res})}{E_{res}}[(1-\nu_{res})(\sigma_{\theta(1)}-p_0) - \nu_{res}(\sigma_{r(1)}-p_0)] \\ \varepsilon^e_{r(1)} = \frac{(1+\nu_{res})}{E_{res}}[(1-\nu_{res})(\sigma_{r(1)}-p_0) - \nu_{res}(\sigma_{\theta(1)}-p_0)] \end{cases} \quad (42)$$

Then, the function $f(r)$ can be expressed as follows:

$$f(r) = \frac{1+\nu_{res}}{E_{res}}[(1-\nu_{res}-\beta\nu_{res})\sigma_{r(1)} + (\beta-\nu_{res}-\beta\nu_{res})\sigma_{\theta(1)} + (2\nu_{res}-1)(1+\beta)p_0] \quad (43)$$

If $\xi_j^{res} \neq 1$, the function $f(r)$ will be rewritten by substituting Eq.(28a) and Eq.(28b) into Eq.(43) as follows:

$$f(r) = \frac{1+\nu_{res}}{E_{res}}[\delta_1^{case3}(\frac{r}{R_0})^{\xi_j^{res}-1} - \delta_2^{case3}] \quad (44)$$

Where, $\delta_1^{case3} = (\sigma_0 + \frac{Y_j^{res}}{\xi_j^{res}-1})[1+\beta\xi_j^{res} - (1+\xi_j^{res})(\nu_{res}+\beta\nu_{res})]$,

$\delta_2^{case3} = (1-2\nu_{res})(1+\beta)[p_0 - Y_j^{res}/(\xi_j^{res}-1)]$.

By substituting Eq.(44) into Eq.(35), the radial displacement in the plastic zone can be derived as follows:

$$u^{case3}_{r(1)} = \frac{1+\nu_{res}}{E_{res}}\frac{1}{r^\beta}[\frac{\delta_1^{case3}}{(\xi_j^{res}+\beta)R_0^{\xi_j^{res}-1}}(r^{\xi_j^{res}+\beta} - R_1^{\xi_j^{res}+\beta}) - \frac{\delta_2^{case3}}{(1+\beta)}(r^{1+\beta}-R_1^{1+\beta})] + u_{R_1}(\frac{R_1}{r})^\beta \quad ...(45)$$

If $\xi_j^{res} = 1$, the function $f(r)$ can be obtained by substituting Eq.(30a) and Eq.(30b) into Eq.(43) as follows:

$$f(r)=\frac{1+\nu_{res}}{E_{res}}[\delta_3^{case3}\ln(\frac{r}{R_0})+\delta_4^{case3}] \quad (46)$$

Where, $\delta_3^{case3}=(1-2\nu_{res})(1+\beta)Y_j^{res}$, $\delta_4^{case3}=(1-\nu_{res}-\beta\nu_{res})\sigma_0+(\beta-\nu_{res}-\beta\nu_{res})(\sigma_0+Y_j^{res})+(2\nu_{res}-1)(1+\beta)p_0$

By submitting Eq.(46) to Eq.(35), the radial displacement for TR and VM criterion in the plastic zone are

$$u_{r(1)}^{case3}=\frac{1+\nu_{res}}{E_{res}}\frac{1}{r^{\beta}}\{\frac{\delta_3^{case3}}{1+\beta}[r^{1+\beta}\ln(\frac{r}{R_0})-R_1^{1+\beta}\ln(\frac{R_1}{R_0})]+[\frac{\delta_4^{case3}}{1+\beta}-\frac{\delta_3^{case3}}{(1+\beta)^2}](r^{1+\beta}-R_1^{1+\beta})\}+u_{R_1}(\frac{R_1}{r})^{\beta} \quad (47)$$

(4) Case 4: The mechanical behaviour of rock mass is closely related to its damage degree in the plastic zone. The higher damage usually leads to larger deformation behaviour. Then, the attenuation of the Young's modulus should be considered in the plastic zone. In this case, the power function attenuation model of Young's modulus along the radius direction is introduced to research the deformation behaviour in the plastic zone:

$$E(r)=E_{res}(r/R_0)^m \text{ and } m=\log(E/E_{res})/\log(R_1/R_0) \quad (48)$$

If $\xi_j^{res}\neq 1$, the function $f(r)$ will be rewritten by substituting Eq. (48) into Eq.(44)as follows:

$$f(r)=\frac{1+\nu_{res}}{E_{res}}(\frac{R_0}{r})^m[\delta_1^{case3}(\frac{r}{R_0})^{\xi_j^{res}-1}-\delta_2^{case3}] \quad (49)$$

By substituting Eq.(49) into Eq.(35), the radial displacement in the plastic zone can be derived. In fact, the expression form is the same with Eq.(45) as follows:

$$u_{r(1)}^{case3}=\frac{1+\nu_{res}}{E_{res}}\frac{1}{r^{\beta}}[\frac{\delta_1^{case3}}{(\xi_j^{res}+\beta-m)R_0^{\xi_j^{res}-m-1}}(r^{\xi_j^{res}+\beta-m}-R_1^{\xi_j^{res}+\beta-m})-\frac{\delta_2^{case3}}{(1+\beta-m)R_0^{-m}}(r^{1+\beta-m}-R_1^{1+\beta-m})]+u_{R_1}(\frac{R_1}{r})^{\beta} \quad (50)$$

If $\xi_j^{res}=1$, the function $f(r)$ can be obtained by substituting Eq.(48) into Eq.(46) as follows:

$$f(r)=\frac{1+\nu_{res}}{E_{res}}(\frac{R_0}{r})^m[\delta_3^{case3}\ln(\frac{r}{R_0})+\delta_4^{case3}] \quad (51)$$

By introducing Eq.(51) to Eq.(35), the expression form for radial displacement is similar to Eq.(47) as follows:

$$u_{r(1)}^{case3}=\frac{1+\nu_{res}}{E_{res}}\frac{1}{r^{\beta}}\{\frac{\delta_3^{case3}R_0^{m}}{1+\beta-m}[r^{1+\beta-m}\ln(\frac{r}{R_0})-R_1^{1+\beta-m}\ln(\frac{R_1}{R_0})]+[\frac{\delta_4^{case3}}{1+\beta-m}-\frac{\delta_3^{case3}}{(1+\beta-m)^2}]R_0^{m}(r^{1+\beta-m}-R_1^{1+\beta-m})\}+u_{R_1}(\frac{R_1}{r})^{\beta} \quad (52)$$

5 Correctness verification and parameter analysis

5.1 A comparison with the traditional solution

Park et al summarized three different definitions for elastic strains in the plastic zone (see case 1~3) and analyzed the deformation law of plastic zone with Mohr-Coulomb criterion [7]. However, the Young's modulus and Poisson's ratio attenuation were ignored. In fact, it can be obtained by taking $E_{res}=E$, $\nu_{res}=\nu$, $\xi_j=\xi_{MC}$ and $Y_j=Y_{MC}$, in this paper. To further verify the correctness of the calculation results, the solution by Park et al. will be presented as a comparison with the solution proposed by this paper. The geometrical and physical parameters for circular opening are shown in Table.2.

Table 2 Geometrical and physical parameters of circular opening

Parameters	(Park et al. 2006)	
	Hard rock	Soft rock
Radius of opening, R_0 (m)	1	1
Initial stress, σ_0 (MPa)	1	1
Internal pressure, p_{in} (MPa)	0	0
Young's modulus, E (MPa)	50000	5000
Poisson's ratio, ν	0.2	0.2
Shear modulus, G (MPa)	20833	2083
c (MPa)	0.173	0.276
φ (deg)	55	35
c_{res} (MPa)	0.061	0.055
φ_{res} (deg)	52	30

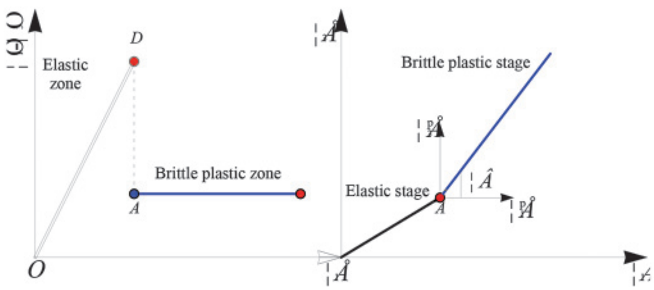


Fig.3 Post-peak failure behaviour of brittle plastic rock mass

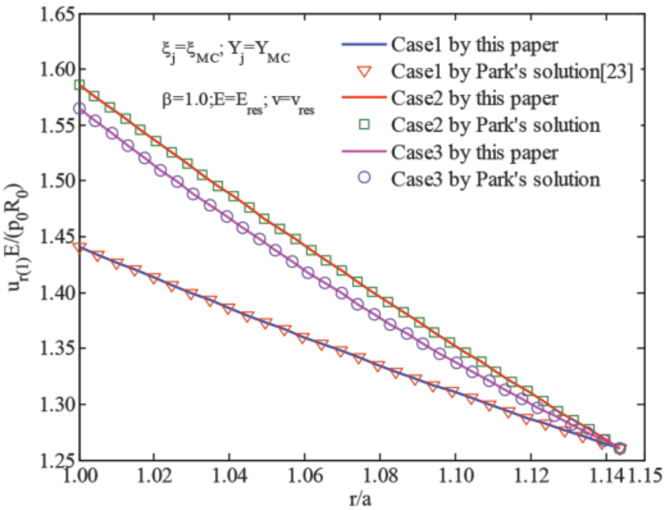
From the Fig.4 it can be seen that the calculation results in this paper is in accordance with Park's closed-form solution. Therefore, the closed-form solution proposed by this paper is correct and can be regarded as an extension of Park's solution.

5.2 Parameters analysis

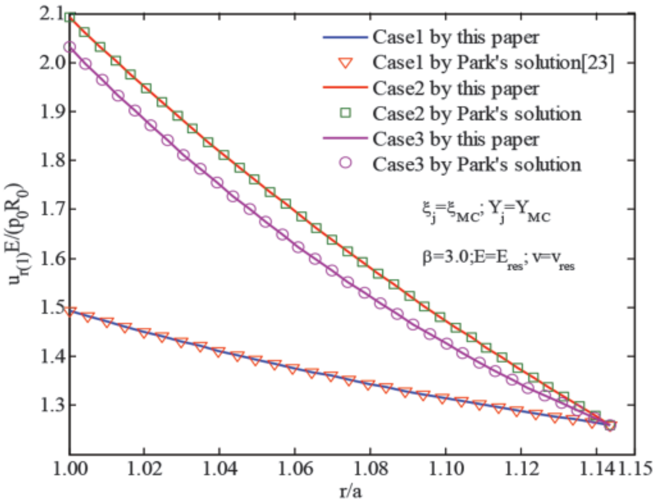
5.2.1 The effect of the strength yield criterions

As previously mentioned, the strength criterions have an extremely important for evaluating the mechanical response of surrounding rock. For studying the influence of strength theoretical effect on the stresses and displacement of surrounding rock, the mechanical and geometrical parameters of circular tunnel are shown in Table 3.

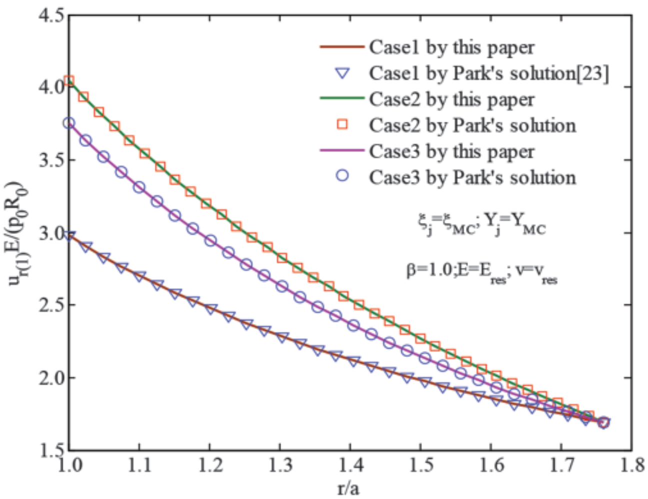
From the Fig.5 and Table 4, it can be seen that the



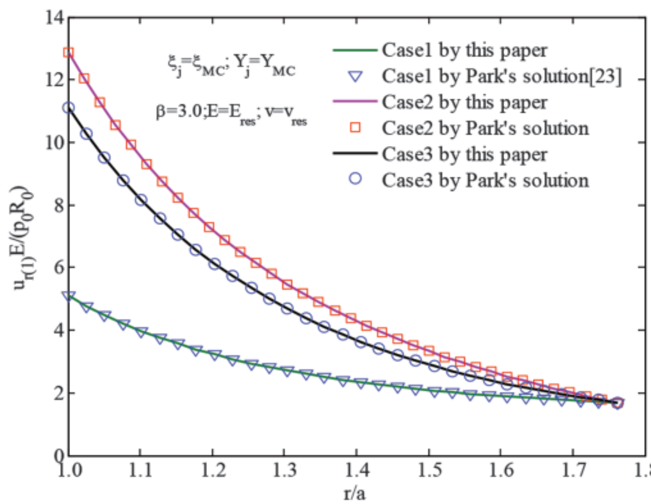
(a) Radial displacement (data: hard rock, β =1.0)



(b) Radial displacement (data: hard rock, β =3.0)



(c) Radial displacement (data: soften rock, β =1.0)



(d) Radial displacement (data: soft rock, β =3.0)

Fig.4 Comparison with the traditional solutions

TABLE 3 THE MECHANICAL AND GEOMETRICAL PARAMETERS OF CIRCULAR OPENING

p_0/MPa	σ_0/MPa	R_0/m	E_{res}=E/GPa	v_{res} = v	c_{res} = c/MPa	φ_{res} = φ/(°)	β
25	0	3	3	0.25	7.2	18.3	1.0

dimensionless values (and) show the characteristics of TR > VM > IDP > MC > MDP > UST0.5 > GSMP > GMC > GLD > GTSS > CDP. Compared to the traditional solution obtained by MC criterion, the dimensionless value (R_1/R_0) calculated by TR and VM criterions obvious increase by 125.26% and 78.53%, respectively. This is mainly because the TR and VM criterions only regards the rock mass as frictionless bonding material and ignores the impact of friction effect on the mechanical properties of the rock mass so that underestimate the bearing capacity of rock mass. Meanwhile, the MC criterion does not take the effect of the intermediate principal stress into account and it is easy to underestimate the bearing capacity of rock mass. Therefore, the deformation of surrounding rock calculated by MC criterion may be slightly larger. The IDP and MDP criterions underestimate the influence of internal principal stresses, so the calculation results may be also larger than the solution obtained by $UST_{0.5}$, GSMP, GMC, GLD, CDP and GTSS criterions. Meanwhile, the result obtained by CDP criterion is minimal compared to other criterion solutions. In fact, CDP criterion overestimates the effect of intermediate principal stresses on rock mass strength and may be not reasonable in practical engineering. In addition, the calculation results (R_1/R_0) obtained by $UST_{0.5}$, GSMP, GMC and GLD criterions are close to each other within the range of 1.294~1.347. The above four criterions seem to be more reasonable considering the effect

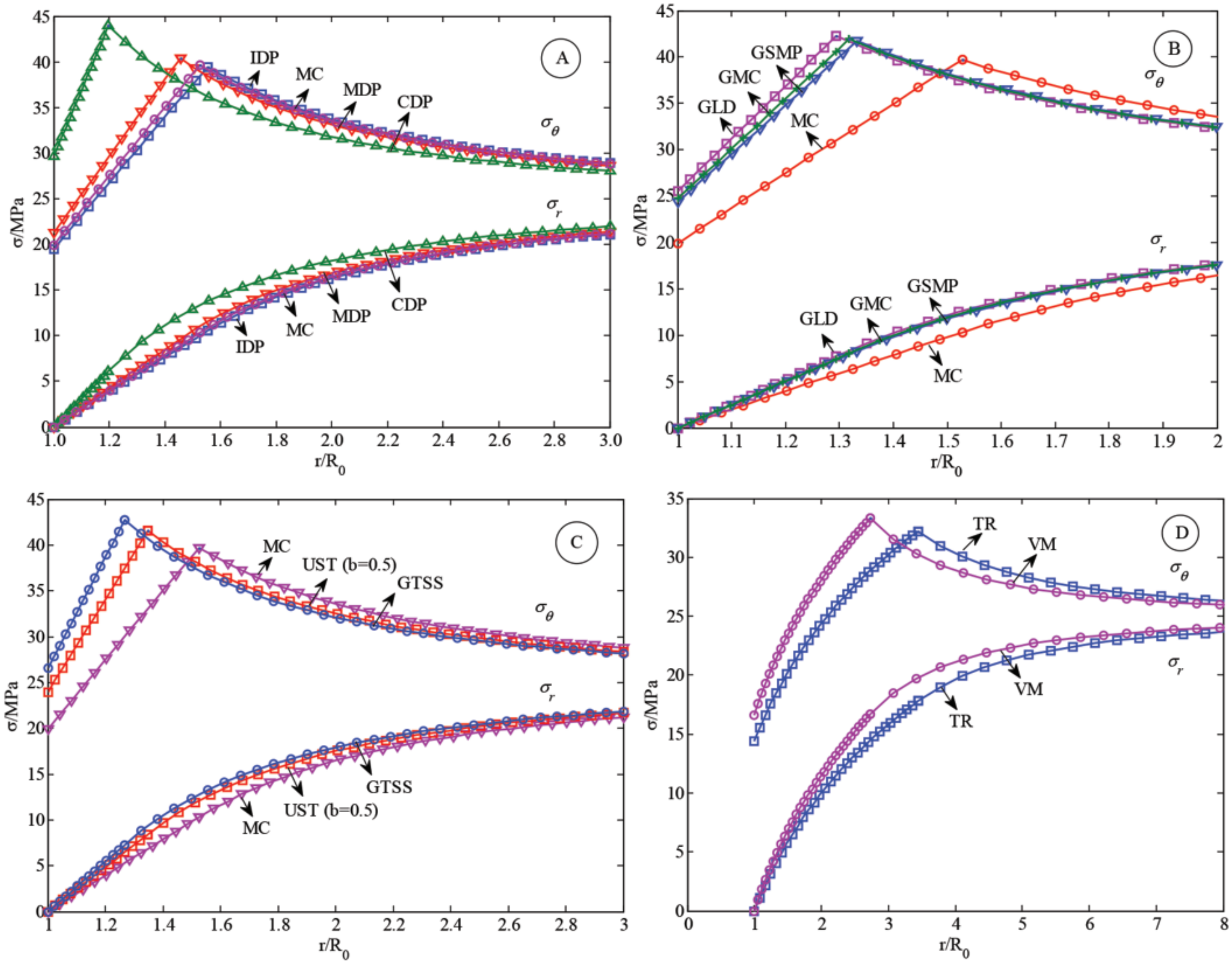


Fig.5 The stresses distributions law under different criterions

of intermediate principal stress on yield strength of rock masses.

From the above analysis, it can be seen that the $UST_{0.5}$, GSMP, GMC and GLD criterions can be strongly recommended for evaluating the mechanics and deformation behaviour of surrounding rock, followed by IDP, MDP, GTSS and MC criterions. The TR, VM and CDP criterions are not recommended to be used for underground engineering.

Table 4 The radial of plastic zone and critical inner pressures under different criterions

Strength criterion	R_1/R_0; (σ_{R1}/p_0)	Strength criterion	R_1/R_0; (σ_{R1}/p_0)
MC	1.528 (0.413)	VM	2.728 (0.667)
GMC	1.318 (0.322)	IDP	1.556 (0.422)
GLD	1.294 (0.308)	MDP	1.456 (0.386)
GSMP	1.332 (0.329)	CDP	1.198 (0.242)
TR	3.442 (0.712)	$UST_{0.5}$	1.347 (0.337)
GTSS	1.266 (0.291)		

5.2.2 The effect of dilation coefficient

As shown in Fig.6, the dilation coefficient has an extremely important influence on the surface displacement of surrounding rock. As the parameter β gradually increase, the dimensionless surface displacement $u_0E/(p_0R_0)$ presents the nonlinear increase characteristics. However, the increasing rate of surface displacement under different yield criterions is significantly different. For instance, as shown in Table 5, β

Table 5 The surface displacement value ($u_0E/(p_0R_0)$) of circular tunnel

	Parameter (β)	MC	GSMP	$UST_{0.5}$	MDP
Case 1	1.0	1.713	1.488	1.503	1.626
	2.0	1.990	1.601	1.626	1.834
	3.0	2.366	1.740	1.778	2.107
Case 2	1.0	1.945	1.607	1.630	1.815
	2.0	2.5867	1.879	1.924	2.302
	3.0	3.567	2.241	2.320	3.011

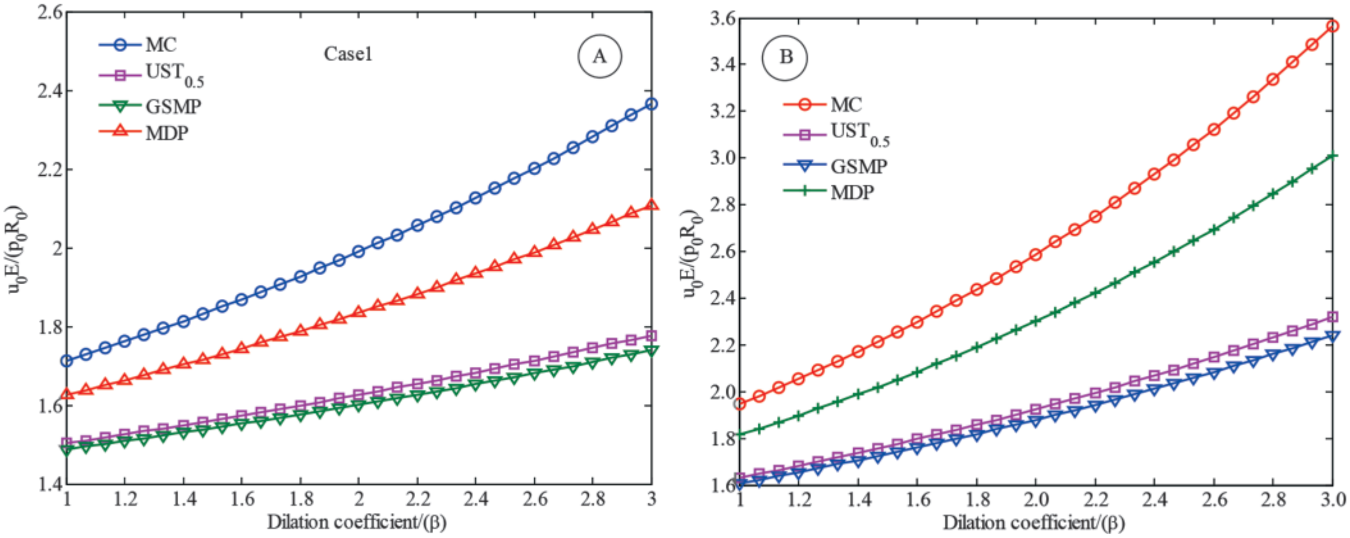


Fig.6 The effect of dilation coefficient on the surface displacement

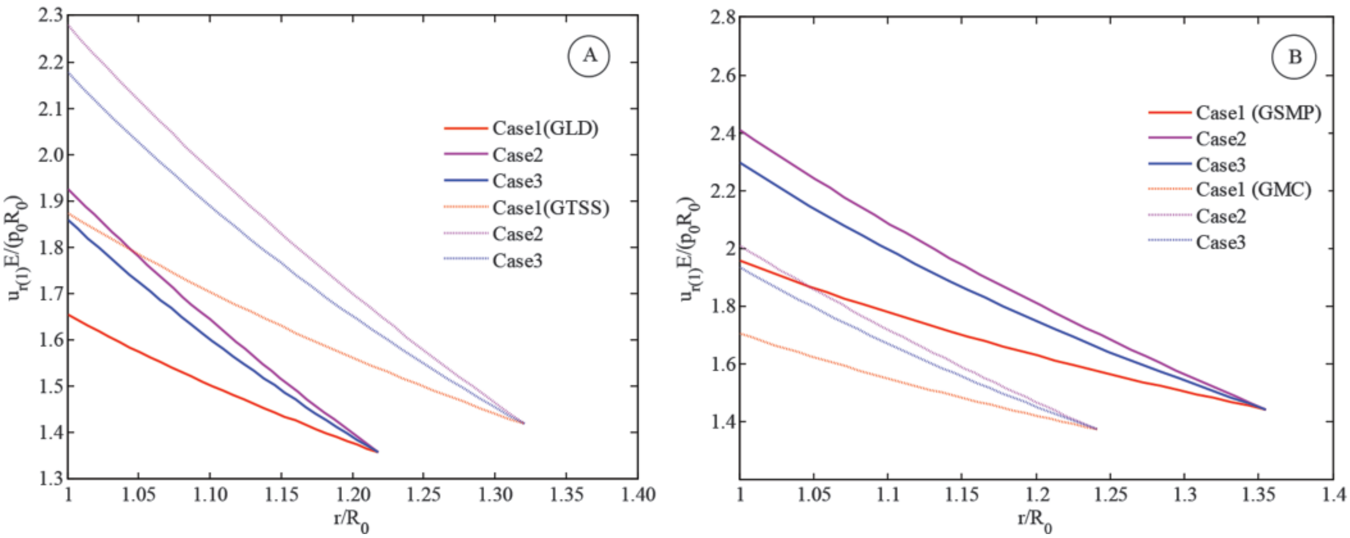


Fig.7 The effect of elastic strain form on the radial displacement

increases from 1.0 to 3.0, the dimensionless value $u_0E/(p_0R_0)$ respectively increase by 38.12% for MC criterion, 16.94% for GSMP criterion, 18.30% for $UST_{0.5}$ criterion and 29.58% for MDP criterion under case 1 condition. Therefore, the effect of dilation coefficient should be taken the supporting parameters and strength design of tunnel into account.

5.2.3 The effect of elastic strain form and rock mass damage

In this study, the geometrical and mechanical parameters of circular tunnel are shown in Table 3 (soft rock). Fig.7 presents the influence of elastic strain form in the plastic zone on the surface displacement of surrounding rock. From the above analysis, the relevant conclusions can be summarized as follows:

- The radial displacement of plastic zone is closely related to the selection of the elastic strain form. Case 2 has the greatest effect on the radial displacement of plastic zone, followed by Case 3. Then, the results obtained by Case 2 is minimal. For example, when $r=R_0$ (tunnel surface), compared with Case 1, the dimensionless surface displacement $u_0E/(p_0R_0)$ respectively increase by 0.272 for Case 2 and 0.206 for Case 3 with an increasing rate of 16.47% for Case 2 and 12.46% for Case 3 under GLD criterion.
- Meanwhile, the influence of elastic strain form on the radial displacement of plastic zone is also closely related to the strength criterion. For example, the dimensionless surface displacements $u_0E/(p_0R_0)$ are respectively 1.926 for Case 2 and 1.860 for Case 3 under GLD criterion. However, the value significantly increases by 18.32 % for Case.2 and 31.71 % for Case.3 under GTSS criterion.

When the rock mass enters the plastic zone, its mechanical behaviour are closely related to the rock damage degree. Generally, the Young's modulus attenuation could be

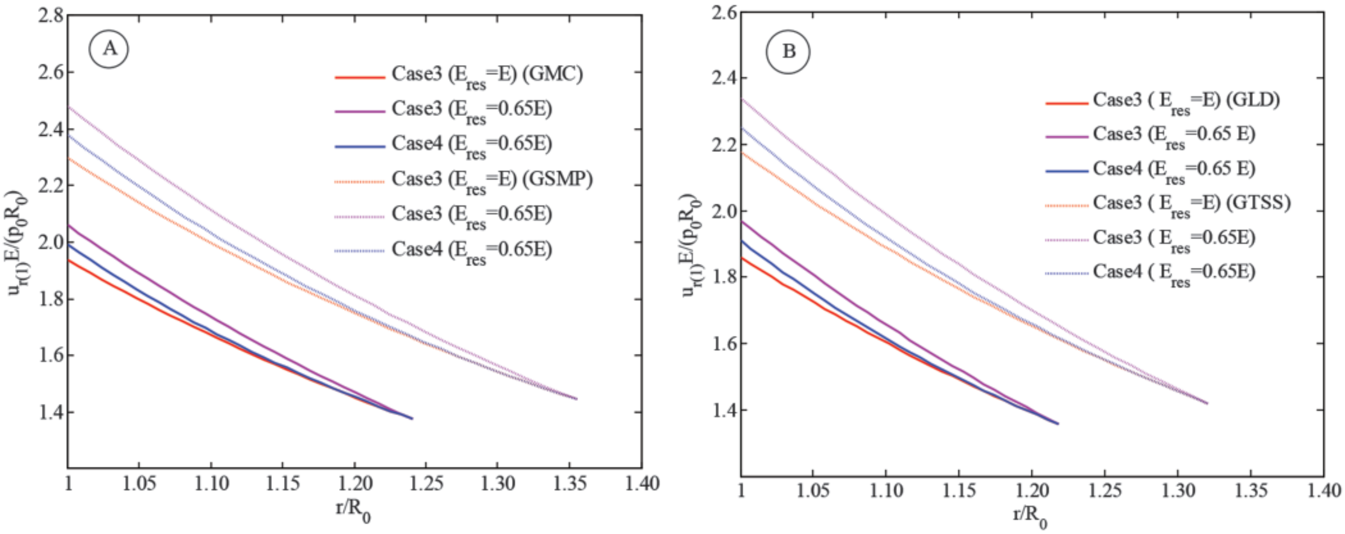


Fig.8. The effect of rock mass damage on the radial displacement

used to indicate the damage degree of rock mass. The influence of rock mass damage degree on the surface displacement of surrounding rock is shown in Fig.8.

- It can be seen that the radial displacement of plastic zone is closely related to the selection of the Young's modulus attenuation model. Compared to the case 3 (E_{res}=E), case 4 has the greatest effect on the radial displacement of plastic zone. Case 3 (E_{res}= 0.65E) is the second. For example, as shown in Table.6, compared with case 3 (E_{res}=E), the dimensionless surface displacement $u_0E/(p_0R_0)$ of Case 3 (E_{res}= 0.65E) and Case 4 increases by 0.163 and 0.074 with an increasing rate of 7.49% and 3.40% respectively under GTSS criterion.
- In addition, the influence of Young's modulus attenuation on the radial displacement of plastic zone is closely related to the strength criterion. For Case 3 (E_{res}= 0.65E) and Case 4, the dimensionless surface displacement $u_0E/(p_0R_0)$ of surrounding rock are 1.971 and 1.911 respectively under GLD criterion. However, when taking GSMP criterion, the value significantly increases by 25.82% and 24.54% than GLD criterion, respectively.

Table 6 Thedimensionless surface displacement ($u_0E/(p_0R_0)$ of circular tunnel

		GLD	GTSS	GMC	GSMP
Case 3	$m = 0$, $E_{res} = E$	1.860	2.177	1.937	2.297
	$m \neq 0$, $E_{res} = 0.65E$	1.971	2.340	2.061	2.480
Case 4	$m \neq 0$, $E_{res} = 0.65E$	1.911	2.251	1.993	2.380

6. Conclusions

By summarizing and simplifying different strength theories, a new unified criterion equation is firstly proposed based on the certain assumption. Then, a new unified closed-form solution for circular opening based on the newly proposed unified criterion equation is deduced. In the plastic zone, four different elastic strain assumptions are applied to solve the plastic zone deformation. Then, the validity of the solution is also verified by comparing with the traditional solution. Finally, the influences of strength criterion effect, dilation coefficient, elastic strain form and rock mass damage on the mechanical response of surrounding rock are discussed in detail. The primary conclusions can be summarized as follows:

1. For the radius of plastic zone, the calculation results obtained by TR and VM criterions which ignore the impact of friction effect are obviously largest, followed by IDP, MC and MDP criterions. The result obtained by CDP criterion which overestimates the effect of intermediate principal stresses on rock mass strength is minimal compared to other criterion solutions. In addition, the calculation results obtained by $UST_{0.5}$, GSMP, GMC and GLD criterions may be given an intermediate range. Therefore, $UST_{0.5}$, GSMP, GMC and GLD criterions can be strongly recommended for evaluating the mechanics and deformation behaviour of surrounding rock, followed by IDP, MDP, GTSS and MC criterions. TR, VM and CDP criterions are not recommended to be used.
2. The influence of dilation coefficient on the surface displacement of surrounding rock is not only closely related to the assumption form of elastic strain in the plastic zone, but also to the strength criterions. As the dilation coefficient gradually increase, the dimensionless surface displacement presents the nonlinear increase characteristics. Therefore, the support parameters design should take the influence of the dilation coefficient into account.
3. The elastic strain assumption forms in the plastic zone have a significant important effect on the deformation of plastic zone under ignoring the effect of rock mass damage. Case 1 gives a smallest deformation and seems

to overestimate the plastic bearing capacity of rock mass; However, Case 2 gives a largest deformation and may be underestimated the plastic bearing capacity of rock mass. Therefore, Case 3 may be more reasonable for evaluating stability of surrounding rock and optimizing support strength design.

4. The effect of Young's modulus attenuation on the deformation of the plastic zone is not only related to the selection of the Young's modulus attenuation model, but also closely related to the strength criterion. If ignoring the continuity of Young's modulus attenuation, the deformation of surrounding rock is easy to be overestimated or underestimated. So, the Young's modulus power function attenuation seems to give more reasonable results.

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