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Multi-objectives optimization of ore grade index based on IPSO

According to the specific characteristic of multi-mental mine, the function models of metal production, gross profit, net present value, rate of return on investment were obtained. Then the multi-objective optimization of ore grade index was achieved under constraints. The model was calculation based on the method of improved particle swarm optimization (IPSO). In the 5 years' evaluation, the new index scheme can increase total profit about 45.25 million yuan, total amount of metal mines about 23,214 tons, net present value about 26.29 million yuan and the rate of return on investment about 11.25 %.

Keywords: Constraints, grade index, improved particle swarm optimization, multi-objectives optimization.

1. Introduction

Due to the scarcity of mineral resources together with the technical and economic requirements in mining process, mining enterprises must benefit from management [1]. With the changes of mineral market conditions and the lever of mine production and technology, the main technical indexes related to production, including deposit industrial index (mainly cutoff grade and industrial grade), mine production scale, loss rate, dilution rate, feed grade, mineral recovery, and concentrate grade, should be carried on the research of real-time dynamic optimization[2,3]. Thereby it could enhance the level of mineral technical management and make mining enterprises obtain the optimum operation results.

According to the characteristics of polymetallic underground mining, multi-objective model of grade index must be studied deeply so that the model can more fully reflect comprehensive effects and be more widely used in order to achieve overall plan of grade programs [4-8]. In addition to economic benefits of production, the social and long-term interests must be considered for the exploitation and construction of mineral deposits [9,10]. Therefore, gross profit, net present value, metal production and investment yield are selected as effectiveness function for multiple objective decision-making [11].

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2. Modelling of objective function of grade index optimization

Objective function models of grade index optimization were built based on mine production and processing technology respectively for Arhada Pd-Zn Mine. For convenience, firstly, grade index of main variables in the models is defined as xij, where the subscripts i, j represent the number of cutoff grade and industrial grade program respectively. Ore reserves consumption is defined as Q(xij) and geology average overall grade is defined as C(xij).

Since the relationships between beneficiation model and grade optimization model can be built in consideration of the recovery rate of all mineral components in optimization process, average grade of each metal geological should be calculated by the regression coefficients fitting respectively. The relation between comprehensive average grade and average grade of each metal are as follows,

(1) The function between comprehensive grade and lead grade can be expressed as:

$$f_1(x) = - 0.0013x^3 + 0.0125x^2 + 0.3248x - 0.0159 \quad ... \quad (1)$$

(2) The function between comprehensive grade and zinc grade can be expressed as:

$$f_2(x) = 0.5645x^{1.0056} \quad ... \quad (2)$$

(3) The function between comprehensive grade and silver grade can be expressed as:

$$f_3(x) = 8.006x^{0.941} \quad ... \quad (3)$$

And the relationship between comprehensive grade and zinc grade is shown in Fig.1.

According to the above basic functions, the objective function models can be established combined with between each index and each objective.

(1) METAL PRODUCTION OBJECTIVE

Arhada Pd-Zn Mine is mainly Zn deposit, Pb and other

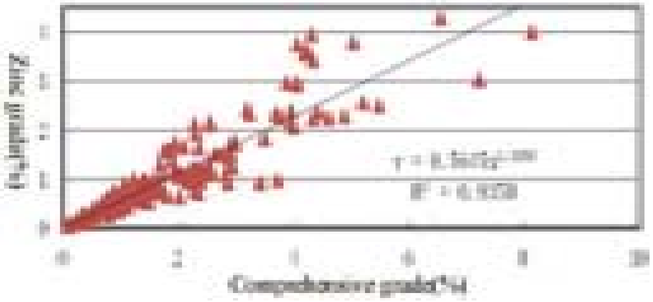

Fig.1 Relation between comprehensive grade and zinc grade

useful components followed. In mine product processing, zinc powder and lead powder containing silver metal are firstly obtained. The objective function of metal production were established taking into account all useful mineral components as measured in zinc equivalents. The expression of metal production in Arhada Pd-Zn Mine is:

$$\max q(x_{ij}) = \sum_t q(x_{ij}) \left(f_1 \times C(x_{ij}) \times \right.$$

$$(1-\rho)\times\delta_k + f_2 \times C(x_{ij})\times(1-\rho)\times\delta_k$$

$$\left. + f_3 C(x_{ij})(1-\rho)\delta_k \right)) \quad \ldots \quad (4)$$

where $q(x_{ij})$ is annual output, t; ρ is dilution rate, %; is loss ratio, %; is the flotation recovery rate of K mine product.

In practice, hopes to achieve a certain metal production value based on the bigger the better. Such goal belongs to maximize the form, In the Matlab graphical display is shown in Fig.2.

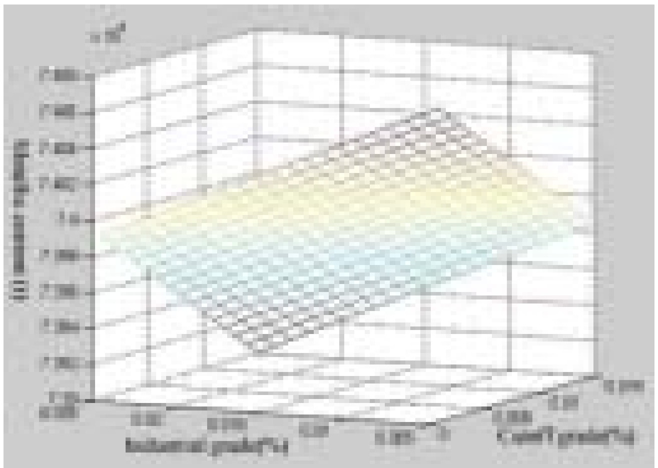

Fig.2 Metal production target model

(2) GROSS PROFIT OBJECTIVE

Gross profit is sales revenue gained by the enterprises by deducting cost including mine cost, sales taxes and surcharge. the mine costs includes financial fees, management fees, cost of mining and mineral processing costs. The sales taxes and surcharge includes value added tax, business tax, special consumption tax and resources tax [12].

The objective function of gross profit can be expressed as:

$$\max p(x_{ij}) = \sum_t q(x_{ij}) \times (\lambda_k \times (1=\alpha) - \theta) \quad \ldots \quad (5)$$

where $p(x_{ij})$ is annual gross profit, million yuan; λ_k is the price of k metal, as measured in zinc equivalents, yuan/t; α is sales tax and surcharge, %; is ore total cost, yuan.

(3) NET PRESENT VALUE OBJECTIVE

Net present value is the main dynamic evaluation index of project profitability during the calculation period. If this index is also used as one of the objective functions of grade index optimization, it can enhance the efficiency of investment evaluation in considering the time value of money and reflect the unity of liquidity and profitability in considering net cash flow of the whole process [13,14]. Due to the stability of mine production and no fixed asset investment in evaluation period, the annual net cash flow is annual total profit. The model is as follows:

$$\max NPV(x_{ij}) = \sum_t p(x_{ij})/(1+u) \quad \ldots \quad (6)$$

where $NPV(x_{ij})$ is annual net present value; u is Benchmark yield.

(4) Return on net assets (ROE)

Investment profit rate refers to the ratio of annual gross profit to total investment, when a project production had reach the design capacity. It is the static index for project profitability [15,16]. In financial evaluation, the rate of return on investment reflects the profitability of the investment. Its model is as follows:

$$\max r(x_{ij}) = \sum_t q(x_{ij}) / (\mu + q(x_{ij}) \times (\lambda_k + \alpha + \theta)) \quad \ldots \quad (7)$$

where $r(x_{ij})$ is the rate of annual return on investment, including investment profit rate, profit and tax investment rate, and selected annual investment profit rate; μ is investment in fixed assets.

3. The constraint conditions of grade index optimization

In the optimization model of ore grade index, all constraint conditions are related to each other. For example, mining production is constrained by annual mining capacity which is also constrained by resource amount and other constraint conditions. On the basis of previous research, the constraint conditions in this research contains conventional constraints, complex constraints and abstract constraints.

(1) RESOURCE CONSTRAINT

Resources, which are divided into geological reserves and recoverable reserves, are generally in the form of geological reserves. In actual optimization of grade index, geological reserves are usually converted into the form of recoverable reserves according to mining loss rate and dilution rate. Only in this way the resource constraint could be more accurate. Its model is as follows:

$$\sum_t Q(x_{ij}) \le Q' \quad \text{... (8)}$$

where Q' is recoverable resources; t is evaluation period.

(2) PRODUCTION TASK CONSTRAINT

According to the objective and actual conditions, the production task of mining area is fixed which is ruled by government or mining enterprises. It embodies not only the combination of social benefits and economic benefits, but also the principle of combination of experience and practice. So in the optimization of grade index, the constraint is that mining quantity of each mining area cannot be smaller than the specific quantity. Its model is as follows:

$$\begin{cases} A_m \le \sum_m Q(x_{ij}) \\ A_t \le \sum_t \sum_m Q(x_{ij}) \end{cases} \quad \text{... (9)}$$

where A_m is the production tasks of NO. m mining area; A_t is the annual production task of all mining areas.

(3) PRODUCTION CAPACITY CONSTRAINT

For a mining enterprise, mining capacity, selecting capacity and transport capacity are all limited. So, in the optimization of grade index, production capacity constraint is one of the constraints which must be taken into account. The production capacities mentioned above are related with each other. And if once one of them is determined, then the other two also corresponding identified. In actual production, mining enterprise arrange the production capacity according to the mining task and production conditions. Its model is as follows:

$$\begin{cases} \sum_m Q(x_{ij}) \le B_m \\ \sum_t \sum_m Q(x_{ij}) \le B_t \end{cases} \quad \text{... (10)}$$

where B_m is the maximum production capacity of the NO. m mining area, B_t is the maximum annual mining capacity, here B = 1.1~1.2A, according to the production provisions of underground metal mine.

(4) ORE QUALITY CONSTRAINT

For single kind of ore, the expression of ore quality constraint is simple. However, the expression of ore quality constraint is complicated in polymetallic mineral deposit. The model must be built according to quality constraints of multi-ores, as shown follows:

$$\beta'_{kl} \le \frac{\sum_{t=1}\sum_{m=1} \varepsilon^l_{tk} Q(x_{ij}) \alpha^l_m}{\sum_{t=1}\sum_{m=1} \delta_{tk} Q(x_{ij})} \le \beta_{kl} \quad \text{... (11)}$$

where, β'_{kl} is No. l minimum quality of No. *k* mineral product, g/t, %; β_{kl} is No. l max quality of No. *k* mineral product, g/t, %; α^l_m is the grade of No. l mineral element in No. *m* mineral area, g/t, %; ε^l_{tk} is recovery rate of No. *k* mineral product of selected ore in No. *t* year, %; δ_{tk} is yield of No. *k* production in No. *t* year, %.

In conclusion, the multi-objective optimization of ore grade index for polymetallic underground mine is obtained which is composed of objective functions and constraint conditions.

$$\begin{cases} \text{Target function:} \\ \max q(x_{ij}) = \sum_t ((1-\rho)/(1-\varphi) Q(x_{ij}) (f_1 \times C(x_{ij}) \\ \times (1-\rho) \times \delta_k + f_2 \times C(x_{ij}) \times (1-\rho) \times \delta_k \\ f_3 \times C(x_{ij}) \times (1-\rho) \times \delta_k \\ \max\ p(x_{ij}) = \sum_t q(x_{ij}) \times (\lambda_k \times (1-\alpha) - \theta) \\ \max \text{NPV}(x_{ij}) = \sum_t p(x_{ij})(1+u)^{-t} \\ \max r(x_{ij}) = \sum_t q(x_{ij}) \Big/ (\mu + q(x_{ij}) \times (\lambda_k + \alpha + \theta)) \\ \text{Constraint conditions:} \\ \sum_t Q(x_{ij}) \le Q' \\ \begin{cases} A_m \le \sum_m Q(x_{ij}) \\ A_t \sum_t \sum_m Q(x_{ij}) \end{cases} \\ \begin{cases} \sum_m Q(x_{ij}) \le B_m \\ \sum_t \sum_m Q(x_{ij}) \le B_t \end{cases} \\ \beta'_{kl} \le \frac{\sum_{t=1}\sum_{m=1} \varepsilon'_{tk} Q(x_{ij}) \alpha^l_m}{\sum_{t=1}\sum_{m=1} \delta_{tk} Q(x_{ij})} \le \beta_{kl} \end{cases} \quad \text{... (12)}$$

4. Calculating of model

The process of calculating model is actually the determination of maximum value of decision-makings in the model of set mine capability and ore grade. It includes four key functions: the relationship of metal production q, gross profit C, net present value ?, rate of return on investment r and cutoff grade c1, lowest industrial grade c2, time t, respectively.

Cutoff grade together with production-grade composes

TABLE 1: COMPARISON BETWEEN CURRENT SCHEME AND OPTIMAL SCHEME

Scheme	Cutoff grade (%)	Industrial grade (%)	Net present value (million yuan)	Total profit (million yuan)	Total production (t)	Rate of return on investment (%)
Current	1.5	2.1	788.10	1102.95	511066	17.53
Optimal	1.1	1.6	814.39	1148.20	534280	28.78
Increment	-	26.29	45.25	23214	11.25	

individual particle swarm of evolutionary computation. Using fitness function of gross profit?net present value?metal production?rate of return on investment to contact with each particle. Then we transfer each target fitness function from master program. Find out the maximum combination of fitness function by using global searching ability of particle swarm optimization.

Particle swarm is assumed to contain thirty fundamental particles. Maximum iteration is 100. The range of cutoff grade is 0.1~1.5 %. The range of optimum grade is 0.5~2.5 %, its maximum speed is 0.6 %. Fitness functions are gross profit, net present value, metal production and rate of return on investment *t* can be selected as 1, 2, …5. According to the data of geological reserves and geologic grade, cutoff grade and lowest industrial grade can be adopted in optimization period. Because that Matlab software can only show 3-D diagram, the coordinates are chosen as gross profit, net present value and metal production. The calculation result is shown in Fig.3.

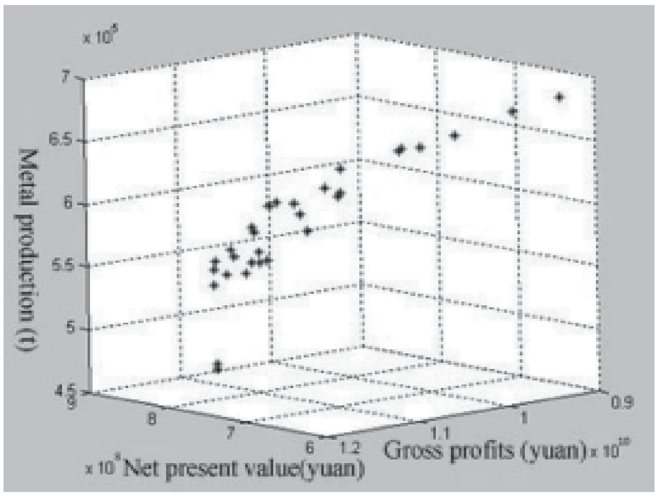

Fig.3 Exporting data of decision goal

5. Analysis of optimization results

The values of decision-making objectives and non-inferior optimal sets are obtained which is comprised 30 units combined cutoff grades and minimum industrial grades. For the above 4 decision-making objectives, the bigger the value it is, the more it contributes to the objectives. In order to determine the optimal scheme among 30 grade combinations, fuzzy comprehensive evaluation is employed in calculation. After each membership grade of the decision-making target is calculated at each grade index, weighted average is obtained by the weight of the target and it follows the comprehensive membership grade. Obviously, the scheme, in which the comprehensive membership grade is biggest, is the optimal scheme and its corresponding value of cutoff grades and minimum industrial grades is the reasonable technical indicator value by optimization.

In this model, target values are optimized by PSO algorithm with variables (such as prices of lead, zinc and silver) and technical index parameters (such as grade and recovery rate); with the cycle repeated, non-inferior optimal sets will be finally obtained and so will its total profit, total production, net present value and rate of return on investment; then on basis of the value of the comprehensive membership grade, the optimal scheme is determined, and if it's valid, its corresponding value of cutoff grades and minimum industrial grades will be calculated; afterwards, the operation objective increment can be obtained in comparison with the current scheme. Comparison between the optimal scheme and the current one is shown in Table 1.

The optimal scheme shows that under the conditions of current production technology and mineral market, the major technical index currently executed in Arhada Pd-Zn mine is not optimal. And proper adjustments should be done, namely, the combination of current mineral deposit industrial index 1.5 %, 2.1 % (cutoff grade and industrial grade respectively) is altered to the combination 1.1 %, 1.6 %. Even ore grade will be slightly decreased, but there is a great rise in ore amount which will significantly improve its overall economic efficiency.

As can be seen from the table, compared with the original index scheme, in the 5 years' evaluation, the new index scheme can increase total profit about 45.25 million yuan , total amount of metal mines about 23,214 tons , net present value about 26.29 million yuan and the rate of return on investment in 11.25 %.

6. Conclusion

Due to the characteristics of multi-metal mine, the function model for the total profit, the net present value, the metal production and the rate of return on investment can be built with constraints set. And then the multi-objective optimization model of ore grade index is obtained. The grade index optimization is modeled and calculated with improved multi-objective particle swarm optimization algorithm. In the 5 years' evaluation period, the optimized scheme enables Arhada Pd-Zn mine to witness a significant rise in resource recovery benefits, economic benefits and benefits of return on investment. Hence, the optimization of mine technical indicators can significantly improve the efficiency of resource utilization as well as the economic benefits of mining exploitation, playing an important and practical role.

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