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Development of a computer aided tool for determination of optimum cut-off grade using dynamic programming based on limited mine capacity of open pit metalliferous deposits

Determination and selection of optimum cut-off grade in case of metalliferous deposits is a very important aspect of mine designing process, which is one of the most challenging problem for surface mining operation and production planning. The cut-off grade of a metalliferous deposit is dynamic in nature thus dynamic programming approach may be considered as one of the suitable methods for solving the cut-off grade determination problems. This paper analyzes an open pit copper mine project considering fixed mine production annually and having no other capacity constraints with respect to milling and refining. A computer tool cut-off grade predictor (COGP) had also been developed based on the dynamic programming algorithm, which iterates through the range of grades to determine the optimum value of the cut-off grade for achieving the maximum value of net present value (NPV). The computer tool was built with the help of Visual Basic 2010 programming language. The software package comprises 3 modules – Input data module, Output result module and Result graphical module.

Keywords: *Dynamic programming, optimum cut-off grade, average grade, open pit copper mine, maximum NPV.*

1. Introduction

In an open pit deposit, determination and selection of the optimum cut-off grade is one of the most fundamental and challenging jobs for mine planners because it draws the line between ore and waste over period of time which directly affects the mine economics and it will vary according to the costs of operation, metallurgical characteristics, stripping ratios (open pit) and the metal in future of the deposits. It significantly influences the profitability and mine life of individual deposits and thereby the rate of mining, milling and refinery operation will be scheduled as per the available quantity of resource.

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Associated with this issue, there exists "a rule of thumb" of mines. It requires that the cut-off grade should decrease (increase) when the rate of metal price increase is greater (smaller) than the rate of discount (Shinkuma and Nishiyama 2000; Shinkuma 2000). Henning (1963) was the first who presented a methodology for calculation of optimum cut-off grade. According to him fixed costs are irrelevant in determination of the cut-off grade, and the maximization of profits implies a descending order of cut-off grades during the project's life, finally reducing to the breakeven value that corresponds to the objective of maximizing the difference between revenue and cost.

In case of any metalliferous deposit there could be different limiting cut-off grades depending on the capacity limitation of mine, mill and smelting and refinery. If the throughput is limited by the mining capacity, the opportunity costs have to be distributed per unit material mined and the corresponding cut-off grade is called the mine limited cut-off grade, similarly for mill and refinery it is defined as mill limited cut-off grade and refinery limited cut-off grade respectively. First three limiting economic cut-off grades are derived by considering that each of the three stages itself limits the total capacity of the operation and the primary objective is to maximize one or more of the followings (Lane 1964) :

1. Total profits
2. Present value of all future profits, and
3. Short term profits.

Cut-off grade is the criterion that discriminates between ore and waste within a given mineral deposit, depending upon the mining method it is used to separate two courses of action, e.g. to mine or to leave, to mill or to dump and then eventual sale [Taylor 1972, 1985]. Multiple cut-off grade optimization had been obtained by achieving maximizing profit per unit time based on genetic algorithms and the results are compared using grid search method and dynamic programming. In the early years of operation, this NPV maximization requires application of the highest cut-off grade that can provide sufficient ore to satisfy the requirements of the processing plant. To achieve the maximum NPV, the cut-off grade must be lowered, thereby lowering the opportunity cost (Cetin and Dowd 2002).

If material grade in the mineral deposit is more than or equal to the cut-off grade, it is classified as ore, otherwise it is classified as waste. The optimum cut-off grade is the grade that maximizes the chosen objective function (Asad and Topal 2011). The optimal cut-off grade depends on all the salient technological features of mining, such as the capacity of extraction and of milling, the geometry and geology of the orebody, and the optimal grade of concentrate to send to the smelter (Barr 2012).

Dowd (1976) proposed a method of dynamic programming application, which includes the cut-off grade as a variable and also involves extension of the method to more general case of stochastic programming – which allows for the forecasting of future market conditions in terms of probability with the optimizing criterion of maximizing present value of the future net profits. Lane (1988) described the problem of cut-off grade optimization as dependent on the opportunity cost of deferring the processing of higher grade ore. Assuming future prices, he outlined a dynamic programming approach that was able to ensure that an optimal cut-off grade is chosen for all points in time. However, it only can be estimated through forecasting involving the element of uncertainty.

Incorporating the revenues and costs, a representative word equation for an elementary cash flow calculation has given below:

- Gross profit (GP) = Gross revenue - (operating expenses + depreciation)
- Net profit (NP) = Gross profit (taxable income) – Tax
- Cash flow (CF) = net profit + depreciation

Dagdelen (1992) also explains similar strategies for increasing the value of breakeven cut-off grade during the initial periods of an open pit mining project. King (2011) presented a variation of cut-off grade policies showing the complexities of Lane's approach taking operating and administrative costs into considerations. Asad and Dimitrakopoulos (2013) gave an extension to Lane's heuristic approach for optimal cut-off grade optimization with geological uncertainty, the heuristic approach suggests a unique cut-off grade policy to maximize NPV based on the possible variations in production from mine, processes, and market/refinery during the mine life based on a stochastic framework from equally probable simulated realizations of the orebody model.

Thompson and Barr (2014) employed a new real options model for determining the optimal cut-off grade of ore under stochastic prices. Narrei and Osanloo (2015) studied the significance of reclamation costs on cut-off grade based upon costs and revenues due to reclamation of waste dumps, tailing dam, mine pit and the revenues generated from selling of valuable materials of wastes.

Optimization and determination of optimal cut-off grade using Lane's approach for a single element through a computer aided application, which was developed by the C++/CLI programming language in VC++ IDE (Githiria et al. 2016). Birch (2016) optimized cut-off grades utilizing risk and mixed integer linear programming in excel solver by maximizing either profit or NPV, allows the way to cut-off grade adjustment by minimizing either waste dilution or lost ore, considering grade uncertainties.

Zarshenas and Saeedi (2017) used Lane's algorithm and remarks that with increase of dilution there was a decrease in average grade and subsequently an increase in cut-off grade considering dilution. The optimum cut-off grade that results into the highest NPV of the project must be chosen. Mohammadi et al. (2017) had determined the optimum cut-off grade modifying Lane's algorithm and objective function of maximizing the NPV of future cash flow using Imperialist Competitive Algorithm (ICA).

However, this optimum value of cut-off grade depends upon several factors like metal price, unit costs of production, processing and the grade distributions of the deposit with the varying capacities of mine, mill and refinery, reserve vis-a-vis mine life. This paper had reported that the optimum cut-off grade for a metalliferous deposit is dynamic in nature, a model had been developed for the determination of cut-off grade using dynamic programming. The model had been developed in visual C#, based on simple algebraic equations and iterative algorithm steps for determination of the optimum cut-off grade and the NPV of the project. For this research work we had included a hypothetical case study of a copper mine to demonstrate the functioning of the tool, where we had considered mine production per annum was the only limiting condition, whereas milling and refining does not have any limiting condition.

2. Theoretical framework

2.1 Operational model

In case of metalliferous open pit mine planning and design, the cut-off grade is mostly affected by three governing capacity of operations i.e. mine, mill and refinery. There are eight different possibilities which can be considered while calculating the optimum cut-off grade. However, in this part of the study, only second possibility (mine production per annum is restricted) had been considered for the calculation of optimum cut-off grade as shown in Table 1.

2.2 Model prerequisite

Prerequisite for the application of a cut-off grade optimization model include the development of ultimate pit limit or pit extent and mineable ore reserves in terms of mineral grade and tonnage distribution in the pit limit and

TABLE 1: EIGHT DIFFERENT POSSIBLE COMBINATIONS BASED ON VARIABLE CONDITION

Module	Mine	Mill	Refinery	Restrictions
1st	√	√	√	Mine, mill and refinery are restricted
2nd*	√			Mine restricted
3rd		√		Mill (plant) restricted
4th			√	Refinery (smelter) restricted
5th	√	√		Mine and mill restricted
6th	√		√	Mine and refinery restricted
7th		√	√	Mill and refinery restricted
8th				All unrestricted

mining, processing and refining stage capacities, the operating costs of these stages, and, other technical and economic parameters including metal price.

2.3 Model assumptions

It has been implicitly assumed that the entire deposit as defined by the grade distribution should be mined out, but only that proportion of material mined that is more than or equal to the cut-off grade should be sent to the concentrator. To simplify the problem, it has been assumed that there is no stock piling option of ore and that, at least on average, all ore mined within any twelve months period will also be processed during that period (Stock piling option has not been considered in the present study, however this option can be taken into account).

Mining capacity has been considered as constant throughout the life of the project irrespective of milling and refining capacity constraints. Further, it has also been assumed that the price of the metal, operating costs, fixed costs, tax rate, interest rate and discount rate remain constant over the period.

2.4 Model formulation

The objective function of cut-off grade optimization is to optimise the total net present value (NPV) of a given mine design. The present value (PV) of a discrete series of cash flow is given by equation [1].

$$PV = \sum_{t=1}^{T} \frac{P_t}{(1+i)^t} \quad \text{... [1]}$$

For a continuous cash flow series, when the cash flow is same every year, PV may be calculated using equation [2]. What follows is the development of a generalised function for P(t) for an open pit mine.

$$PV = P * \frac{\{(1+i)^n - 1\}}{i * (1+i)^n} \quad \text{... [2]}$$

Mining operations generate revenue by excavating and processing material to create a saleable product. Revenue is calculated as the product of recoverable ore grade, tonnes of ore processed and the selling price of the product. During operation, mine incurs both fixed costs and variable costs. Variable costs can be further broken down into those associated with excavating and treating ore and those associated with excavating and handling waste. Typical variable ore costs may include: drilling and blasting, hauling, crushing, floatation, tailings disposal among others. Variable waste costs may include drilling and blasting, dumping, remediation and others. Fixed costs include general, administrative costs and any other costs that are independent of production level. Capital costs can simply be discounted and subtracted from the discounted cash flow to determine NPV. Now comprising all the above parameters, sequentially, the equation for the calculation of NPV has given in the equation [3]:

$$NPV = PV - CC \quad \text{... [3]}$$

Subject to: $Qm_n < M \forall n$,

$Qc_n < C \forall n$,

$Qr_n < R \forall n$

where:

P = cash flow

t = time (year) of cash flow

T = total time of the project

i = discount rate

n = period (year) indicator

M = mining capacity (tonnes/year)

C = mill capacity (tonnes/year)

R = refinery capacity (tonnes/year)

CC = capital cost

Q_m = actual quantity of material mined (tonnes/year)

Q_c = actual quantity of ore processes (tonnes/year)

Q_r = actual quantity of final metal produced (tonnes/year)

Assuming that the grade-tonnage distribution of a mineral inventory consists of V grade increments i.e. (g_1, g_2), (g_2, g_3), (g_3, g_4)... ... (g_{v-1}, g_v) and for each grade increment, there exist t_v tonnes of material. In general, if v represents grade increment (g_{v-1}, g_v) and g_{v*} is considered as the cut-off grade, then the quantity of ore T_{ore}, quantity of waste T_{waste} and the average grade of ore $\bar{g}$ are given in the equations [4], [5] and [6] respectively:

$$T_{ore}(g_{v*}) = \sum_{v=v^*}^{V} (t_v) \quad \text{... [4]}$$

$$T_{waste}(g_{v*}) = \sum_{v=1}^{v^*} (t_v) \quad \text{... [5]}$$

$$\bar{g}(g_{v*}) = \frac{\sum_{v=v^*}^{V} \left[t_v \left(\frac{g_v + g_{v+1}}{2} \right) \right]}{T_{ore}(g_v)} \quad \text{... [6]}$$

If y is the overall metallurgical recovery then

$y = y_m * y_c * yr$

where

y_m = mining recovery,

y_c = mill recovery,

y_r = refinery recovery.

Based on the cut-off grade g_{v*}, amount of production in different stages i.e. mine, mill and refinery are subsequently determined by any one of the following conditions:

(1) Set:

$Q_m = M$, $Q_c = Q_m [T_{ore}/(T_{ore} + T_{waste})]$ and $Q_r = Q_c \bar{g}\, y$ where, $\bar{g}$ is average grade of ore.

(2) If $Q_c > C$ from condition 1, then set:

$Q_c = C$, $Q_m = Q_c [(T_{ore} + T_{waste})/T_{ore}]$ and $Q_r = Q_c \bar{g}\, y$

(3) If $Q_r > R$ from condition 2, then set:

$Q_r = R$, $Q_c = Q_r / \bar{g}\, y$ and $Q_m = Q_c [(T_{ore} + T_{waste})/T_{ore}]$

(4) Life of the project will be the complex life of the mine-mill-refinery, as: Life of the mine-mill-refinery complex = $(T_{ore} + T_{waste})/Q_m$

(5) Revenues can be calculated as:

Gross revenue = Q_r * selling price of metal per tonne

(6) Depreciation, in this model, has been assumed to be straight line function of investment with zero salvage value:

Depreciation = Investment/life of the project

(7) Profits are calculated as:

Gross profit = gross revenue – (operating expenses + other costs including general administration cost + fixed costs + interests of the capitals, etc.) – depreciation

Total taxes = gross profit * tax rate

Net profit = gross profit – total taxes = gross profit (1 – tax rate)

(8) Finally the value of operating cash flow is to be calculated as: cash flow = net profit + depreciation

(9) NPV of the project is calculated by evaluating the present value (PV) of all the cash flows using the equation [1] and then subtracting the discounted values of all the investments from it.

3. Development of computer model

A computer model has been developed based on the method for determination of the optimum value of the cut-off grade using the formulae as discussed. The computer model has greater operating flexibility and graphical user interface (GUI) based on user interface programme. It has been based on the simple programming algorithm which iterates through the range of grades to determine the optimum value of the cut-off grade that will give the maximum value of NPV. The developed software and the different modules are discussed below.

3.1 Brief details about the software tool

The followings have been used in the development of this software:

- Language used to develop the computer package is C# 4.0, ideal programming language for development of scientific and business applications which requires great extent of calculation speed and accuracy of data.
- Framework of development is Microsoft. Net Framework 4.0, it supports current version and higher (with back version compatibility).
- Intergraded development interface (IDE) used in this software is Microsoft Visual Studio 2010.
- Database used in this software is internal. Net framework database and dataset.
- This software package has completely GUI based environment which is very user friendly.
- It supports both 32 bit and 64 bit processor.
- Supports all versions of Microsoft Windows (XP/Vista/7/8) and other platforms (LINUX and Mac) which support Microsoft .Net Framework 4.0 compatibility and higher version of the framework.
- Size of the software is compact and can be installed in seconds.
- Supports the low speed capacity computer with as minimum RAM as up to 64MB.
- Results can be printed or exported to the text file which is supported by Microsoft Notepad.

Further, detailed specifications and requirements for C# programming language and development framework can be accessed on the website of Microsoft Visual Studio.

3.2 Modules of the software tool

The software package comprises three modules which are:

1. Input data module
2. Output result module
3. Result graphical module to present the results graphically

3.2.1. Input data module

The input module consists of all the input functions such as the grade range, quantity of material, the various fixed costs and variable costs. The tool is user friendly in nature and it will ask the user whether the values are correctly entered or not and then only it processes the data and moves to graphical module. The following data are to be entered as input data.

1. Grade ranges
2. Quantity of material
3. Cost data for mine, plant, and smelter and refinery.
   (a) Fixed annual costs
   (b) Investment for mine, plant, smelter and refinery
   (c) Variable costs

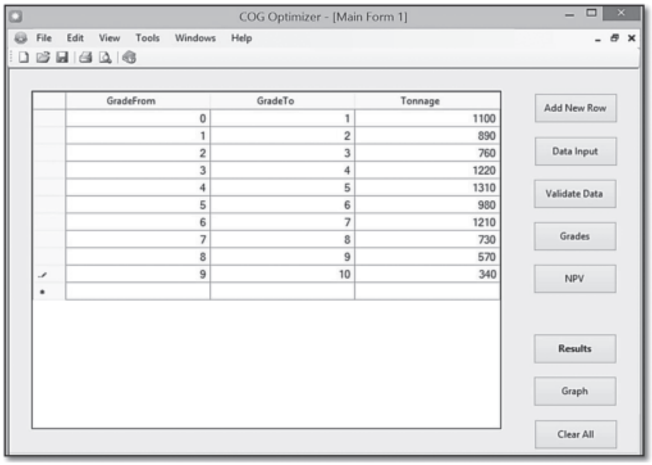

Fig.1 Mineral Inventory data input window in the main window of the software

4. Limiting/maximum capacities of mine, concentrator, smelter and refinery.
5. Other data which includes:
   (a) Sale value of the metal
   (b) Discount rate/year
   (c) Yield/recovery in plant
   (d) Handling loss.

This data input can be categorized into two parts:

1. Mineral inventory data – The mineral inventory input data of grade class intervals and the tonnage distribution for each given class interval. After input data, the data have been validated and rearranged for given grade interval for the whole range starting from zero to the maximum value of the grade in the mineral inventory data. This data can be filled in the data-grid-view (GUI form) given on the main window as shown in Fig.1.
2. Costs and other parameters input – This part of the input module can be accessed by clicking on the "Data Input" button present on the main form window. The Data Input wizard consists of five sections of different input data which are:

(a) Costs inputs for (mine, mill, refinery)

(b) Additional input

(c) Rates input

3.2.2. Output result module

This module gives the final optimum cut-off grade of the deposit in addition to the optimized NPV of the entire operation. The output of the result module has been divided into four sections as shown below:

(1) Mine section

(2) Mill section

(3) Refinery section

(4) Profits section

3.2.3. Result graphical module

The tool provides the following graphical presentations to the user:

(1) Grade distribution v/s tonnage available

(2) Cut-off grade v/s cumulative tonnage

(3) Cut-off grade v/s average grade

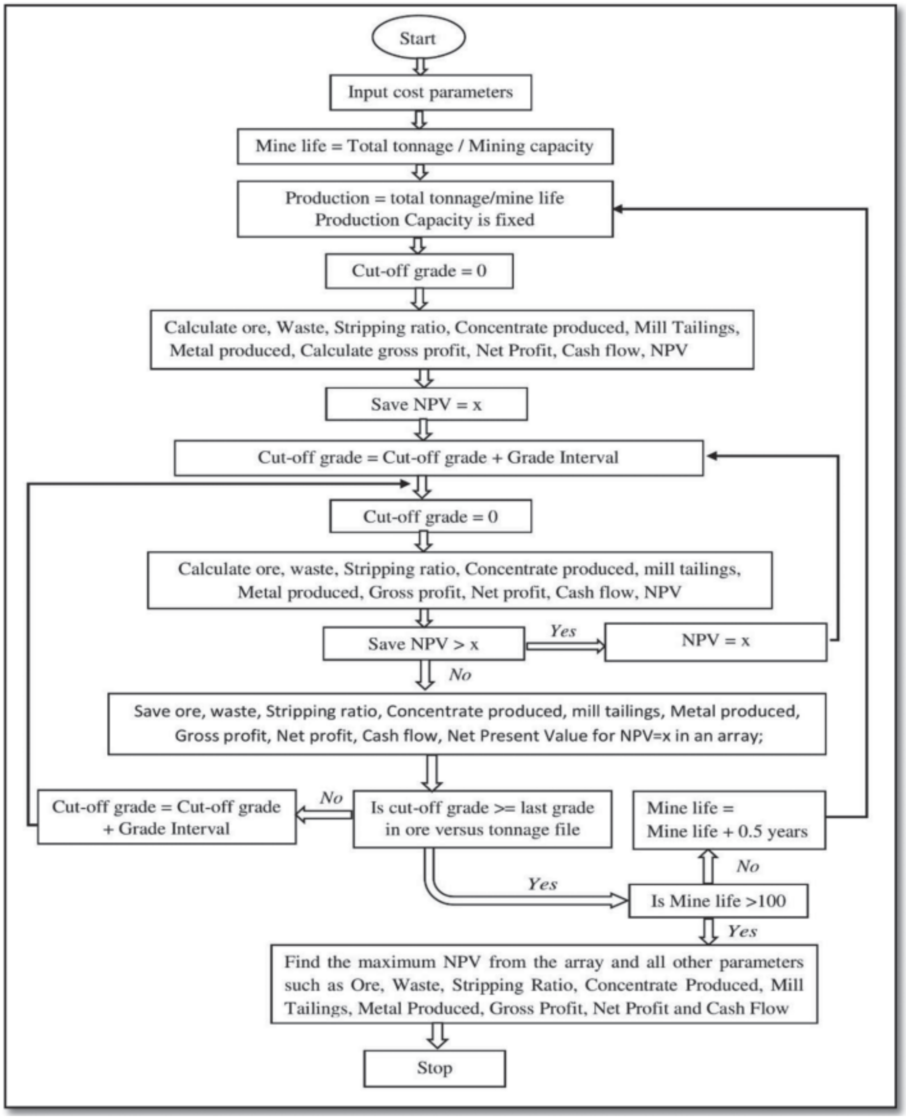

Fig.2 Flowchart of software algorithm

Table 2: Grade tonnage distribution of deposit

	Grade		
	From	To	Tonnage (in tonnes)
1.	0	1	1100.00
2.	1	2	890.00
3.	2	3	760.00
4.	3	4	1220.00
5.	4	5	1310.00
6.	5	6	980.00
7.	6	7	1210.00
8.	7	8	730.00
9.	8	9	570.00
10.	9	10	340.00

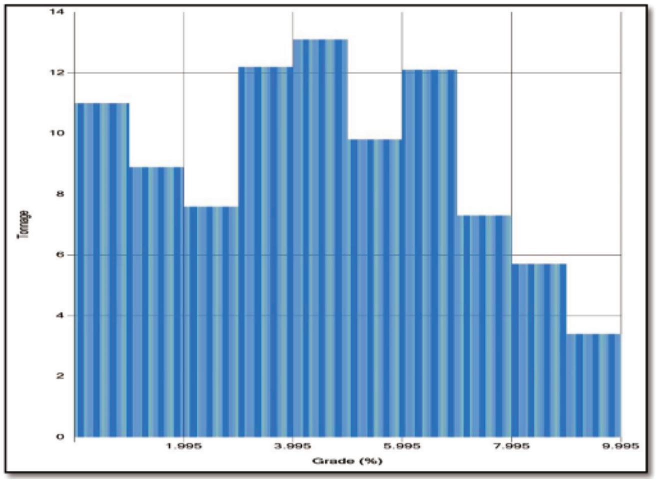

Fig.3 Tonnage versus grade distribution of deposit

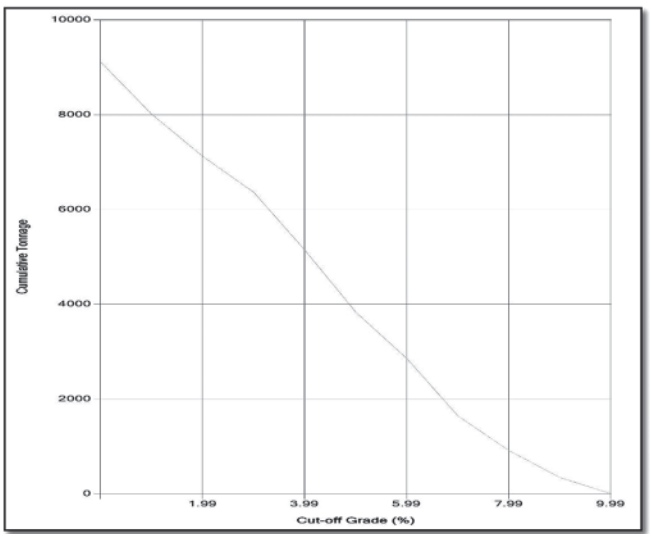

Fig.4 Cumulative tonnage versus cut-off grade

Table 3: Technical and economic data (Rupees) for this case study

Mining		
Particulars	Unit	Values
Maximum mining capacity	tonnes/year	500
Percentage mining recovery	%	90
Specific investment cost for mining	Rs./tonne/year	1200
Unit cost of excavation	Rs./tonne of ore and waste	100
Fixed mining cost	Rs./year	1000
Average ore transportation cost	Rs./tonne/Km	5
Average waste transportation cost	Rs./tonne/Km	5
Average ore transportation haul distance	Km	0.5
Average waste transportation haul distance	Km	2.0
Mill		
Maximum mill capacity	tonnes/year	Unrestricted
Mill concentrate grade	%	26
Percentage mill Recovery	%	85
Specific investment cost for mill	Rs./tonne/year	400
Unit cost of ore processing	Rs./tonne	300
Fixed mill cost	Rs./year	2400
Average concentrate transportation cost	Rs./tonne/Km	5
Average mill tailing transportation cost	Rs./tonne/Km	5
Average concentrate transportation haul distance	Km	2
Average mill tailing transportation haul distance	km	1
Refinery		
Maximum refinery capacity	tonnes/year	Unrestricted
Percentage refinery Recovery	%	98
Specific investment cost for refinery	Rs./tonne/year	1500
Unit cost of refining	Rs./tonne of metal produced	8000
Fixed refinery cost	Rs./year	4800
Average metal transportation cost	Rs./tonne/Km	5
Average slag transportation cost	Rs./tonne/Km	5
Average metal transportation haul distance	Km	15
Average slag transportation haul distance	Km	1
Additional costs		
Other costs including general administration cost (percentage of the total operating cost)	%	15
Rates		
Selling price of metal	Rs/tonne	500000
Tax rate	%	40
Interest rate of investment	%	20
Discount rate	%	15
Grade interval precision		
Grade Interval increment	%	0.01

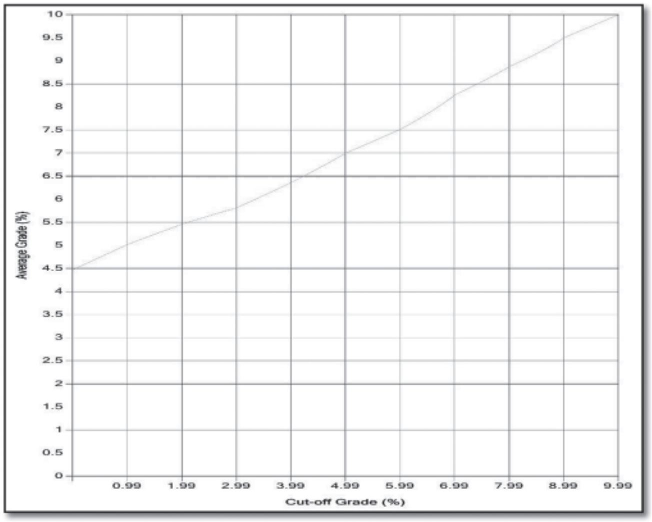

Fig.5 Average grade verses cut-off grade

4. Software algorithm

The main purpose of this software is to compute the optimum value of cut-off grade for the maximum NPV found with respect to the increasing grade value by an iterative algorithm using the arrays to save and manipulate the data. The flow chart of software algorithm is shown in Fig.2.

5. Case study: hypothetic mine

A simplified hypothetical case study is given here as a worked example with relevant data i.e. grade-tonnage distribution, technical and economic data. To simplify the problem the grade tonnage distribution (mineral inventory) data is given in the Table 2.

5.1. Input data

In addition to the grade tonnage deposit (mineral inventory) data, the required technical and economic data has been given in Table 3, which are to be required in the calculation and software input.

5.2 Results output

The above mentioned data was provided to the software and processed by the optimizer, after which the final optimized output of every sections are generated and listed in Table 4:

5.3 Graphs of the results output

GUI of the output data module for the above results output as mentioned in the graphical result module has been shown through graphs depicted in the Figs.3, 4 and 5 respectively.

6. Discussions of results

In this second model of cut-off grade optimization, the COGP tool used dynamic cut-off grade approach to mine the higher cut-off grade during the early years and lower cut-off grade in the later years for faster recovery of the capital invested with the projection of NPV maximization of future cash flows. The final output quantity of mill and refinery are dependent only on

Table 4: Results output as reported from cut-off grade optimizer

Particulars	Unit	Values
Mining		
Actual mine production rate (ore & waste)	tonne/year	500.000
Mid Mill cut-off grade	%	0.110
Mine-Mill-Refinery complex Life	years	18.000
Tonnage of ore produced per year	tonne/year	499.389
Tonnage of waste handled per year	tonne/year	6.722
Total tonnage of ore production	tonne	8989.000
Total tonnage of waste handled	tonne	121.000
Overall stripping ratio		0.013
Mill		
Actual ore processing rate	tonne/year	444.023
Average mill head grade	%	4.533
Tonnage of concentrate produced per year	tonne/year	65.804
Tonnage of mill tailing produced per year	tonne/year	378.219
Total tonnage of concentrate produced	tonnes	1184.465
Total tonnage of mill tailing produced	tonnes	6807.950
Refinery		
Actual metal production rate	tonne/year	16.77
Tonnage of slag produced per year	tonne/year	49.04
Total tonnage of metal produced	tonnes	301.80
Total tonnage of slag produced	tonnes	882.66
Profits		
Total Investment	Rs.	802759.3586
Depreciation	Rs./year	44597.7421
Revenue generated per year	Rs./year	8383379.3212
Annual Operating and maintenance cost	Rs./year	379155.6022
Annual Other costs including GAC	Rs./year	48385.5133
Annual tax	Rs./year	3183850.3907
Net Profit per year	Rs./year	4775775.5861
Annual Cash Flow	Rs./year	4820373.3283
Present value of Cash Flow	Rs.	29539083.2555
Net Present Value (NPV)	Rs.	28736323.8969
Total Revenue generated	Rs.	150900827.7818
Total Net Profit	Rs.	85963960.5500

mining capacity because it was the only fixed capacity among them. The cut-off grade simulation satisfies the production constraints and accommodated the grade-tonnage distribution.

So, it is very clear from the results we got after simulation is that at mine-mill-refinery complex life (years) of 18.00 and average grade (%) of 4.533, we are getting the optimum cut-off grade (%) of 0.11 with maximum NPV of Rs.28736323.8969. NPV starts declining after achieving its peak value and dipped down by the end of mine life with depletion of the reserves by time.

The cut-off grade optimization maximizes undiscounted profit for a given mining operation thus increasing the return of investment. The discounted cash-flows increased in the early years and then decreased later as the ore deposit got depleted.

7. Conclusion and recommendations

In this paper, a block model of a hypothetical copper deposit has been used as input to generate a grade-tonnage distribution of the deposit. The grade tonnage distribution and current economic parameters are then used to create an optimum cut-off grade policy using COGP tool. Cut-off grade optimisation is done to generate a production schedule which is used further for after-cash analysis. This computer model offers definite conceptual advantage and will overcome the shortcomings of other methodologies. The developed computer tool has designed to be user friendly and can work in any Windows based platform. The computer tool computes the optimum cut-off grade based on optimum NPV and gives the various costs associated with mine, mill and refinery.

The computer is able to produce the result in seconds and thus it will turn out to be an effective industry oriented tool. Further study is required to find how COGP applies all other complex mining situations into the cut-off grade optimisation algorithm. For instance, an investigation into how COGP applies situations such as stockpiling, blending and orebody with multiple minerals is necessary. The intension of this model is to introduce a general method for the solution. This method is capable of handling technically more acceptable relationships and definitions.

In conclusion, this research paper provides an insight into dynamic programming application in optimising cut-off grade. The research identified how COGP tool can optimise cut-off grade and its applicability in daily mine planning of any given mine project. However, it is critical that the material classified as waste today could become economical to be processed in future depending on mining conditions and economics, and accordingly things need to be changed.

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