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Reliability analysis of shear strength parameters of rock mass derived using the Hoek-Brown criterion

Determining the mechanical parameters of a rock mass is a difficult but crucial matter in studies pertaining to stability. In this study, the Hoek-Brown criterion is used to derive the shear strength parameters of a rock mass; the parameters are subsequently optimized through reliability analyses, including the nonlinear Mohr-Coulomb envelope, optimized slopes, and least square variance methods. Further, through a case study of the Jianshan open pit mine, the c and φ values of the orebody were comparatively studied using the proposed method. The nonlinear Mohr-Coulomb envelope method and the optimized slopes method can attain reliability values exceeding 80%, as required by Chinese standards.

Keywords: *Hoek-Brown criterion, non-linear Mohr-Coulomb envelope, least square variance, reliability.*

1. Introduction

The mechanical parameters of a rock mass are the fundamental parameters for rock stability evaluation. Therefore, determining reasonable mechanical parameters of a rock mass in practice is a crucial matter. Mechanical experimentation in a laboratory is an effective way to acquire these parameters, but the properties of intact rock cannot represent those of a rock mass owing to the complexity of the mechanical characteristics of a jointed rock mass. Normally [1], the most accurate way to obtain these parameters is to conduct large-scale in situ tests [2], however, this method is prohibitive because of the time consumed, high cost, and other factors [3].

In this study, considering the effect of discontinuities and structures of rock masses, groundwater, and size effect [4], n sets of (σ_i, τ_i) values were acquired through intact rock tests by using the Hoek-Brown criterion. Then the shear strength parameters of rock mass are derived through regression analysis, and finally, the reliability of these shear strength parameters are analyzed using the reliability theory. The application of the reliability theory for analyzing the shear strength parameters of a rock mass yielded results for different probabilities; this method is applicable in rock engineering design [5]. In China, the stipulated probability of rock mass shear strength parameters is 80% for hydropower structures (GB50199-94, 1994). Three regression methods, namely, nonlinear Mohr envelope, least square method and predominant slope method are employed in this study.

2. Hoek-Brown criterion

By studying the statistical analysis of a large amount of triaxial test data and results from in situ experiments for rock mass and considering both the Griffith theory and modified Griffith theory, Hoek and Brown proposed Equation 1, in 1980, to express the relationship between principal stresses when a rock mass fails. This relationship is also known as the Hoek-Brown criterion [6].

$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2} \quad \ldots \quad (1)$$

where σ_1 stands for the maximum principal stress when a rock mass fails; σ_3 is the minimum principal stress; σ_c represents

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the uniaxial compressive strength of the intact rock; *m* and *s* are material constants, and also can be described as the functions of the geological strength index (GSI) and are expressed as shown in the following equation:

$$\left.\begin{aligned} m &= m_i e^{\frac{GSI-100}{28-14D}} \\ s &= e^{\frac{GSI-100}{9-3D}} \end{aligned}\right\} \quad \ldots \quad (2)$$

where *D* is the parameter of the disturbing degree of rock mass, with the value ranging between 0 and 1, where 0 represents the undisturbed state; m_i stands for the Hoek-Brown constant of the intact rock [7], which can be acquired from laboratory tests; GSI is the geological strength index proposed by Kaiser and Brown in 1995, and it is used to estimate the value of rock strength under different geological conditions[8]. Normally, the value of GSI is determined in terms of the geological condition and the structural and surface properties of the rock mass. However, it may be difficult to determine GSI if the rock structures are not quantitatively described. Consequently, the rock mass rating (RMR) can be used to quantitatively classify the rock mass. Bieniawski[9] showed that the value of GSI could be derived from the modified RMR index [10].

The Mohr envelope, namely the normal and shear stresses on the failure plane, can be derived from the following equation [11]:

$$\left.\begin{aligned} \sigma &= \sigma_3 + \frac{\tau_m{}^2}{\tau_m + \frac{m\sigma_c}{8}} \\ \tau &= (\sigma - \sigma_3)\sqrt{1 + \frac{m\sigma_c}{4\tau_m}} \\ \tau_m &= \frac{(\sigma_1 - \sigma_3)}{2} \end{aligned}\right\} \quad \ldots \quad (3)$$

After substituting σ_1 and σ_3 into equation (3), the (σ_i, τ_i) of the *n* points can be computed and plotted on an σ_i-τ_i plane. Regression analysis of the *n* sets of (σ_i, τ_i) can determine the shear strength parameters used for Mohr's criterion.

3. Methods to determine the shear strength parameters of a rock mass

3.1 Nonlinear Mohr-Coulomb envelope method

As the shear strength of rock mass, especially disturbed rock mass, tends to be nonlinear, Hoek proposed a nonlinear relation equation (4):

$$\tau = A\sigma_c(\sigma / \sigma_c - T)^B \quad \ldots \quad (4)$$

where A and B are constants. Equation (4) can be rewritten as equation (5):

$$y = ax + b \quad \ldots \quad (5)$$

where $y = ln\tau/\sigma_c$, $x = ln(\sigma/\sigma_c - T)$, $a = B$, $b = lnA$, and $T = \frac{1}{2}\left(m - \sqrt{m^2 + 4s}\right)$.

Constants A and B can be determined using the least square method:

$$\left.\begin{aligned} \ln A &= \sum y / n - B\left(\sum x / n\right) \\ B &= \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}} \end{aligned}\right\} \quad \ldots \quad (6)$$

The coefficient of fitting is as follows:

$$r^2 = \frac{\left[\sum xy - (\sum x \sum y)/n\right]^2}{\left[\sum x^2 - (\sum x)^2/n\right]\left[\sum y^2 - (\sum y)^2/n\right]} \quad \ldots \quad (7)$$

From equation (4), when $\sigma = 0$, $\tau = c_m$, the cohesion of the rock mass is as given below.

$$c_m = A\sigma_c(-T)^B \quad \ldots \quad (8)$$

The instantaneous tangential friction angle at any point σ_i on the nonlinear Mohr envelope can be derived from equation (4):

$$\phi_i = \arctan[AB(\frac{\sigma_i}{\sigma_c} - T)^{B-1}] \quad \ldots \quad (9)$$

The overall average friction angle of rock mass can be expressed by equation (10):

$$\phi_m = \frac{\sum_{i=1}^{n} \phi_i}{n} \quad \ldots \quad (10)$$

In rock slope engineering, the maximum value of lateral confining stress $\sigma_{3\max}$ can be determined by equation (11):

$$\sigma_{3\max} = 0.72\sigma_{mc}(\frac{\sigma_{mc}}{\gamma H})^{-0.91} \quad \ldots \quad (11)$$

where γ represents the unit weight of the rock mass; *H* is the slope height; σ_{mc} is the uniaxial compressive strength of the rock mass, and it can be computed by the equivalent Mohr-Coulomb strength parameters:

$$\sigma_{mc} = \frac{2c\cos\phi}{1 - \sin\phi} \quad \ldots \quad (12)$$

Through the iterative processes from equations (1) to (12), the shear strength parameters of rock mass can be derived for Mohr's criterion.

3.2 Author information optimized slope method

A σ_i-τ_i graph is plotted in a σ-τ plane, and then the optimized slopes of the σ_i-τ_i curve are determined based on the distribution and trend of the group data [12]. Thus, the range of the *c* and *f* values is obtained. The lower end of the range is recommended as the shear strength parameter of the rock mass. The limitation of this method is the subjectiveness in selecting *c* and *f*.

3.3 Least square variance method

The univariate linear regression equation (13) can be statistically optimized to determine *c* and *f* using the least square variants of *n* points (σ_i,τ_i).

$\tau = cf + \sigma$ ... (13)

4. Reliability analysis

Reliability analysis [13] can overcome the limitation that the probabilistic distribution of *c* and *f* cannot be quantitatively determined by the three aforementioned methods. The basic assumption of this reliability analysis model is that each (σ_i, τ_i) point has a pair of corresponding (c_i, φ_i) that satisfies equation (14):

$$\tau_i = \sigma_i \tan \phi_i + c_i \quad \text{... (14)}$$

Apparently, the probability distribution of *c* and φ can be determined once n sets of (c_i, φ_i) are acquired. However, there are 2*n* unknowns with only n equation. To solve this problem, the reliability model is set up in accordance with the maximum-likelihood criterion: (c_i, φ_i) satisfy equation (14) at the highest probability. According to the reliability theory, the following limiting equation holds:

$$g(\tau_i, \sigma_i) = \tau_i - \sigma_i \tan\phi_i - c_i = 0 \quad \text{... (15)}$$

Therefore, the value of the design check point of equation 15 is the solution for (c_i, φ_i). The process employs the distribution of *c* and φ_i and an iterative algorithm calculation procedure is as follows:

(1) Assume a distribution pattern of *c* and φ, and determine the initial mean value, variance, and correlation coefficient.

(2) Calculate the values of the design check points c_i, φ_i (i = 1, 2, ……, n) by employing equation (15).

(3) For the result obtained in step 2, calculate the mean, variance, and correlation coefficient, and compare them with the initial values in step 1. If they are equal, go to the next step; else, use the calculated result as the initial value, and repeat step 2.

4.1 Modified first-order second-moment method

The reliability index β and $P^*(x^*1, x^*2, ..., x^*n)$ can be calculated using the following equation:

$$\left.\begin{aligned} x_i^* &= \overline{x_i} - \alpha\beta\delta_i \quad i = (1,2,\cdots)n \\ \alpha_i &= \frac{\delta_i \left.\frac{\partial g}{\partial x_i}\right|_{x_i^*}}{\sqrt{\sum_{i=1}^{n}\left(\delta_i \left.\frac{\partial g}{\partial x_i}\right|_{x_i^*}\right)^2}} \left(i = 1,2,\cdots n\right) \\ \beta &= \frac{g(x_1^*, \cdots, x_n^*) + \sum_{i=1}^{n}(\overline{x_i} - x_i^*)\left.\frac{\partial g}{\partial x_i}\right|_{x_i^*}}{\sum_{i=1}^{n}\alpha\delta_{ii}\left.\frac{\partial g}{\partial x_i}\right|_{x_i^*}} \end{aligned}\right\} \quad \text{... (16)}$$

Note that the coordinates of *P** are not known before solving the above equation, and therefore, an iterative algorithm should be employed. The equation for $g\,(x^*1, x^*2, ..., x^*n) = 0$ cannot be satisfied because of the assumed values of *P**, and hence, $g\,(x^*1, x^*2, ..., x^*n)$ should be added to the numerator of the equation for calculating β.

4.2 Reliability analysis

A probability model for c and φ, where $\varphi = \frac{180^{\circ}}{\pi} \, arc \tan f$, has to be established.

Assuming that the distributions of *c* and φ are normal, and assuming [14] that *c* and φ are related to each other (Cai et al., 2016), the following procedure is adopted:

(1) Set the initial values of the average and variance of *c* as $\overline{\varphi_0}$ and δ_1^0, respectively, and those of φ as $\overline{c_0}$ and δ_2^0, respectively; their initial correlation coefficient is ρ_0.

(2) Further, *c* and φ are represented by two independent normal variables of x_1 and x_2, as *c* and φ are correlated.

$$\left.\begin{aligned} \phi &= \delta_1^0 x_1 - \delta_1^0 x_2 + \overline{\phi_0} \\ c &= \delta_2^0 x_1 + \delta_2^0 x_2 + \overline{c_0} \end{aligned}\right\} \quad \text{... (17)}$$

where x_1 and x_2 are independent normal variables, and the mean value is $\overline{X_1} = \overline{X_2} = 0$. Their standard deviations are as follows:

$$\left.\begin{aligned} \delta_{x_1} &= \sqrt{1 + \rho_0} \\ \delta_{x_2} &= \sqrt{1 - \rho_0} \end{aligned}\right\} \quad \text{... (18)}$$

Equation (15) is rewritten as follows:

$$\begin{aligned} g(x_1, x_2) = \tau_i - \sigma_i \tan[\frac{\pi}{180}(\delta_1^0 x_1 - \delta_1^0 x_2 + \overline{\phi_0})] \\ -(\delta_2^0 x_1 + \delta_2^0 x_2 + \overline{c}_0) \end{aligned} \quad \text{... (19)}$$

$$\frac{\partial g}{\partial x_1} = \frac{-\frac{\pi}{180}\sigma\delta_1^0}{\cos^2[\frac{\pi}{180}(\delta_1^0 x_1 - \delta_1^0 x_2 + \overline{\phi_0})]} - \delta_2^0 \quad \text{... (20)}$$

$$\frac{\partial g}{\partial x_2} = \frac{\frac{\pi}{180}\sigma\delta_1^0}{\cos^2[\frac{\pi}{180}(\delta_1^0 x_1 - \delta_1^0 x_2 + \overline{\phi_0})]} - \delta_2^0 \quad \text{... (21)}$$

(3) Set the initial value to x_1 and x_2, $X_1^* = \overline{X_1} = 0$, $X_2^* = \overline{X_2} = 0$.

(4) Calculate for $\left.\frac{\partial g}{\partial x_i}\right|_{x_i^*}$, i = 1, 2.

(5) Calculate $\alpha_{xi} = \frac{\delta_{x_i}\left.\frac{\partial g}{\partial x_i}\right|_{x_i^*}}{\sqrt{\sum_{i=1}^{n}\left(\delta_{x_i}\left.\frac{\partial g}{\partial x_i}\right|_{x_i^*}\right)^2}}, (i2= \;)$, and then,

subsequently calculate β.

(6) Calculate the new $X_{i1}^* = \overline{X}_i - \alpha_{xi}\beta\delta_{xi}\ (i=1,2)$.

(7) Calculate $e_i = \left|X_{i1}^* - X_i^*\right|$, $i = 1,2$. If e_i is less than 0.00001, go to the next step, else, let $X_i^* = X_{i1}^*\ (i=1,2)$, and repeat steps (4) to (7).

(8) The design check points $\left(X_{11}^*, X_{21}^*\right)$ are calculated by using equation (19). As there are n reference points (σ_i, τ_i) corresponding to n design check points $\left(X_{11}^*, X_{21}^*\right)$, n sets of ($c_i$, φ_i) are derived as follows:

$$\left.\begin{aligned} \phi_i &= \delta_1^0 x_{11}^* - \delta_1^0 x_{21}^* + \overline{\phi_0} \\ c_i &= \delta_2^0 x_{11}^* + \delta_2^0 x_{21}^* + \overline{c_0} \end{aligned}\right\} \quad \text{...} \quad (22)$$

(9) The mean, variance, and correlation coefficient of c and φ can be derived from the results obtained using equation (22):

$$\overline{\phi} = \sum_{i=1}^{n} \frac{\phi_i}{n},\ \delta_1 = \sqrt{\frac{\sum_{i=1}^{n}(\phi_i - \overline{\phi})^2}{n-1}}$$

$$\overline{c} = \sum_{i=1}^{n} \frac{c_i}{n},\ \delta_2 = \sqrt{\frac{\sum_{i=1}^{n}(c_i - \overline{c})^2}{n-1}}$$

$$\rho = \frac{\sum_{i=1}^{n}(\phi_i - \overline{\phi})(c_i - \overline{c})}{\sqrt{\sum_{i=1}^{n}(\phi_i - \overline{\phi})^2}\sqrt{\sum_{i=1}^{n}(c_i - \overline{c})^2}}$$

$$e_1 = \left|\overline{\phi} - \overline{\phi_0}\right|, e_2 = \left|\overline{c} - \overline{c_0}\right|, e_2 = \left|\delta_1 - \delta_1^0\right|, e_2 = \left|\delta_2 - \delta_2^0\right|,$$

$$e_2 = \left|\rho - \rho_0\right|.$$

If e_i (i = 1,2,…,5) is less than 0.001, go to the next step; else, set $\overline{\varphi_0} = \overline{\varphi}$, $\overline{c_0} = \overline{c}$, $\delta_1^0 = \delta_1$, $\delta_2^0 = \delta_2$ and $\rho_0 = \rho$, and go back to step (2). Thus, n sets of (c_i, φ_i) and their statistical parameters are determined.

The statistical analysis of the calculated results shows that the correlation coefficient ρ tends to approach 1.0, indicating the correlation of c and φ. Therefore, the probability distribution model of c and φ can be determined as follows:

$$\left.\begin{aligned} \phi &= \overline{\phi} + \delta_1\theta \\ c &= \overline{c} + \delta_2\theta \end{aligned}\right\} \quad \text{...} \quad (23)$$

where θ is a variable of the standard normal distribution with a mean value of 0 and variance 1.0. The distribution of β is reflected by θ. Thus, the required reliability of c and φ is a function of θ, as shown in equation (24):

$$p = \Phi(-\theta) = 1 - \Phi(\theta) = 1 - \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\theta} e^{-\frac{x^2}{2}} dx \quad \text{...} \quad (24)$$

5. Case study

5.1 Project overview

The Jianshan phosphorus open pit mine contains marine sedimentary phosphorus deposits. At present, the overall height of the slope is 310 m. The strike of the orebody is approximately east-west, and the dip angle is 45°. The ore body is 21 m thick. The footwall of the orebody is 46 m thick with a dip angle of 46° and consists of dolomite and sandy dolomite. There are clear bedding planes of lower cohesion in the rock and orebody.

5.2 Rock quality evaluation

The physical and mechanical properties of the rock are listed in Table 1. The RMR results of the orebody and country rock are listed in Table 2.

Table 1: Physical and mechanics parameters of the rock in laboratory

Type	Density (g/cm^3)	Uniaxial compressive strength (MPa)	Uniaxial tensile strength (MPa)
Orebody	2.77	82.89	3.82
Dolomite and sandy dolomite	2.78	218.94	6.23

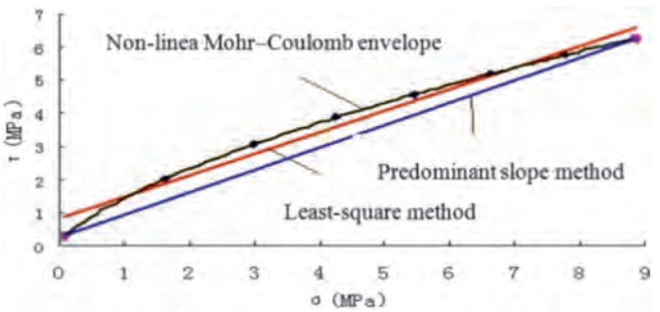

Fig.1 Comparison of the σ-τ fitted curve

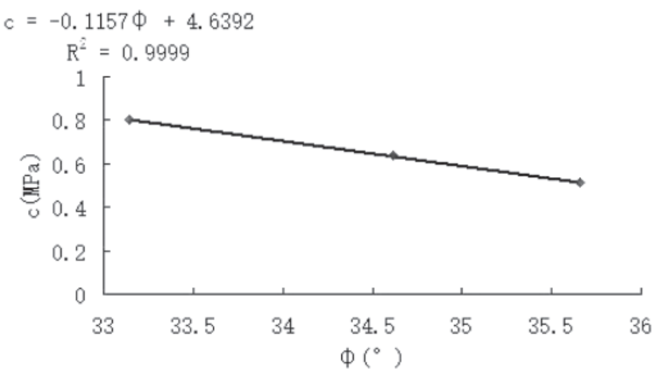

Fig.2 Correlation curve of c and φ

TABLE 2: RMR EVALUATION RESULTS OF THE ROCK MASS

Type	Classification parameters						
	UCS	RQD	Joint spacing	Joint state	Groundwater	Corrected value	RMR
Orebody	8	7	8	20	10	-11	42
Dolomite and sandy dolomite	13	8	8	18	10	-11	46

TABLE 3: RESULTS AND COMPARISON OF THE RESULTS OF RELIABILITY ANALYSIS OF C AND φ IN TERMS OF THE NON-LINEAR MOHR-COULOMB ENVELOPE METHOD

Type	Reliability method								Conventional method	
	Standard deviation		50% guarantee rate		80% guarantee rate		90% guarantee rate		Non-linear Mohr-Coulomb envelope method	
	δ1	δ2	φ/ (°)	c/MPa	φ/ (°)	c/MPa	Φ/ (°)	c/MPa	φ/ (°)	c/MPa
Orebody	0.32537	0.32537	35.655	0.5134	35.379	0.2368	35.236	0.09366	35.284	0.1419
Dolomite and sandy dolomite	0.36609	0.36609	40.937	1.2632	40.626	0.9521	40.465	0.791	40.236	0.5623

TABLE 4: RESULT AND COMPARISON CHART OF RELIABILITY ANALYSIS OF C AND φ IN TERMS OF PREDOMINANT SLOPE METHOD

Type	Reliability method								Conventional method	
	Standard deviation		50% guarantee rate		80% guarantee rate		90% guarantee rate		Predominant slope method	
	δ1	δ2	φ/ (°)	c/MPa	φ/ (°)	c/MPa	φ/ (°)	c/MPa	φ/ (°)	c_{min}/MPa
Orebody	0.30059	0.30059	34.617	0.6356	34.362	0.3801	34.229	0.2478	34.22	0.2385
Dolomite and sandy dolomite	0.35562	0.35562	40.505	1.3392	40.203	1.0369	40.046	0.8804	40.03	0.8642

TABLE 5: RESULT AND COMPARISON CHART OF RELIABILITY ANALYSIS OF C AND φ IN TERMS OF LEAST SQUARES METHOD

Type	Reliability method								Conventional method	
	Standard deviation		50% guarantee rate		80% guarantee rate		90% guarantee rate		Least square method	
	δ1	δ2	φ/ (°)	c/MPa	φ/ (°)	c/MPa	φ/ (°)	c/MPa	φ/ (°)	c/MPa
Orebody	0.29452	0.29452	33.146	0.8039	32.896	0.5536	32.766	0.424	33.148	0.8055
Dolomite and sandy dolomite	0.35403	0.35403	39.242	1.5562	38.941	1.2553	38.785	1.0995	39.245	1.5593

5.3 RESULTS AND DISCUSSION

In this study, σ_3 is increased from 0 to σ_{3max} by seven equal increments [15]. D = 0.7 according to the RMR and the degree of excavation disturbance. The m_i value of the orebody is determined as 11.368 and 9.313 for the country rock based on the results of the laboratory tests and from the reference, respectively [16]. The values of m and s were derived from equations (2) and (3). The shear strength parameters are determined using equations (4) to (13). The reliability analysis is conducted using equations (14) to (24). The results are presented in Tables 4 and 5. The σ-τ curve fitted for the orebody is shown in Fig.1 and the corresponding correlation of c and φ is given in Fig.2, which shows a strong linear correlation between c and φ.

From Tables 3, 4, and 5, it is seen that the reliability of the shear strength parameter reach 80-90%, 90%, and 50% for the nonlinear Mohr-Coulomb envelope method, optimized slope method, and least square variance method, respectively. In other words, the reliability of the results of the first two methods can meet the requirements of GB50199-94 (1994) in China.

6. Conclusion

Based on the Hoek-Brown criterion, the uniaxial compressive strength of the rock mass, and RMR, sets of (σ_i, τ_i) are determined, and the probability distributions of c and φ are quantitatively analyzed using reliability analysis. Further, c and φ are determined with the required reliability that complies with the Chinese standard GB50199-94 (1994). A case study demonstrated that the reliability analysis method is feasible and reasonable for determining shear strength parameters in rock engineering.

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