Study on phase sensitive detection technology of multi-frequency signals in electrical logging

Based on the principle of phase sensitive detection (PSD) technology, the multi-frequency digital PSD algorithm is deduced to achieve the constrained relationship of sampling frequency, sampling number and signal frequency in the PSD of multi-frequency signals. The results of simulation experiment show that the amplitude detection error is controlled within 0.5% and the phase detection error is controlled within 0.2% when the signal to noise ratio (SNR) is as low as -10dB.

Keywords: *Electrical logging, multi-frequency signals, PSD*

1. Introduction

In petroleum electrical logging, by measuring the amplitude and phase of the response signals with the same frequency, the apparent resistivity of the target stratum is quantitatively calculated so that the parameter of stratum is determined to achieve the purpose of logging. In short, excitation signals with different frequencies correspond to investigation depths of different strata. When multiple-frequency signals excite at the same time, not only the logging efficiency can be improved, but the multi-dimensional information of strata can be obtained to improve fine exploration capability. Whether the multi-frequency logging signals are accurately detected directly affects the accuracy of the measurement of the target stratum parameters. Common amplitude and phase detection methods include PSD technology and fast fourier transform (FFT). Compared to FFT that requires high-cost hardware, PSD technology does not ask for much for hardware and determines the amplitude and phase of the frequency signal through phase-locked detection. In the real-time detection system, acquisition can be performed while calculation can be conducted. Calculation results do not require a large amount of memory space, and the calculation amount is small. Therefore, PSD technology is often used in the detection of petroleum logging signals. Based on the principle of PSD technology, this study deduces the algorithm formula of PSD of multi-frequency signals, analyzes the performance of the algorithm, and further verifies the correctness of the algorithm through simulation.

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2. Principle of PSD

2.1 Principle of PSD

The principle of digital PSD is shown in Fig.1, where s(*n*) is detected signal, cos(*n*), sin(*n*) and s(*n*) are orthogonal reference signals of the same frequency. Through detection the magnitude A_m and phase φ of the signal can be obtained.

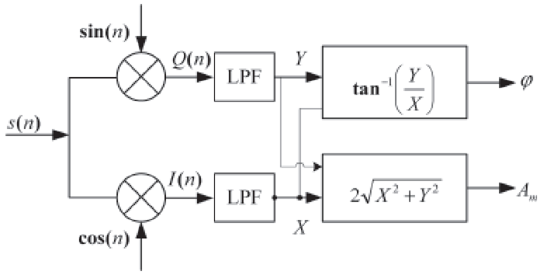

Fig.1 Principle of digital PSD

Assume that,

$$\mathbf{s(n)} = s_{DC} + A_m \,\mathbf{cos}\left(\frac{2\pi fn}{f_s} + \varphi\right), 0 \le n \le N-1 \quad \text{... (1)}$$

$$\mathbf{cos(n)} = \mathbf{cos}\left(\frac{2\pi fn}{f_s}\right),\ 0 < n < N-1 \quad \text{... (2)}$$

$$\sin(n) = \sin\left(\frac{2\pi fn}{f_s}\right),\ 0 < n < N-1 \quad \text{... (3)}$$

Where, s_{DC} is DC component of the signal, f is the frequency of detected signal, f_s is sampling frequency of system and N is sampling number.

$$I(n) = s(n)\cos(n) = \frac{1}{2}A_m\cos\phi + \frac{1}{2}A_m\cos\left(\frac{4\pi fn}{f_s} + \phi\right) + s_{DC}\cos\left(\frac{2\pi fn}{f_s}\right) \quad \text{... (4)}$$

$$Q(n) = s(n)\sin(n) = -\frac{1}{2}A_m\sin\phi + \frac{1}{2}A_m\sin\left(\frac{4\pi fn}{f_s} + \phi\right) + s_{DC}\sin\left(\frac{2\pi fn}{f_s}\right) \quad \text{... (5)}$$

The calculation results of Formulas (4) and (5) include a DC signal, a signal with the same frequency, and two frequency-doubling signals. The amplitude and phase information of the detection signal is contained in the DC term, which can be obtained by suitable low-pass filtering. The DC amount s_{DC} in the original signal does not affect the detection result. After *I(n)* and *Q(n)* pass low-pass filtering, it can be obtained:

$$X = \frac{1}{2} A_m \cos\varphi \qquad \text{... (6)}$$

$$Y = -\frac{1}{2} A_m \sin\varphi \qquad \text{... (7)}$$

So the amplitude and phase are detected as:

$$A_m = 2\sqrt{X^2 + Y^2} \qquad \text{... (8)}$$

$$\varphi = -\tan^{-1}\left(\frac{Y}{X}\right) \qquad \text{... (9)}$$

2.2 Principle of multi-frequency PSD

Assume that the multi-frequency signals to be detected is:

$$s_1(n) = s_{DC} + \sum_{i=1}^{M} A_i \cos\left(\frac{2\pi f_i n}{f_s} + \varphi_i\right),\ 0 < n < N-1 \qquad \text{... (10)}$$

Where, M is the number of frequency. According to the principle of PSD, when the detection frequency is f_i, it is necessary to multiply this signal by orthogonal signals cos(n) and sin(*n*) with the frequency of f_i. For the detection of a signal with M frequencies, the signal shall be multiplied by the orthogonal signal of the same frequency in turn. Each detection can only detect the amplitude and phase of a frequency signal. The signal of *M* frequencies needs to be detected by PSD for *M* times. The frequency to be checked is set as f_k, then:

$$\begin{aligned} I_k(n) = s_1(n)\cos(n) &= s_{DC}\cos\frac{2\pi f_k n}{f_s} \\ &+ \frac{1}{2}\left[\sum_{i=1}^{M} A_i \cos\left(\frac{2\pi(f_i + f_k)n}{f_s} + \phi_i\right)\right] \\ &+ \frac{1}{2}\left[\sum_{i=1}^{M} A_i \cos\left(\frac{2\pi(f_i - f_k)n}{f_s} + \phi_i\right)\right] \end{aligned} \qquad \text{... (11)}$$

$$\begin{aligned} Q_k(n) = s_1(n)\sin(n) &= s_{DC}\sin\frac{2\pi f_k n}{f_s} \\ &+ \frac{1}{2}\left[\sum_{i=1}^{M} A_i \sin\left(\frac{2\pi(f_i + f_k)n}{f_s} + \phi_i\right)\right] \\ &- \frac{1}{2}\left[\sum_{i=1}^{M} A_i \sin\left(\frac{2\pi(f_i - f_k)n}{f_s} + \phi_i\right)\right] \end{aligned} \qquad \text{... (12)}$$

$I_k(n)$ and $Q_k(n)$ include a f_k, $M(f_i+f_k)$ and $M(f_i-f_k)$. When f_k is equal to f_i, $I_k(n)$ and $Q_k(n)$ include DC component that contain amplitude and phase information, enabling detection of A_k and φ_k.

2.3 Design of optimal filtering

To detect the magnitude A_k and phase φ_k of a signal from Formulas (4)-(5) and (11)-(12), the key is to design an optimal filter that filters out irrelevant frequency components and preserves DC components containing amplitude and phase information. The filter in this study adopts digital average and the principle is shown in Fig.2.

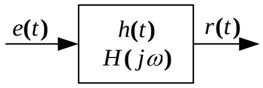

Fig.2 The principle of averaging filter

$$\begin{aligned} r(t) &= \frac{1}{N}\sum_{i=0}^{N-1} e(t - iT_s) = \frac{1}{N}\sum_{i=0}^{N-1} e(t) * \delta(t - iT_s) \\ &= e(t) * \frac{1}{N}\sum_{i=0}^{N-1}\delta(t - iT_s) \end{aligned} \qquad \text{... (13)}$$

Where, *N* is average number of points and T_s is sampling interval. The impulse response function *h*(*t*) of this system is:

$$h(t) = \frac{1}{N}\sum_{i=0}^{N-1}\delta(t - iT_s) \qquad (14)$$

Fourier transform is conducted on Formula (14) to obtain the frequency response function $H(j\omega)$:

$$\begin{aligned} H(j\omega) &= \int_{-\infty}^{+\infty} h(t)e^{-j\omega t}dt \\ &= \int_{-\infty}^{+\infty}\frac{1}{N}\sum_{i=0}^{N-1}\delta(t - iT_s)e^{-j\omega t}dt \\ &= \frac{1}{N}\sum_{i=0}^{N-1} e^{-j\omega(iT_s)} = \frac{1}{N}\frac{1 - e^{-jN\omega T_s}}{1 - e^{-j\omega T_s}} \\ &= \frac{1}{N}\frac{\sin(N\omega T_s/2)}{\sin(\omega T_s/2)}e^{-j(N-1)\omega T/2} \end{aligned} \qquad (15)$$

The corresponding amplitude frequency response is:

$$\left|H(j\omega)\right| = \frac{1}{N}\left|\frac{\sin(N\omega T_s/2)}{\sin(\omega T_s/2)}\right| \qquad (16)$$

Set $\omega = 2\pi f$ and substitute $T_s = 1/f_s$ into Formula (16), then:

$$\left|H(jf)\right| = \frac{1}{N}\left|\frac{\sin(N\pi f/f_s)}{\sin(\pi f/f_s)}\right| \qquad (17)$$

In Formula (17), when it meets

$$\frac{Nf}{f_s} = k \text{ or } f = \frac{kf_s}{N}\quad 1 \le k < 0.5N \text{ and } k \text{ is integer} \qquad (18)$$

Then, $|H(jf)| = 0$

That is to say, when the frequency *f*, sampling frequency f_s, and sampling number (the average number of filtering points) *N* contained in the excitation signale (*t*) satisfy the requirement of Formula (18) (namely, sampling one (*t*) for the entire period), the response of the frequency signal to the filter is zero. This property is used to filter out the multi-frequency AC signals in Formulas (11) and (12), and the DC signals containing the amplitude and phase information to be measured are preserved so as to achieve the purpose of detecting determined frequency signals.

When N = 10, the corresponding response curve of Formula (17) is shown in Fig.3. When the frequency f is 0.1 f_s, 0.2f_s, 0.3f_s, 0.4f_s, 0.5f_s, its response frequency is zero and the bandwidth of the filter is 0.0886 f_s. Theoretically, it can arbitrarily reduce f_s/N (frequency resolution) to obtain filter of any narrow bandwidth. When the excitation signale (t) contains white noise, in-band noise affects the accurate detection of the signal.

3. Design of multi-frequency detection algorithm

Petroleum logging systems require real-time detection of multi-channel and multi-frequency signals, so effective design of the algorithm is critical. As shown in Fig.1, PSD algorithm must implement the multiplication of N data and the addition of N data in turn. When the signal to be detected contains M frequencies, the calculated amount of PSD algorithm is M times of that of the single-frequency PSD algorithm. Assume that the signal to be measured contains M frequencies, the sampling frequency is f_s, and the sampling number is N, it can obtain according to the principle of multi-frequency PSD:

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_M \end{bmatrix} = \frac{1}{N} \begin{bmatrix} \cos\left(\frac{2\pi f_1 \cdot 0}{f_s}\right) & \cos\left(\frac{2\pi f_1 \cdot 1}{f_s}\right) & \cdots & \cos\left(\frac{2\pi f_1 \cdot (N-1)}{f_s}\right) \\ \cos\left(\frac{2\pi f_2 \cdot 0}{f_s}\right) & \cos\left(\frac{2\pi f_2 \cdot 1}{f_s}\right) & \cdots & \cos\left(\frac{2\pi f_2 \cdot (N-1)}{f_s}\right) \\ \vdots & \vdots & \vdots & \vdots \\ \cos\left(\frac{2\pi f_M \cdot 0}{f_s}\right) & \cos\left(\frac{2\pi f_M \cdot 1}{f_s}\right) & \cdots & \cos\left(\frac{2\pi f_M \cdot (N-1)}{f_s}\right) \end{bmatrix} \begin{bmatrix} s_1(0) \\ s_1(1) \\ \vdots \\ s_1(N-1) \end{bmatrix} \quad \text{... (19)}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_M \end{bmatrix} = \frac{1}{N} \begin{bmatrix} \sin\left(\frac{2\pi f_1 \cdot 0}{f_s}\right) & \sin\left(\frac{2\pi f_1 \cdot 1}{f_s}\right) & \cdots & \sin\left(\frac{2\pi f_1 \cdot (N-1)}{f_s}\right) \\ \sin\left(\frac{2\pi f_2 \cdot 0}{f_s}\right) & \sin\left(\frac{2\pi f_2 \cdot 1}{f_s}\right) & \cdots & \sin\left(\frac{2\pi f_2 \cdot (N-1)}{f_s}\right) \\ \vdots & \vdots & \vdots & \vdots \\ \sin\left(\frac{2\pi f_M \cdot 0}{f_s}\right) & \sin\left(\frac{2\pi f_M \cdot 1}{f_s}\right) & \cdots & \sin\left(\frac{2\pi f_M \cdot (N-1)}{f_s}\right) \end{bmatrix} \begin{bmatrix} s_1(0) \\ s_1(1) \\ \vdots \\ s_1(N-1) \end{bmatrix} \quad \text{... (20)}$$

Among them, $X_1 \cdots X_M$ and $Y_1 \cdots Y_M$ are the calculation results of M frequencies, and $s_1(0) \cdots s_1(N-1)$ is N sampling data. Formulas (19) and (20) achieved at a multiplication and data accumulation respectively. Formulas (19) and (20) can be combined into:

$$\begin{bmatrix} X_1 + jY_1 \\ X_2 + jY_2 \\ \vdots \\ X_m + jY_m \end{bmatrix} = \frac{1}{N} \begin{bmatrix} e^{j\frac{2\pi f_1 \cdot 0}{f_s}} & e^{j\frac{2\pi f_1 \cdot 1}{f_s}} & \cdots & e^{j\frac{2\pi f_1 \cdot (N-1)}{f_s}} \\ e^{j\frac{2\pi f_2 \cdot 0}{f_s}} & e^{j\frac{2\pi f_2 \cdot 1}{f_s}} & \cdots & e^{j\frac{2\pi f_2 \cdot (N-1)}{f_s}} \\ \vdots & \vdots & \vdots & \vdots \\ e^{j\frac{2\pi f_m \cdot 0}{f_s}} & e^{j\frac{2\pi f_m \cdot 1}{f_s}} & \cdots & e^{j\frac{2\pi f_m \cdot (N-1)}{f_s}} \end{bmatrix} \begin{bmatrix} s_1(0) \\ s_1(1) \\ \vdots \\ s_1(N-1) \end{bmatrix} \quad \text{... (21)}$$

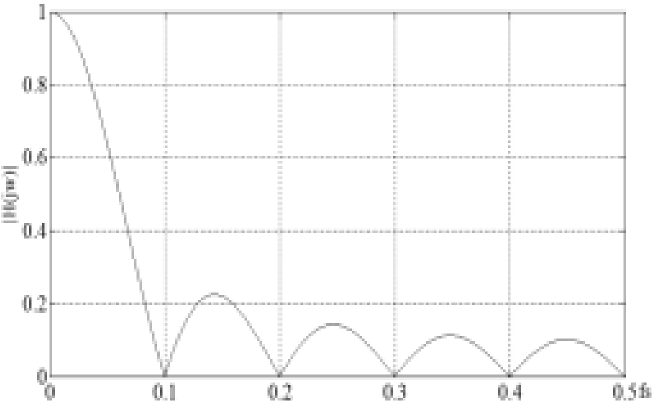

Fig.3 Amplitude-frequency response of the N=10 average filter

The amplitude and phase of I frequency in the detected signal are:

$$A_{mi} = 2\sqrt{X_i^{\,2} + Y_i^{\,2}} \quad i = 1 \ldots M \quad \text{... (22)}$$

$$\varphi_i = -\tan^{-1}\left(\frac{Y_i}{X_i}\right) \quad i = 1 \ldots M \quad \text{... (23)}$$

4. Simulation experiment

The experiment uses three frequency signals 25.325kHz, 50.65kHz and 101.3kHz selected by MIT array inductance gauge to simulate. The noise is random white noise and the sampling frequency of the signal is 1.005MS/s, the sampling number Is 4000, and the selected parameters satisfy the requirement of Formula (18) (sampling for the entire period). The mean value measured for 1000 times is taken as detection value. In the Matlab software environment, the Formulas (19)-(23) are used and the simulation result is shown in Table 1, where the SNR is defined as the ratio of the power of the signal to the noise. The selected frequencies in Table 2 are 25.335kHz, 50.66kHz and 101.31kHz, and the sampling frequency and the sampling number stay unchanged. At this time, the parameters election does not meet the requirement of Formula

TABLE 1 DETECTION RESULTS OF SAMPLING FOR THE ENTIRE PERIOD

SNR(dB)	detection error of amplitude and phase (%)		
	25.325kHz	50.65kHz	101.3kHz
Inf	$<10^{-12}/<10^{-11}$	$<10^{-12}/<10^{-11}$	$<10^{-12}/<10^{-12}$
20	0.004/0.015	0.011/0.013	0.012/0.009
10	0.018/0.013	0.020/0.005	0.011/0.021
0	0.187/0.056	0.147/0.006	0.042/0.004
-10	0.430/0.020	0.172/0.025	0.211/0.125

TABLE 2 DETECTION RESULTS OF SAMPLING FOR PARTIAL PERIOD

SNR(dB)	detection error of amplitude and phase (%)		
	25.335kHz	50.66kHz	101.31kHz
Inf	0.040/0.060	0.020/0.050	0.010/0.040
20	0.039/0.060	0.012/0.060	0.012/0.025
10	0.043/0.073	0.044/0.053	0.043/0.057
0	0.061/0.034	0.128/0.172	0.032/0.011
-10	0.165/0.135	0.489/0.150	0.361/0.009

(18). The mean value measured for 1000 times is taken as detection value. It can be seen from the detection results in Table 1 that the detection error of amplitude and phase of the three frequencies is less than 10^{-13} when there is no noise, which can verify the correct derivation of the detection algorithm. The detection error of amplitude is controlled within 0.5 and the detection error of phase is controlled within 0.2% when the SNR is as low as -10dB. It can be seen from the detection results in Table 2 that the maximum detection error of amplitude is 0.04% and the maximum detection error of phase is 0.06% when there is no noise. The detection result is basically the same as the error when the SNR is 20dB and 10dB. The detection error of Table 1 is smaller under the same SNR condition. For example, at 10dB, the amplitude error of 25.325kHz inTable 2 is more than twice of that in Table 1, and the phase error is more than 5 times.

4. Conclusions

Based on PSD technology, this study deduces the PSD algorithm for multi-frequency signals, analyzes the response characteristics of the digital average filter in the frequency domain, and obtains the constrained relationship of signal frequency, sampling frequency, and sampling number in the PSD of multi-frequency signals. Under the constraint condition, there is no effect between the frequency detection of multi-frequency signals. The experimental results show that the detection error of amplitude is controlled within 0.5% and the detection error of phase is controlled within 0.2% when the SNR is as low as-10dB. Therefore, the SNR of the collected signal is increased as much as possible in the design of the actual logging instrument data acquisition system. The acquisition parameter selection shall satisfy the constrained relationship, thereby realizing the high-precision detection of multi-frequency signals.

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