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Relationship between fractal dimension and shear strength of Vertisols and red earth

In order to study the effect of cracks in Vertisols and red earth on shear strength, the experimenters used imaging software and fractal dimension based on box dimension algorithm ideological and digital image technology in Vertisols and red earth in Fujian, China. Under different moisture content, direct shear tests are carried out on the samples, the relationship between the fractal dimension and shear strength is given. The results show that with the increase in the moisture content, the shear strength of the samples would decrease. The shear strength of two kinds of soils has a good correlation with fractal dimension and shear strength, which could be an identification index of the strength of two soils. There is a positive correlation of fractal dimension (D) and shear strength (τ), which can be formulated as $\tau = a + bD + cD^2$.

Keywords: Fractal dimension, shearing strength, multi-scale numerical model.

1. Introduction

Vertisols developed on the weathering products of the basalt, distributed over hilly land and terraces along Fujian coast have been distinguished into the lateritic red earths. Their properties are obviously different. Vertisols and red earth properties, soil fertility, agricultural production characters such as nutrient status, soil adsorption, moisture capacity, tillage, cropping system, fertilization effect, irrigation measures and crop yield and quality have many reports and the related achievements come out.

Cracking is a common natural phenomenon that occurred in Vertisols and red earth. The development of cracks in liners and covers will provide preferential flow paths for water infiltration and dramatically increase the hydraulic conductivity, resulting in the failure of anti-seepage systems. In addition, cracks will induce zones of weakness in a soil mass, leading to the reduction of the soil shear strength and the increase of the soil compressibility[1]. Moreover, cracks will probably cause the instability of slopes, foundations, and other structures related to Vertisols. Therefore, better understanding of soil cracking formation and development can facilitate the analysis of a wide spectrum of geotechnical, environmental and geological problems[2-7]. Shear strength is the principal engineering property of soil, which controls the stability and bearing capacity of a soil mass under structural loads. The shear strength parameters are determined by field or laboratory tests before engineering design or safety assessment. Conventional Mohr–Coulomb theory is widely used for fully saturated soils. Numerous researchers applied axis-translation technique to control and measure the matric suction for unsaturated direct shear test, ring shear test, and triaxial shear test[8-14].

In this paper, by comparison with the lateritic red earth, there are difference in use and management. We used the fractal theory which has the unique superior to the complex system to analyze soil particle composition pellet fractal and the cracks fractal characteristics. The calculated Vertisols and red earth cracks fractal dimension is the fundamentary for the cracked soil permeability by MATLAB. By comparing Vertisols and red earth, a set of direct shear facility is designed and applied. The testing results of Vertisols and red earth are studied and analyzed. We study on Vertisols and red earth with the achievements of engineering mechanics, it can provide more availability for further research on the soils.

2. Theory and experiments

2.1 Fractal Theory

Fractal theory is a new discipline that it is a powerful tool for dealing with the theory of natural and engineering irregular pattern. Fractal dimension (D) is an important parameter of fractal; it will not be able to quantitative description or difficult to quantitatively describe the nonlinear complex object with a relatively simple quantitative method fully express. As generally refers to the box dimension of fractal dimension, it is to use a square lattice ($\delta \times \delta$) to cover the fractal curve, the size of the grid is changing. Given the size of the box, the box cover fractal curve can be calculated as total N, assuming that the first step i cover the use the grid of ($C \times \delta_i$), box number is $N_i(\delta_i)$, in the first step $i + 1$ to cover for the grid of ($\delta_{i+1} \times \delta_{i+1}$), box number is $N_{i+1}(\delta_{i+1})$, one can find in any two scales required box is the ratio of the number and the ratio of the length relations which are

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as follows:

$$N_{i+1}/N_i = \left(\delta_i/\delta_{i+1}\right)^D \tag{1}$$

Promoted to the general situation, can be obtained:

$$D = \log\left(N_{i+1}/N_i\right) \Big/ \log\left(\delta_{i+1}/\delta_i\right) \tag{2}$$

So as long as is known any two box size and the corresponding number "box", can be directly calculated as fractal dimension:

$$\log N + \log a - D\log\delta \tag{3}$$

In the process of covering, it will get a set of data (G, N), as log-log graph, the slope s is equal to the set of fractal dimension (s = D).

The digital image values graphic information in matrix storage mode, write the Matlab simplified digital image box dimension calculation as is shown in Fig. 1[15].

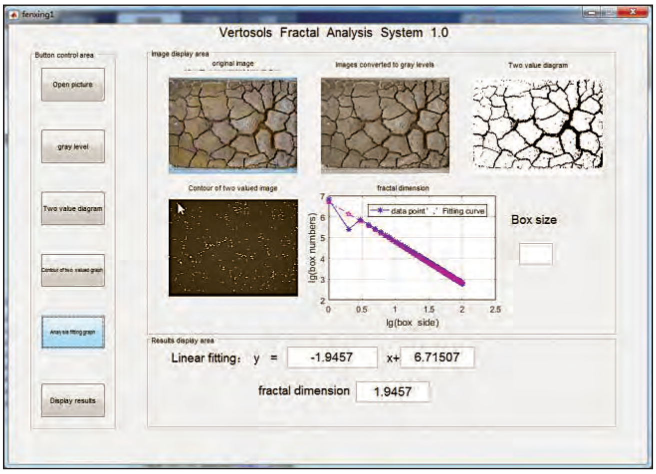

Fig. 1: Fractal Feature analysis on vertisols and red earth fracture ansyssystem interface

2.2 Shear strength test

Shear strength is the principal engineering property of soil, which controls the stability of a soil mass under structural loads. Accurate determination of the soil shear strength parameters (angle of internal friction and cohesion) is a major interest in the design of different geotechnical structures. These parameters can be determined either in the laboratory or in the site. The triaxial compression and direct shear tests are the most common tests for determining the angle of internal friction and cohesion values in the laboratory. Special care must be taken to establish loading condition actually existing in the ground and to duplicate this condition in the laboratory.

The direct shear apparatus used in this study was modified from unsaturated direct shear box and set up in the geotechnical laboratory of Putian University. Vertisols and red earth are used to perform the tests for determining the shear strength parameters. The vertical stress magnitudes of 0kPa have been applied. Vertisols and red earth are sheared separately under the constant horizontal displacement velocity of 0.5 mm/min until the horizontal deformation reaches the limit. The shear test is performed under two different cases (methods), namely: when constant vertical stress (q = const.) is applied; when constant sample volume (h = const.) is applied. The shear tests have been performed for Vertisols and red earth. The peak soil shearing strength has been determined according to the maximum ratio of tangential and normal stresses, id est. according to $\tau/\sigma = \max$. The sample was quickly dismantled from the shear box for the determination of final water content. The general test arrangement of direct shear test is shown in Fig. 2.

Fig. 2: Overview of direct shear test system

3. Analysis

3.1 The Relationship between moisture content (A) and fractal dimension (D)

Based on the experimental data, the relationships of the relationship between moisture content and fractal dimension of Vertisols and red earth are non-linear, as shown in Figs. 3 and 4. The impact on the fractal dimension of the moisture content is a simple quadratic polynomial relationship. It is easy to build the mathematical model of fractal dimension with the moisture content. The moisture content is fitted as independent variables, while the fractal dimension is fitted as a function of the dependent variable respectively by data processing software Origin. According to Figs. 3 and 4, there are a positive correlation of fractal dimension and the moisture content, which can be formulated as respectively:

$$D = -0.00112\alpha^2 + 0.04243\alpha + 1.50729 \text{ (red earth)} \quad (4)$$

$$D = -0.00027\alpha^2 + 0.0095\alpha + 1.88256 \text{ (Vertisols)} \quad (5)$$

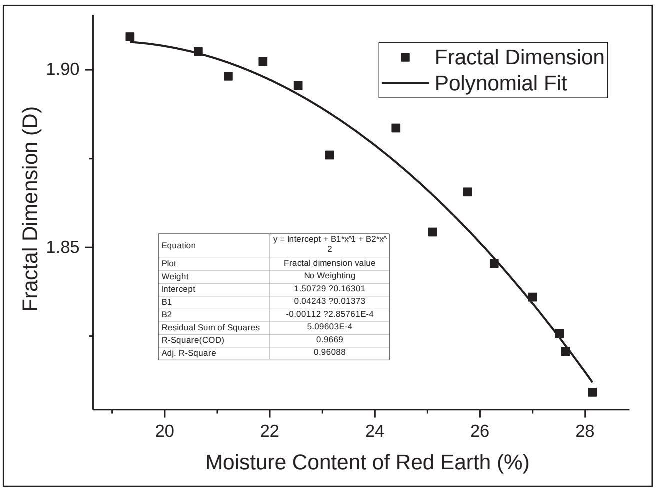

Fig. 3: Relationship between Vertisols surface moisture content and fractal dimension value

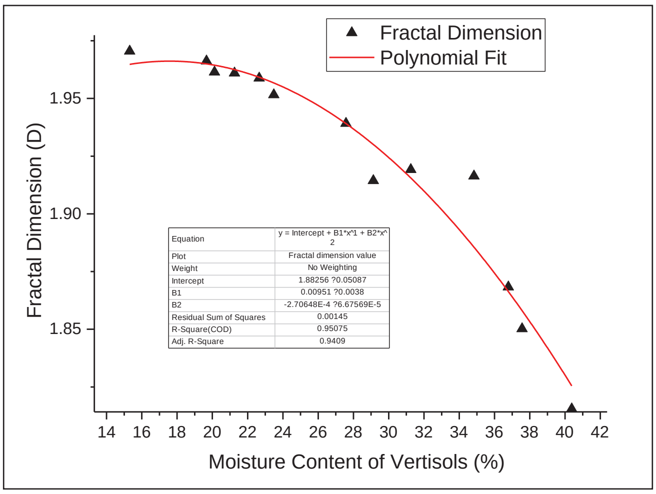

Fig. 4: Relationship between Vertisols surface moisture conten and fractal dimension value

3.2 Relationship between moisture content (α) and shear strength (τ)

According to experimental results, The relationship between moisture content and shear strength of Vertisols and red earth are shown separately in Figs. 5 and 6. It is observed that the shear strength of Vertisols and red earth reveal an approximately linear behaviour for moisture content. The slope of the linear relationship slightly increases as the moisture content reduction. Eq. (6) and Eq. (7) are formulas between moisture content and shear strength of Vertisols and red earth by data processing software Origin.

$$\tau = -0.79263\alpha + 27.29 \text{ (red earth)} \quad (6)$$

$$\tau = -1.9945\alpha + 67.41 \text{ (Vertisols)} \quad (7)$$

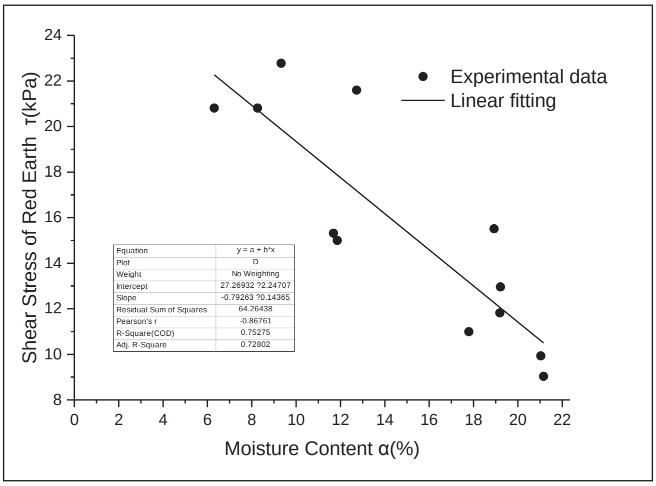

Fig. 5: Relationship between Vertisols surface moisture conten and shear strength

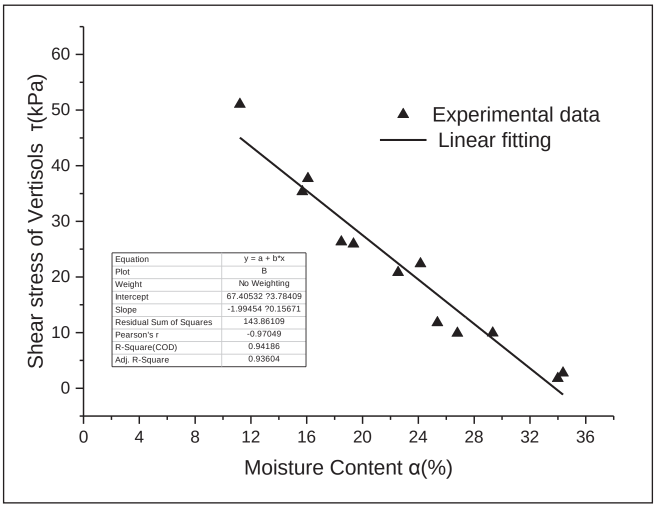

Fig. 6: Relationship between Vertisols surface moisture content and shear strength

3.3 Relationship between fractal dimension (D) and shear strength (τ)

Based on 3.1 and 3.2, the relationships between fractal dimension and shear strength of Vertisols and red earth are shown in Table 1. The relation curves of fractal dimension and shear strength of Vertisols and red earth are shown in Figs. 7 and 8. The characteristic that the shear stress increases with increasing fractal dimension. For red earth, the shear stress is from 5.0 to 11.9 when fractal dimension is from 1.8092 to 1.9093. For Vertisols, the shear stress is from 0 to 36.9 when fractal dimension is from 1.9164 to 1.9705. It is clear from the data in this figure that differs significantly between red earth and Vertisols. The fractal dimension and shear stress of red earth is smaller than Vertisols. Based on Table 1, Eq. (8) of red earth and Eq. (8) of Vertisols are presented respectively.

Table 1. The relationship between fractal dimension (D) and shear strength (τ)

Order number	red earth		Vertisols	
	D	τ (kPa)	D	τ (kPa)
1	1.8092	5.0	1.9164	0.0
2	1.8207	5.4	1.9192	5.1
3	1.8258	5.5	1.9144	9.3
4	1.836	5.9	1.9392	12.4
5	1.8455	6.4	1.9516	20.6
6	1.8656	6.9	1.9588	22.2
7	1.8543	7.4	1.961	25.0
8	1.8836	7.9	1.9614	27.3
9	1.876	8.9	1.9662	28.2
10	1.8956	9.4	1.9705	36.9
11	1.9023	9.9		
12	1.8982	10.5		
13	1.9051	10.9		
14	1.9093	11.9		

$$\tau = 1507.213 - 1675D + 466.92D^2 \text{ (red earth)} \quad (8)$$

$$\tau = 31248.44 - 32693.62D + 8552.72D^2 \text{ (Vertisol)} \quad (7)$$

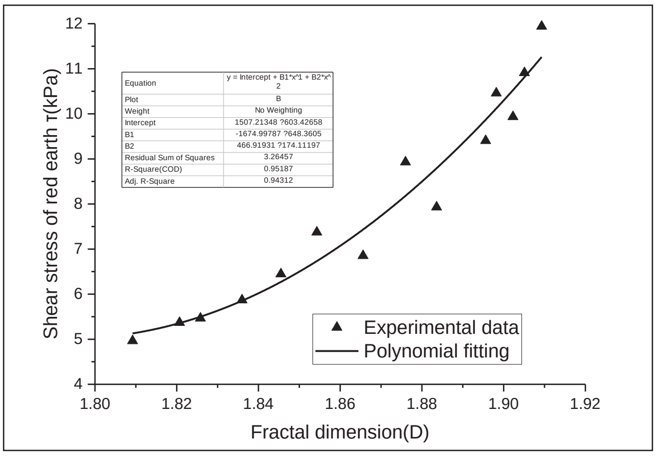

Fig. 7: Relationship between fractal dimension value and shear strength of red earth

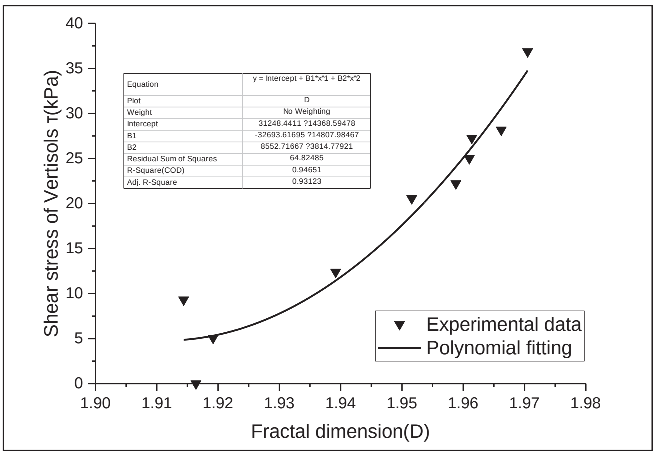

Fig. 8: Relationship between fractal dimension value and shear strength of Vertisols

4. Conclusions

By studying the effects of moisture content on shear strength and fractal dimension of Vertisols and red earth, the relationship between the fractal dimension and shear strength is analyzed. The main conclusions are as follows:

(1) With the increase of moisture content, the fractal dimension and shear strength of Vertisols and red earth would decrease.

(2) There is a strong correlation between the fractal dimension and shear strength of Vertisols and red earth. The formulas of Vertisols and red earth can be written respectively as under:

$\tau = 31248.44 - 32693.62D + 8552.72D^2$ (Vertisols)
and
$\tau = 1507.213 - 1675D + 466.92D^2$ (red earth)

(3) The shear strength will reinforce with an increase in the fractal dimension.The fractal dimension can be used as one of the indicators in discrimination of the shear strength of Vertisols and red earth.

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