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Linear, Parabolic, and Inverted Parabolic Temperature Gradients Impact on Double-Diffusive Rayleigh-Darcy Convection: a composite system with couple stress fluid

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Abstract

The influence of linear, parabolic and inverted parabolic temperature gradients on the onset of double-diffusive Rayleigh-Darcy convection is theoretically investigated. The composite system is constrained horizontally by adiabatic and free-free thermal boundaries, and appropriate interfacial boundary conditions are used to connect fluid-porous layers. The regular perturbation approach is used to determine the critical Rayleigh number expression for different temperature gradients. Graphs are used to investigate the significance of a variety of dimensionless characteristics. The couple stress parameter, couple stress viscosity ratio, solute Rayleigh number, and solute diffusivity ratio clearly have a stabilizing effect on the system, whereas the Darcy number and thermal diffusivity ratio destabilize it.

Keywords: *Double-Diffusive Convection, Couple stress fluid, Thermal Rayleigh number, Solute Rayleigh number, Composite system.*

1. Introduction

Many industrial processes and natural phenomena include coupled heat and mass transport in setups that combine a clear fluid with a porous material. Thermal insulations, filtration processes, nuclear waste storage, drying processes, dendritic solidification, spreading on porous substrates, biofilm growth, gasification of biomass, fuel cells, and in the context of environmental concerns geothermal systems, groundwater pollution are all examples of processes involving heat and mass transfer.

For many years, the topic of double-diffusive convection has been investigated in this context. Nield [1968] was the first to investigate the start of thermohaline convection, and he was followed by several others Taunton et al. [1972], Tuner and Huppert [1981], Rudraiah et al. 1982] and Schmitt [1994]. Their research concentrated on saturated porous media. When a fluid layer rests above a porous material, a composite system exists. Double diffusive convection in a composite system with a Newtonian fluid is investigated with a focus on the effect of temperature and salinity gradients and the significance of dimensionless factors in determining the commencement of Rayleigh-Benard or Marangoni convection. Chen and Chen [1998] investigated finger convection in a horizontal porous layer positioned underneath a fluid layer. Gobin et al. [1998] examined double-diffusive convection in a composite fluid-

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porous layer. Sumithra [2012, 2014, 2020] investigated the mathematical modeling of hydrothermal development of crystals and magneto Marangoni double-diffusive convection in composite layer and expanded the work to include the Soret effect in Darcy-Benard double-diffusive Marangoni convection. Gangadharaiah[2021] explored double- diffusive convection driven by surface tension in a fluid-porous system.

Stokes conceived and hypothesized the hypothesis of couple-stress fluid in [1966]. The application of couple-stress fluid is to the study of the process of lubrication of synovial joints, which has become the primary focus of scientific research, and it was discovered that the synovial fluid in human joints behaves similarly to a couple-stress fluid. Sharma et al. [2002] examined the couple-stress fluid penetrated with suspended particles heated from below and discovered that the couple stress parameter stabilizes convection. Sharma and Chandel [2004] investigated the effect of suspended particles on couple-stress fluid heated and soluted from the bottom in a porous media. It has been discovered that solute gradient and couple-stress stabilize the system, whereas solute and medium permeability destabilize it. Malashetty et al. [2010] and Shivakumara et al. [2011] investigated the linear and nonlinear stability of double-diffusive convection in a couple stress-saturated porous layers. The existence of couple stresses is demonstrated to delay the beginning of convection. Malashetty et al. [2011] and Srivastava et al. [2013] investigated thermo-solutal convection in a fluid- saturated anisotropic porous media with couple-stress fluid saturation. It was discovered that raising the couple stress parameter improved the system's flow stability. Gaikwad and Kouser analyzed the linear and nonlinear stability of double-diffusive convection in fluid-saturated porous layers with an internal heat source. Harfash et al. [2019] examined the influence of couple stresses on double-diffusive convection in a reactive fluid. In this system, there are two opposing effects: the temperature gradient, which causes instability, and the salt gradient, which promotes the system's stability.

Double-diffusive Rayleigh-Darcy convection in a composite system with a couple stress fluids is the focus of this investigation. Saturated porous media as a single layer has attracted much attention, but we will concentrate on composite layers with a special property of a two- layer system in this paper. In this scenario, we assume that normal velocity and stress, tangential velocity and stress, and coupled stress are all continuous in the fluid-porous zones. The Navier-Stokes equation governs momentum transfer in the fluid area for couple stress fluid, while Darcy law is used in the porous zone. It is the goal of this research to examine the effects of linear, parabolic, and inverted parabolic temperature gradients on the commencement of double-diffusive Rayleigh-Darcy convection, which is compared graphically to examine the system's stability.

2. Mathematical formulation

Consider a composite system with a height 2 h and bounded by free-free surfaces on both sides. At the center of the composite system is where the Cartesian coordinate system starts. From this point, the horizontal axis and vertical axis are directed. Couple stress fluid occupies Region 1, and the same fluid saturated Darcy porous medium occupies Region 2. The lower and upper free surfaces are kept at different constant temperatures and solute concentrations. Gravitational force acts vertically downward.

The governing equations in Region 1:

The conservation of mass, momentum, energy, concentration and the equation of state are specified for fluid layer as:

$$\nabla \cdot \vec{q}_1 = 0 \quad \text{... [1]}$$

$$\rho_0\left(\frac{\partial \vec{q}_1}{\partial t} + \left(\vec{q}_1 \cdot \nabla\right)\vec{q}_1\right) = -\nabla P_1 - \rho_0\,\vec{g} + \mu\nabla^2\vec{q}_1 - \mu_1'\,\nabla^4\vec{q}_1 \quad \text{... [2]}$$

$$\frac{\partial T_1}{\partial t} + \left(\vec{q}_1 \cdot \nabla\right)T_1 = \kappa_{T1}\nabla^2 T_1 \quad \text{... [3]}$$

$$\frac{\partial C_1}{\partial t} + \left(\vec{q}_1 \cdot \nabla\right)C_1 = \kappa_{c1}\nabla^2 C_1 \quad \text{... [4]}$$

$$\rho_1 = \rho_0\left(1 - \alpha_{T1}\left(T_u - T_0\right) + \alpha_{c1}\left(C_u - C_0\right)\right) \quad \text{... [5]}$$

The governing equations in Region 2

The conservation of mass, momentum, energy, and equation of state are specified for porous layer as:

$$\nabla \cdot \vec{q}_2 = 0 \quad \text{... [6]}$$

$$\frac{\rho_0}{\varphi}\frac{\partial \vec{q}_2}{\partial t} = -\nabla P_2 - \rho_0\,\vec{g} - \frac{\mu}{K}\vec{q}_2 + \frac{\mu_2'}{K}\nabla^2\vec{q}_2 \quad \text{... [7]}$$

$$\mathrm{M}\frac{\partial T_2}{\partial t} + \left(\vec{q}_2 \cdot \nabla\right)T_2 = \kappa_{T2}\nabla^2 T_2 \quad \text{... [8]}$$

$$\mathrm{N}\frac{\partial C_2}{\partial t}+\left(\vec{q}_2\cdot\nabla\right)C_2=\kappa_{C2}\nabla^2 C_2 \quad ...[9]$$

$$\rho_2=\rho_0\left(1-\alpha_{T2}\left(T_l-T_0\right)+\alpha_{C2}\left(C_l-C_0\right)\right) \quad ...[10]$$

Here $\vec{q}(u,v,w)$, P, $\vec{g}$, μ, μ', T, C, κ, ρ, α_T, α_C, φ, K, C_p, M, and N denotes velocity, pressure, gravitational force, viscosity, couple-stress fluid viscosity, temperature, solute concentration, density, coefficient of thermal expansion, coefficient of soluteconcentration expansion, porosity, permeability, specific heat, ratio of heat capacities, ratio of solute concentration capacities. $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ denotes Laplacian operator, ρ_0 denotes density at a reference temperature $T = T_0$, The subscripts 1 and 2 refer to fluid and porous regions respectively.

Assuming that the fundamental steady state of the composite system is quiet, and it is determined that the temperature and concentration distributions will be as follows:

$$T_{1b}(z_1)=T_0-\left(\frac{T_0-T_u}{h_1}\right)z_1 \quad \text{and} \quad C_{1b}(z_1)=C_0-\left(\frac{C_0-C_u}{h_1}\right)z_1 \qquad 0\le z_1\le h_1 \quad ...[11]$$

$$T_{2b}(z_2)=T_0-\left(\frac{T_l-T_0}{h_2}\right)z_2 \quad \text{and} \quad C_{2b}(z_2)=C_0-\left(\frac{C_l-C_0}{h_2}\right)z_2 \qquad -h_2\le z_2\le 0 \quad ...[12]$$

here, $T_0=\frac{\kappa_{T1}h_2T_u+\kappa_{T2}h_1T_l}{\kappa_{T1}h_2+\kappa_{T2}h_1}$, $C_0=\frac{\kappa_{C1}h_2C_u+\kappa_{C2}h_1C_l}{\kappa_{C1}h_2+\kappa_{C2}h_1}$ and subscript b denote interfacial temperature and basic state.

Small perturbations of this kind are familiarized with in order to evaluate the system's level of steadiness.

$$q_1=q_1',\ P_1=P_{1b}\left(z_1\right)+P_1',\ T_1=T_{1b}\left(z_1\right)+\theta_1,\ C_1=C_{1b}\left(z_1\right)+\phi_1,\ \ \rho_1=\rho_{1b}\left(z_1\right)+\rho_1' \quad ...[13]$$

$$q_2=q_2',\ P_2=P_{2b}\left(z_2\right)+P_2',\ T_2=T_{2b}\left(z_2\right)+\theta_2,\ C_2=C_{2b}\left(z_2\right)+\phi_2,\ \ \rho_2=\rho_{2b}\left(z_2\right)+\rho_2' \quad ...[14]$$

The disturbances brought about by the primed amounts must be minimal. By substituting equations (13) and (14) into equations (1) through (10) the resulting equations are linearized. Applying curl twice eliminates the pressure term from Equations (2) and (7). The variables are non-dimensionalized using T_0-T_u, C_0-C_u, h_1, h_1^2/κ_{T1}, and κ_{T1}/h_1 as the units of temperature, concentration, length, and velocity in the fluid layer. The corresponding characteristic quantities for the porous layer are T_l-T_0, C_l-C_0, h_2, h_2^2/κ_{T2}, and κ_{T2}/h_2. The non-dimensionalized equations of Region 1 and Region 2 are found with three unknowns w, θ and ϕ. Assuming w, θ and ϕ are periodic waves, normal mode solutions may be expressed as

$$\left(w_1,\theta_1,\phi_1\right)=\left(W_1\left(z_1\right),\Theta_1\left(z_1\right),\Phi_1\left(z_1\right)\right)e^{i_1\left(l_1x_1+m_1y_1\right)-\omega_1t_1} \quad ...[15]$$

$$\left(w_2,\theta_2,\phi_2\right)=\left(W_2\left(z_2\right),\Theta_2\left(z_2\right),\Phi_2\left(z_2\right)\right)e^{i_2\left(l_2x_2+m_2y_2\right)-\omega_2t_2} \quad ...[16]$$

here, w is the frequency, l and m are the wave number components in x and y directions. The following ordinary differential equations are produced by substituting the aforementioned formulas for the non-dimensionalized partial differential equations:

$$\left(C_1\left(D_1^2-a_1^2\right)^3-\left(D_1^2-a_1^2\right)^2-\left(D_1^2-a_1^2\right)\frac{\omega_1}{\mathrm{Pr}_1}\right)W_1\left(z_1\right)=-Ra_1^T a_1^2\Theta_1\left(z_1\right)+Ra_1^S a_1^2\Phi_1\left(z_1\right) \quad ...[17]$$

$$\left(D_1^2-a_1^2-\omega_1\right)\Theta_1\left(z_1\right)=-W_1 f_1\left(z_1\right) \quad ...[18]$$

$$\left(D_1^2-a_1^2-\omega_1\right)\Phi_1\left(z_1\right)=-W_1\left(z_1\right) \quad \text{... [19]}$$

$$\left(\left(D_2^2-a_2^2\right)-C_2\left(D_2^2-a_2^2\right)^2-\left(D_2^2-a_2^2\right)\frac{Da\,\omega_2}{\Pr_2}\right)W_2\left(z_2\right)=-Ra_2^T\,a_2^2\,\Theta_2\left(z_2\right)+Ra_2^S\,a_2^2\,\Phi_2\left(z_2\right) \quad \text{... [20]}$$

$$\left(D_2^2-a_2^2-\mathrm{M}\,\omega_2\right)\Theta_2\left(z_2\right)=-W_2\,f_2\left(z_2\right) \quad \text{... [21]}$$

$$\left(D_2^2-a_2^2-\mathrm{N}\,\omega_2\right)\Phi_2\left(z_2\right)=-W_2\left(z_2\right) \quad \text{... [22]}$$

here $D=\dfrac{d}{dz}$, $a_1=\sqrt{l_1^2+m_1^2}$, $Da=\dfrac{K}{h_2^2}$, $C_1=\dfrac{\mu'}{\mu\,h_1^2}$, $\Pr_1=\dfrac{\mu}{\rho_0\,\kappa_1}$, $Ra_1^T=\dfrac{\rho_0\,g\,\alpha_{T1}\left(T_0-T_u\right)h_1^3}{\mu\,\kappa_{T1}}$, and $Ra_1^S=\dfrac{\rho_0\,g\,\alpha_{C1}\left(C_0-C_u\right)h_1^3}{\mu\,\kappa_{T1}}$ denotes differential operator, wave number, Darcy number couple stress parameter, prandtl number, thermal Rayleigh number and solute Rayleigh number in fluid layer, $C_2=\dfrac{\mu_2'}{\mu\,h_2^2}$, $\Pr_2=\dfrac{\phi\,\mu}{\rho_0\,\kappa_2}$, $Ra_2^T=\dfrac{\rho_0\,g\,\alpha_{T2}\left(T_l-T_0\right)h_2^3}{\mu\,\kappa_{T2}}$ ·, and $Ra_2^S=\dfrac{\rho_0\,g\,\alpha_{C2}\left(C_l-C_0\right)h_2^3}{\mu\,\kappa_{T2}}$ denotes the above corresponding terms in porous layer.

We limit the study to stationary convection and apply equations (17) to (22) to arrive at the following equations since the concept of exchange of stability applies in this case because there are no processes to make the system oscillatory.

$$\left(C_1\left(D_1^2-a_1^2\right)-1\right)\left(D_1^2-a_1^2\right)^2W_1\left(z_1\right)=-Ra_1^T\,a_1^2\,\Theta_1\left(z_1\right)+Ra_1^S\,a_1^2\,\Phi_1\left(z_1\right) \quad \text{... [23]}$$

$$\left(D_1^2-a_1^2\right)\Theta_1=-W_1\;f_1\left(z_1\right) \quad \text{... [24]}$$

$$\left(D_1^2-a_1^2\right)\Phi_1\left(z_1\right)=-W_1\left(z_1\right) \quad \text{... [25]}$$

$$\left(C_2\left(D_2^2-a_2^2\right)-1\right)\left(D_2^2-a_2^2\right)W_2\left(z_2\right)=Ra_2^T\,a_2^2\,\Theta_2\left(z_2\right)-Ra_2^S\,a_2^2\,\Phi_2\left(z_2\right) \quad \text{... [26]}$$

$$\left(D_2^2-a_2^2\right)\Theta_2=-W_2\,f_2\left(z_2\right) \quad \text{... [27]}$$

$$\left(D_2^2-a_2^2\right)\Phi_2\left(z_2\right)=-W_2\left(z_2\right) \quad \text{... [28]}$$

The following are the conditions that define the boundary of the composite system:

$$\left.\begin{array}{l}
W_1(1)=0,\quad D_1^2\,W_1(1)=0,\quad D_1^4\,W_1(1)=0,\quad D_1\Theta_1(1)=0,\quad D_1\Phi_1(1)=0\\
W_1(0)=\dfrac{\hat{h}}{\hat{\kappa}_T}W_2(1),\quad D_1W_1(0)=\dfrac{\hat{h}^2}{\hat{\kappa}_T}D_2W_2(1),\\
\left(C_1\left(D_1^2-a_1^2\right)^2-D_1^2+3a_1^2\right)D_1W_1(0)=\dfrac{\hat{h}^4}{Da\,\hat{\kappa}_T}\left(1-C_2\left(D_2^2-a_2^2\right)\right)D_2W_2(1)\\
(D_1^2+a_1^2)W_1(0)=\dfrac{\hat{h}^3}{\hat{\mu}\,\hat{\kappa}_T}\left(D_2^2+a_2^2\right)W_2(1),\left(D_1^4-a_1^4\right)W_1(0)=\dfrac{\hat{h}^5}{\hat{\mu}_c\,\hat{\kappa}_T}\left(D_2^4-a_2^4\right)W_2(1),\\
\Theta_1(0)=\hat{T}\,\Theta_2(1),\quad D_1\Theta_1(0)=D_2\,\Theta_2(1),\quad \Phi_1(0)=\hat{S}\,\Theta_2(1),\quad D_1\Phi_1(0)=D_2\,\Phi_2(1)\\
W_2\left(0\right)=0,\quad D_2^2\,W_2\left(0\right)=0,\quad D_2\,\Theta_2(0)=0,\quad D_2\,\Phi_2(0)=0
\end{array}\right\} \quad \text{... [29]}$$

Where, $\hat{\kappa}_T = \kappa_1/\kappa_2$, $\hat{\mu}_c = \mu_1'/\mu_2'$, $\hat{\mu} = \mu_1/\mu_2$, and $\hat{h} = h_1/h_2$ denotes thermal diffusivity ratio, couple stress viscosity ratio, viscosity ratio, and depth ratio respectively and also $\hat{T} = \hat{\kappa}_T/\hat{h}$ $\hat{S} = \hat{\kappa}_C/\hat{h}$.

3. Solution by regular perturbation technique

Perturbation methods rely on the assumption of small parameters. Here we select 'a_1' as the wavenumber, which is negligibly small. The terms W, Θ and Φ are expanded in series to 'a_1' , the set of terms having equal powers in 'a_1' are solved until the solution is obtained.

$$\begin{bmatrix} W_1 \\ \Theta_1 \\ \Phi_1 \end{bmatrix} = \sum_{k=0}^{\infty} a_1^{2k} \begin{bmatrix} W_{1k} \\ \Theta_{1k} \\ \Phi_{1k} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} W_2 \\ \Theta_2 \\ \Phi_2 \end{bmatrix} = \sum_{k=0}^{\infty} \left(\frac{a_1}{\hat{h}}\right)^{2k} \begin{bmatrix} W_{2k} \\ \Theta_{2k} \\ \Phi_{2k} \end{bmatrix} \qquad [30]$$

In order to determine velocities W_1 and W_2 of Region 1, and Region 2, Eqs. (23) to (28) are solved by using composite system's boundary conditions.

The zero-order of a_1^2 solution of the equation is given by:

$$W_1(z_1) = W_2(z_2) = 0,\ \Theta_1(z_1) = \hat{T},\ \ \Phi_1(z_1) = \hat{S},\ \Theta_2(z_2) = \Phi_2(z_2) = 1. \qquad \text{... [31]}$$

The first-order in a_1^2, Eqs. (19) – (22) then reduce to

$$D_1^6 W_1(z_1) - \eta_1^2 D_1^2 W_1(z_1) + \eta_1^2 Ra_1^T \hat{T} - \eta_1^2 Ra_1^S \hat{S} = 0 \qquad \text{... [32]}$$

$$D_1^2 \Theta_1 - \hat{T} + W_1 f_1(z_1) = 0 \qquad \text{... [33]}$$

$$D_1^2 \Phi_1 - \hat{S} + W_1(z_1) = 0 \qquad \text{... [34]}$$

$$D_2^4 W_2(z_2) - \eta_2^2 D_2^2 W_2(z_2) - \eta_2^2 Ra_2^T + \eta_2^2 Ra_2^S = 0 \qquad \text{... [35]}$$

$$D_2^2 \Theta_2 - 1 + W_2 f_2(z_2) = 0 \qquad \text{... [36]}$$

$$D_2^2 \Phi_2 - 1 + W_2(z_2) = 0 \qquad \text{... [37]}$$

and the composite system's boundary conditions (23) – (25) becomes

$$\left.\begin{aligned}
& W_1(1) = D_1^2 W_1(1) = D_1^4 W_1(1) = D_1\Theta_1(1) = D_1\Phi_1(1) = 0, \\
& W_1(0) = \frac{1}{\hat{h}\,\hat{\kappa}_T} W_2(1),\ \ D_1 W_1(0) = \frac{1}{\hat{\kappa}_T} D_2 W_2(1),\ \ D_1^2 W_1(0) = \frac{\hat{h}}{\hat{\mu}\,\hat{\kappa}_T} D_m^2 W_2(1), \\
& D_1^4 W_1(0) = \frac{\hat{h}^3}{\hat{\mu}_c\,\hat{\kappa}_T} D_2^4 W_2(1),\ \ \left(C_1 D^5 - D^3\right) W_1(0) = \left(\frac{\hat{h}^2}{Da\,\hat{\kappa}_T}\right)\left(D_2 - C_2 D_2^3\right) W_2(1) \\
& \Theta_1(0) = \frac{\hat{T}}{\hat{h}^2}\Theta_2(1),\ \ D_1\Theta_1(0) = \frac{1}{\hat{h}^2} D_2\Theta_2(1),\ \Phi_1(0) = \frac{\hat{S}}{\hat{h}^2}\Phi_2(1),\ \ D_1\Phi_1(0) = \frac{1}{\hat{h}^2} D_2\Phi_2(1) \\
& W_2(0) = D_2^2 W_2(0) = D_2\Theta_2(0) = D_2\Phi_2(0) = 0,
\end{aligned}\right\} \qquad \text{... [38]}$$

The general solution of equations (32) and (35) are given by each

$$W_1(z_1)=\begin{pmatrix}\Gamma_7+z_1\Gamma_9+z_1^2\Gamma_{11}+\\ z_1^3\Gamma_{13}+\operatorname{Cosh}(\eta_1 z_1)\Gamma_{15}\\ +\operatorname{Sinh}(\eta_1 z_1)\Gamma_{17}+\dfrac{\hat{T}}{24}z_1^4\end{pmatrix}Ra_1^T+\begin{pmatrix}\Gamma_8+z_1\Gamma_{10}+z_1^2\Gamma_{12}+\\ z_1^3\Gamma_{14}+\operatorname{Cosh}(\eta_1 z_1)\Gamma_{16}\\ +\operatorname{Sinh}(\eta_1 z_1)\Gamma_{18}+\dfrac{\hat{S}}{24}z_1^4\end{pmatrix}Ra_1^S \quad \text{... [39]}$$

$$W_2(z_2)=\begin{pmatrix}\operatorname{Sinh}(\eta_2 z_2)\Gamma_1+z_2\Gamma_3+\\ \left(\dfrac{\operatorname{Cosh}(\eta_2 z_2)}{\eta_2^2}-\dfrac{1}{\eta_2^2}-\dfrac{z_2^2}{2}\right)\Delta_1\end{pmatrix}Ra_1^T+\begin{pmatrix}\operatorname{Sinh}(\eta_2 z_2)\Gamma_2+z_2\Gamma_4+\\ \left(\dfrac{\operatorname{Cosh}(\eta_2 z_2)}{\eta_2^2}-\dfrac{1}{\eta_2^2}-\dfrac{z_2^2}{2}\right)\Delta_2\end{pmatrix}Ra_1^S \quad \text{... [40]}$$

where

$\eta_1^2=\dfrac{1}{C_1},\ \eta_2^2=\dfrac{1}{C_2}$ $\quad \Delta_1=\dfrac{\hat{\alpha}_T\, Da\,\hat{\kappa}_T^2}{\hat{h}^4},$ $\quad \Delta_2=\dfrac{\hat{\alpha}_C\, Da\,\hat{\kappa}_C^2}{\hat{h}^4}$

$\Gamma_1=\dfrac{(P_{21}P_{23}-P_{19}P_{25})}{(P_{20}P_{23}-P_{19}P_{24})},$ $\quad \Gamma_2=\dfrac{(P_{22}P_{23}-P_{19}P_{26})}{(P_{20}P_{23}-P_{19}P_{24})}$ $\quad \Gamma_3=\dfrac{(P_{21}-P_{20}\Gamma_1)}{P_{19}},$

$\Gamma_4=\dfrac{(P_{22}-P_{20}\Gamma_2)}{P_{19}},$ $\quad \Gamma_5=\dfrac{\Delta_1}{\eta_2^2},$ $\quad \Gamma_6=-\dfrac{\Delta_2}{\eta_2^2}$

$\Gamma_7=\left(\dfrac{\Gamma_3}{\hat{h}\,\hat{\kappa}_T}+P_{16}\Gamma_1+P_{17}\right),$ $\quad \Gamma_8=\left(\dfrac{\Gamma_4}{\hat{h}\,\hat{\kappa}_T}+P_{16}\Gamma_2+P_{18}\right),$ $\quad \Gamma_9=\left(\dfrac{\Gamma_3}{\hat{\kappa}_T}+P_{13}\Gamma_1+P_{14}\right),$

$\Gamma_{10}=\left(\dfrac{\Gamma_4}{\hat{\kappa}_T}+P_{13}\Gamma_2+P_{15}\right)$ $\quad \Gamma_{11}=(P_{10}\Gamma_1+P_{11}),$ $\quad \Gamma_{12}=(P_{10}\Gamma_2+P_{12}),$

$\Gamma_{13}=\left(-\dfrac{\hat{h}^2\,\Gamma_3}{6Da\,\hat{\kappa}_T}+P_7\Gamma_1+P_8\right),$ $\quad \Gamma_{14}=\left(-\dfrac{\hat{h}^2\,\Gamma_4}{6Da\,\hat{\kappa}_T}+P_7\Gamma_2+P_9\right),$ $\quad \Gamma_{15}=(P_1\Gamma_1+P_2),$

$\Gamma_{16}=(P_1\Gamma_2+P_3),$ $\quad \Gamma_{17}=(P_4\Gamma_1+P_5),$ $\quad \Gamma_{18}=(P_4\Gamma_1+P_6),$

$P_1=\dfrac{\hat{h}^3\,\eta_2^4}{\eta_1^4\,\hat{\mu}_c\,\hat{\kappa}_T}\operatorname{Sinh}(\eta_2),$ $\quad P_2=\dfrac{\hat{h}^3\,\eta_2^2}{\eta_1^4\,\hat{\mu}_c\,\hat{\kappa}_T}\operatorname{Cosh}(\eta_2)\Delta_1-\dfrac{\hat{T}}{\eta_1^4},$

$P_3=-\left(\dfrac{\hat{h}^3\,\eta_2^2}{\eta_1^4\hat{\mu}_c\,\hat{\kappa}_T}\operatorname{Cosh}(\eta_2)\Delta_2-\dfrac{\hat{S}}{\eta_1^4}\right),$ $\quad P_4=-\dfrac{\operatorname{Cosh}(\eta_1)}{\operatorname{Sinh}(\eta_1)}P_1,$

$$P_5 = -\left(\frac{\text{Cosh}(\eta_1)}{\text{Sinh}(\eta_1)} P_2 + \frac{\hat{T}}{\eta_1^4 \,\text{Sinh}(\eta_1)}\right), \qquad P_6 = -\left(\frac{\text{Cosh}(\eta_1)}{\text{Sinh}(\eta_1)} P_3 - \frac{\hat{S}}{\eta_1^4 \,\text{Sinh}(\eta_1)}\right),$$

$$P_7 = \frac{\hat{h}^2 \,\eta_2 \,\text{Cosh}(\eta_2)}{6 Da\, \hat{\kappa}_T}\left(C_2\, \eta_2^2 - 1\right) - \frac{\eta_1^3\left(1 - C_1\, \eta_1^2\right)}{6} P_4,$$

$$P_8 = -\left(\frac{\hat{h}^2\, \Delta_1}{6 Da\, \hat{\kappa}_T}\left(\frac{\text{Sinh}(\eta_2)\left(-C_2\, \eta_2^2 + 1\right)}{\eta_2} - 1\right) - \frac{\eta_1^3\left(-1 + C_1 \eta_1^2\right)}{6} P_5\right)$$

$$P_9 = \left(\frac{\hat{h}^2 \Delta_2}{6 Da\, \hat{\kappa}_T}\left(\frac{\text{Sinh}(\eta_2)\left(1 - C_2\, \eta_2^2\right)}{\eta_2} - 1\right) + \frac{\eta_1^3\left(C_1\, \eta_1^2 - 1\right)}{6} P_6\right),$$

$$P_{10} = \frac{\hat{h}}{2\hat{\mu}\, \hat{\kappa}_T} \eta_2^2 \,\text{Sinh}(\eta_2) - \frac{\eta_1^2}{2} P_1, \qquad P_{11} = \frac{\hat{h}}{2\hat{\mu}\, \hat{\kappa}_T}\left(\text{Cosh}(\eta_2) - 1\right)\Delta_1 - \frac{\eta_1^2}{2} P_2,$$

$$P_{12} = -\left(\frac{\hat{h}}{2\hat{\mu}\, \hat{\kappa}_T}\left(\text{Cosh}(\eta_2) - 1\right)\Delta_2 + \frac{\eta_1^2}{2} P_3\right), \qquad P_{13} = \frac{\eta_2}{\hat{\kappa}_T} \text{Cosh}(\eta_2) - \eta_1 P_4,$$

$$P_{14} = \frac{\Delta_1}{\hat{\kappa}_T}\left(\frac{\text{Sinh}(\eta_2)}{\eta_2} - 1\right) - \eta_1 P_5, \qquad P_{15} = -\left(\frac{\Delta_2}{\hat{\kappa}_T}\left(\frac{\text{Sinh}(\eta_2)}{\eta_2} - 1\right) + \eta_1 P_6\right),$$

$$P_{16} = \frac{1}{\hat{h}\, \hat{\kappa}_T} \text{Sinh}(\eta_2) - P_1, \qquad P_{17} = \frac{\Delta_1}{\hat{h}\, \hat{\kappa}_T}\left(\frac{\text{Cosh}(\eta_2) - 1}{\eta_2^2} - \frac{1}{2}\right) - P_2,$$

$$P_{18} = -\left(\frac{\Delta_2}{\hat{h}\, \hat{\kappa}_T}\left(\frac{\text{Cosh}(\eta_2) - 1}{\eta_2^2} - \frac{1}{2}\right) + P_3\right), \qquad P_{19} = \frac{1}{\hat{h}\, \hat{\kappa}_T} + \frac{1}{\hat{\kappa}_T} - \frac{\hat{h}^2}{6 Da\, \hat{\kappa}_T},$$

$$P_{20} = \left(P_{16} + P_{13} + P_{10} + P_7 + \text{Cosh}(\eta_1) P_1 + \text{Sinh}(\eta_1) P_4\right),$$

$$P_{21} = -\left(P_{17} + P_{14} + P_{11} + P_8 + \text{Cosh}(\eta_1) P_2 + \text{Sinh}(\eta_1) P_5 + \frac{\hat{T}}{24}\right),$$

$$P_{22} = -\left(P_{18} + P_{15} + P_{12} + P_9 + \text{Cosh}(\eta_1) P_3 + \text{Sinh}(\eta_1) P_6 - \frac{\hat{S}}{24}\right),$$

$$P_{23} = -\frac{\hat{h}^2}{Da\, \hat{\kappa}_T}, \qquad P_{24} = \left(2P_{10} + 6P_7 + \eta_1^2 \;\text{Cosh}(\eta_1) P_1 + \eta_1^2 \;\text{Sinh}(\eta_1) P_4\right),$$

$$P_{25} = -\left(2P_{11} + 6P_8 + \eta_1^2 \;\text{Cosh}(\eta_1) P_2 + \eta_1^2 \;\text{Sinh}(\eta_1) P_5 + \frac{\hat{T}}{2}\right)$$

$$P_{26} = -\left(2P_{12} + 6P_9 + \eta_1^2 \ \mathrm{Cosh}(\eta_1) P_3 + \eta_1^2 \ \mathrm{Sinh}(\eta_1) P_6 - \frac{\hat{S}}{2}\right)$$

4. Solvability condition

Integrating equations (39) and (40) at intervals $z = 0$ to $z = 1$, using the relevant boundary conditions and adding the resulting equation yields, the following solvability condition:

$$\begin{pmatrix} \int_0^1 W_1 f_1(z_1) dz_1 + \frac{1}{\hat{h}^2}\int_0^1 W_2 f_2(z_2) dz_2 + \\ \tau_2 \int_0^1 W_1(z_1) dz_1 + \frac{\tau_1}{\hat{h}^2}\int_0^1 W_2(z_2) dz_2 \end{pmatrix} = \frac{\hat{\kappa}_T}{\hat{h}} + \frac{1}{\hat{h}^2} + \left(\frac{\hat{\kappa}_C}{\hat{h}} + \frac{1}{\hat{h}^2}\right)\tau_1 \tau_2 \quad \text{... [41]}$$

here, $f_1(z_1)$, and $f_2(z_2)$ take different forms according to the basic temperature gradients. We denote $f_1(z_1) = f_2(z_2) = 1$ for linear temperature gradient (LTG), $f_1(z_1) = 2z_1$ and $f_2(z_2) = 2z_2$ for parabolic temperature gradient (PTG), $f_1(z_1) = 2(1 - z_1)$ and $f_2(z_2) = 2(1 - z_2)$ for inverted parabolic temperature gradient (ITG).

The following are the expression obtained for the critical Rayleigh numbers linear (Ra_{C1}), parabolic (Ra_{C2}) and inverted parabolic (Ra_{C3}) temperature gradients, respectively.

$$Ra_{C1} = \frac{\frac{\hat{\kappa}_T}{\hat{h}} + \frac{1}{\hat{h}^2} + \left(\frac{\hat{\kappa}_C}{\hat{h}} + \frac{1}{\hat{h}^2}\right)\tau_1 \tau_2 - \Sigma_2 Ra_S}{\Sigma_1} \quad \text{... [42]}$$

$$Ra_{C2} = \frac{\frac{\hat{\kappa}_T}{\hat{h}} + \frac{1}{\hat{h}^2} + \left(\frac{\hat{\kappa}_C}{\hat{h}} + \frac{1}{\hat{h}^2}\right)\tau_1 \tau_2 - \Sigma_4 Ra_S}{\Sigma_3} \quad \text{... [43]}$$

$$Ra_{C3} = \frac{\frac{\hat{\kappa}_T}{\hat{h}} + \frac{1}{\hat{h}^2} + \left(\frac{\hat{\kappa}_C}{\hat{h}} + \frac{1}{\hat{h}^2}\right)\tau_1 \tau_2 - \Sigma_6 Ra_S}{\Sigma_5} \quad \text{... [44]}$$

Here,

$$\Sigma_1 = (1+\tau_2)\Pi_1 + \frac{1}{\hat{h}^2}(1+\tau_1)\Pi_3 \qquad \Sigma_2 = (1+\tau_2)\Pi_2 + \frac{1}{\hat{h}^2}(1+\tau_1)\Pi_4$$

$$\Pi_1 = \left(\Gamma_7 + \frac{\Gamma_9}{2} + \frac{\Gamma_{11}}{3} + \frac{\Gamma_{13}}{4} + \left(\frac{\mathrm{Sinh}(\eta_1)}{\eta_1}\right)\Gamma_{15} + \left(\frac{\mathrm{Cosh}(\eta_1) - 1}{\eta_1}\right)\Gamma_{17} + \frac{\hat{T}}{120}\right)$$

$$\Pi_2 = \left(\Gamma_8 + \frac{\Gamma_{10}}{2} + \frac{\Gamma_{12}}{3} + \frac{\Gamma_{14}}{4} + \left(\frac{\mathrm{Sinh}(\eta_1)}{\eta_1}\right)\Gamma_{16} + \left(\frac{\mathrm{Cosh}(\eta_1) - 1}{\eta_1}\right)\Gamma_{18} + \frac{\hat{S}}{120}\right)$$

$$\Pi_3 = \frac{1}{\hat{h}^2}\left(\frac{\Gamma_3}{2} + \left(\frac{\mathrm{Sinh}(\eta_2)}{\eta_2^3} - \frac{1}{\eta_2^2} - \frac{1}{6}\right)\Delta_1 + \left(\frac{\mathrm{Cosh}(\eta_2)-1}{\eta_2}\right)\Gamma_1\right)$$

$$\Pi_4 = \frac{1}{\hat{h}^2}\left(\frac{\Gamma_4}{2} + \left(\frac{\mathrm{Sinh}(\eta_2)}{\eta_2^3} - \frac{1}{\eta_2^2} - \frac{1}{6}\right)\Delta_2 + \left(\frac{\mathrm{Cosh}(\eta_2)-1}{\eta_2}\right)\Gamma_2\right)$$

$$\Sigma_3 = (1+\tau_2)\Pi_5 + \frac{1}{\hat{h}^2}(1+\tau_1)\Pi_7 \qquad \Sigma_4 = (1+\tau_2)\Pi_6 + \frac{1}{\hat{h}^2}(1+\tau_1)\Pi_8$$

$$\Pi_5 = 2\left(\frac{\Gamma_7}{2} + \frac{\Gamma_9}{3} + \frac{\Gamma_{11}}{4} + \frac{\Gamma_{13}}{5} + \left(\frac{\mathrm{Sinh}(\eta_1) - \mathrm{Cosh}(\eta_1) + 1}{\eta_1}\right)\Gamma_{15} + \left(\frac{\mathrm{Cosh}(\eta_1) - \mathrm{Sinh}(\eta_1)}{\eta_1}\right)\Gamma_{17} + \frac{\hat{T}}{144}\right)$$

$$\Pi_6 = 2\left(\frac{\Gamma_8}{2} + \frac{\Gamma_{10}}{3} + \frac{\Gamma_{12}}{4} + \frac{\Gamma_{14}}{5} + \left(\frac{\mathrm{Sinh}(\eta_1) - \mathrm{Cosh}(\eta_1) + 1}{\eta_1}\right)\Gamma_{16} + \left(\frac{\mathrm{Cosh}(\eta_1) - \mathrm{Sinh}(\eta_1)}{\eta_1}\right)\Gamma_{18} + \frac{\hat{S}}{144}\right)$$

$$\Pi_7 = \frac{2}{\hat{h}^2}\left(\frac{\Gamma_3}{3} + \left(\frac{\mathrm{Sinh}(\eta_2) - \mathrm{Cosh}(\eta_2) + 1}{\eta_2^3} - \frac{1}{2\eta_2^2} - \frac{1}{8}\right)\Delta_1 + \left(\frac{\mathrm{Cosh}(\eta_2) - \mathrm{Sinh}(\eta_2)}{\eta_2}\right)\Gamma_1\right)$$

$$\Pi_8 = \frac{2}{\hat{h}^2}\left(\frac{\Gamma_4}{3} + \left(\frac{\mathrm{Sinh}(\eta_2) - \mathrm{Cosh}(\eta_2) + 1}{\eta_2^3} - \frac{1}{2\eta_2^2} - \frac{1}{8}\right)\Delta_2 + \left(\frac{\mathrm{Cosh}(\eta_2) - \mathrm{Sinh}(\eta_2)}{\eta_2}\right)\Gamma_2\right)$$

$$\Sigma_5 = (2+\tau_2)\Pi_1 + (1+\tau_1)\Pi_3 - \Pi_5 - \Pi_7 \qquad \Sigma_6 = (2+\tau_2)\Pi_2 + (1+\tau_1)\Pi_4 - \Pi_6 - \Pi_8$$

5.0 Results and discussions

The regular perturbation approach is used to estimate the eigenvalue to evaluate the stability of double-diffusive Rayleigh-Darcy convection in a composite system for LTG, PTG and ITG. For dimensionless factors such as couple stress parameter, solute Rayleigh number, ratio of couple stress viscosity, thermal diffusivity ratio, molecular diffusivity ratio of thermal to heat Darcy number, and thermal diffusivity ratio, variations in the critical Rayleigh number Ra_{C1}, Ra_{C2} and Ra_{C3} against the depth ratio for all three temperature gradients are plotted to depict the system's stability characteristics. We set the viscosity ratio $\hat{\mu} = 1$ and the coefficient of thermal expansion ratio $\hat{\alpha}_T = \hat{\alpha}_C = 1$.

Figures (1a, 1b, 1c) demonstrate the variance in critical Rayleigh number Ra_{C1}, Ra_{C2} and Ra_{C3} against depth ratio for LTG, PTG, and ITG. As a result, it can be deduced from the comparative plot that the ITG has the most stabilizing effect, the LTG has a moderate stabilizing effect, and the PTG has a destabilizing effect on DDRDC. As the solute Rayleigh number $Ra_S = 100$ is fixed for all three temperature gradients and the critical thermal Rayleigh number starts at the same value at the commencement of convection, a linear relationship between the solute and thermal Rayleigh numbers is implied.

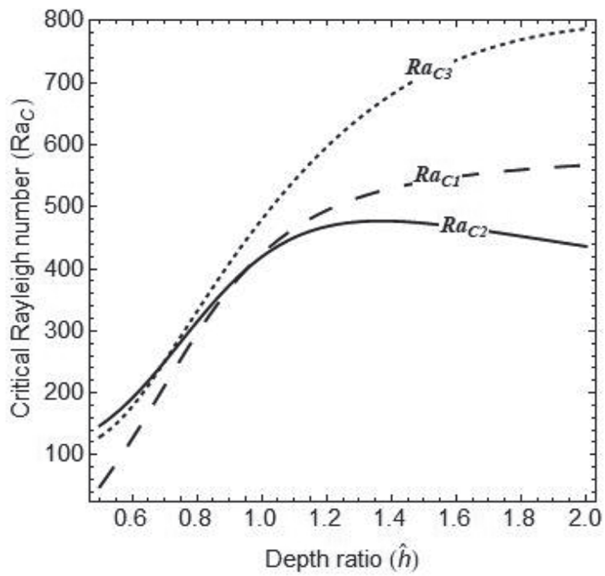

Figure 1: The combined effect Ra_{C1} , Ra_{C2} and Ra_{C3}

For LTG, PTG, and ITG, the variation in critical thermal Rayleigh number Ra_{C1}, Ra_{C2} and Ra_{C3} versus depth ratio for different values of $\Lambda = 0.4$, $\Lambda = 0.6$, and $\Lambda = 0.8$ is depicted in Figures (2a, 2b, 2c). The graph demonstrates that the parameter Λ has a stabilizing effect on DDRDC because as the value of Λ increases so does Ra_{C1}, Ra_{C2} and Ra_{C3}. Increasing the value of Λ increases the fluid's viscosity and consequently delays the onset of DDRDC. This enhances the system's stability across all three temperature gradients.

For LTG, PTG, and ITG, the variation in critical thermal Rayleigh number Ra_{C1}, Ra_{C2} and Ra_{C3} versus depth ratio for different values of $Da = 0.0003$, $Da = 0.003$, and $Da = 0.03$ is depicted in Figures (3a, 3b, 3c). The graph demonstrates that the parameter has a destabilizing effect on DDRDC because an increase in causes a decrease in Ra_{C1}, Ra_{C2} and Ra_{C3}. Increasing the value of Da accelerates the onset of DDRDC by increasing the permeability of the system. This causes the system to be unstable across all three temperature gradients.

For LTG, PTG, and ITG, the variation in critical thermal Rayleigh number Ra_{C1}, Ra_{C2} and Ra_{C3} versus depth ratio for different values of $\hat{\mu}_C = 0.6$, $\hat{\mu}_C = 0.8$, and $\hat{\mu}_C = 1.0$ is depicted in Figures (4a, 4b, 4c). The graph demonstrates that the parameter $\hat{\mu}_C$ has a stabilizing effect on DDRDC because an increase in $\hat{\mu}_C$ causes an increase in Ra_{C1}, Ra_{C2} and Ra_{C3}. Increasing the value of $\hat{\mu}_C$ delays the onset of DDRDC by increasing the system's couple stress viscosity. As a result, the system remains stable across all three temperature gradients.

For LTG, PTG, and ITG, the variation in critical thermal Rayleigh number Ra_{C1}, Ra_{C2} and Ra_{C3} versus depth ratio for different values of $\hat{\kappa}_T = 0.4$, $\hat{\kappa}_T = 0.6$, and $\hat{\kappa}_T = 0.8$ is depicted in Figures (5a, 5b, 5c). The graph demonstrates that the parameter $\hat{\kappa}_T$ has a destabilizing effect on DDRDC because a rise in $\hat{\kappa}_T$

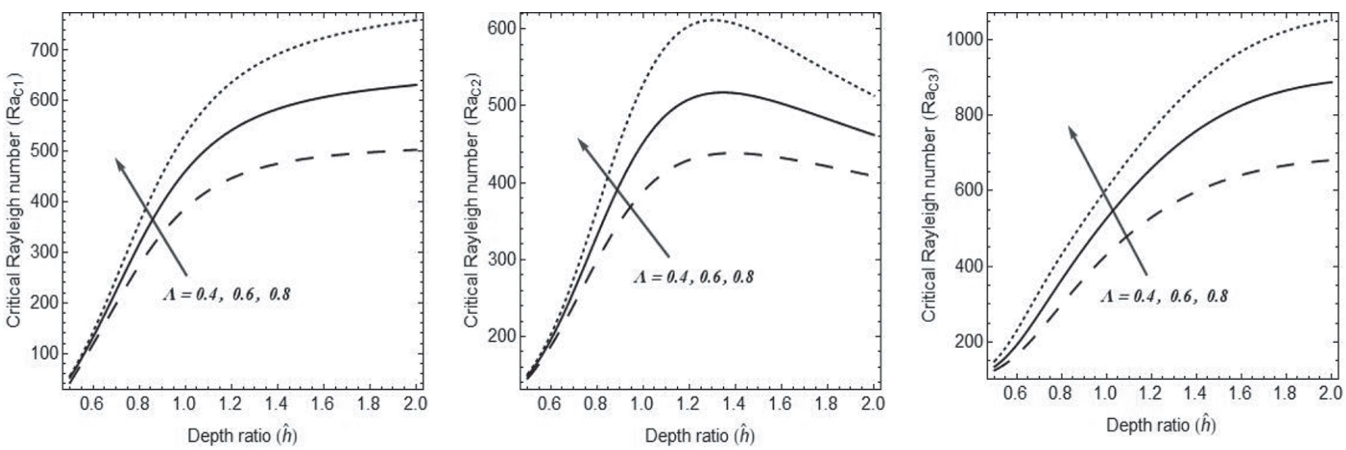

Figure: (2a, 2b, 2c): Significance of Λ

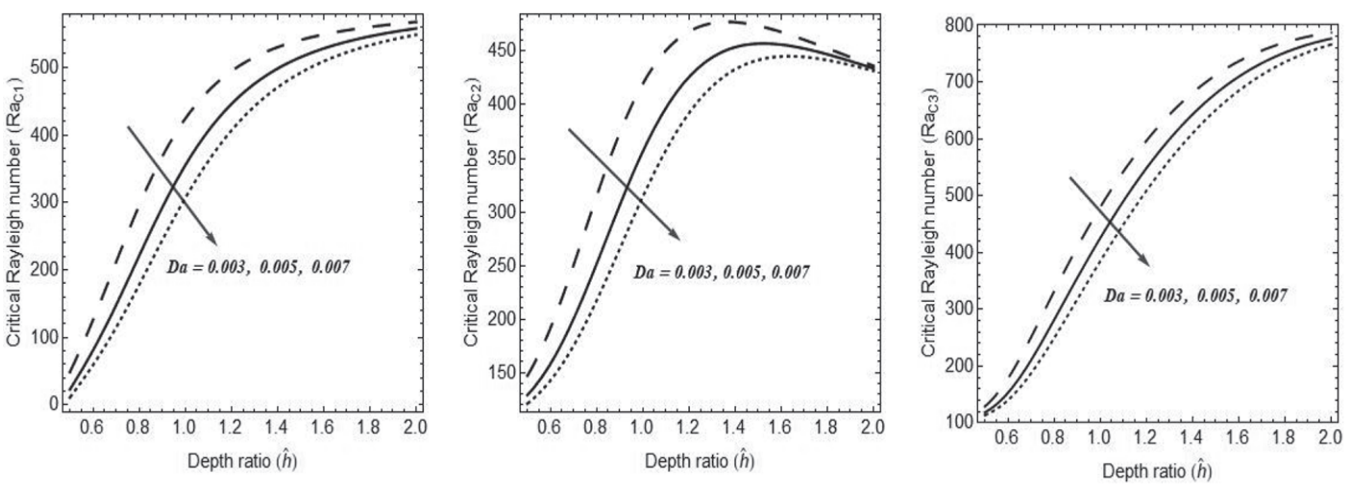

Figure. (3a, 3b, 3c): Significance of *Da* induces a drop in Ra_{C1}, Ra_{C2} and Ra_{C3}. Increasing the value of $\hat{\kappa}_T$ hastens the onset of DDRDC by increasing the thermal density gradient of the system. Consequently, the system continues to be unstable over all three temperature gradients.

For LTG, PTG, and ITG, the variation in critical thermal Rayleigh number Ra_{C1}, Ra_{C2} and Ra_{C3} versus depth ratio for different values of $\hat{\kappa}_C = 0.4$, $\hat{\kappa}_C = 0.6$, and $\hat{\kappa}_T = 0.8$ is depicted in Figures (6a, 6b, 6c). The graph demonstrates that the parameter $\hat{\kappa}_C$ stabilizes DDRDC since an increase in $\hat{\kappa}_C$ produces an increase in Ra_{C1}, Ra_{C2} and Ra_{C3}. Increasing the value of $\hat{\kappa}_C$ delays the onset of DDRDC by reducing the system's solute density gradient. As a result, the system remains stable throughout all three temperature gradients.

For LTG, PTG, and ITG, the variation in critical thermal Rayleigh number Ra_{C1}, Ra_{C2} and Ra_{C3} versus depth ratio for different values of $R_{aS} = 100$, $R_{aS} = 200$, and R_{aS} is depicted in Figures (7a, 7b, 7c). The graph demonstrates that the parameter RaS stabilizes DDRDC because as the value of R_{aS} increases so do Ra_{C1}, Ra_{C2} and Ra_{C3}. This suggests that a system with a higher solute concentration in the fluid requires a larger buoyancy force to induce convective motion, which in turn delays the initiation of DDRDC. This enhances the system's stability across all three temperature gradients.

For LTG, PTG, and ITG, the variation in critical thermal Rayleigh number Ra_{C1}, Ra_{C2} and Ra_{C3} versus depth ratio for different values of $\tau = 0.4$, $\tau = 0.6$, and $\tau = 0.8$ is depicted in Figures (8a, 8b, 8c). The graph demonstrates that the parameter τ has a stabilizing effect on DDRDC because an increase in τ causes an increase in Ra_{C1}, Ra_{C2} and Ra_{C3}. By increasing τ in the system, the development of DDRDC can be delayed. Thus, the system's stability is maintained over the three temperature gradients.

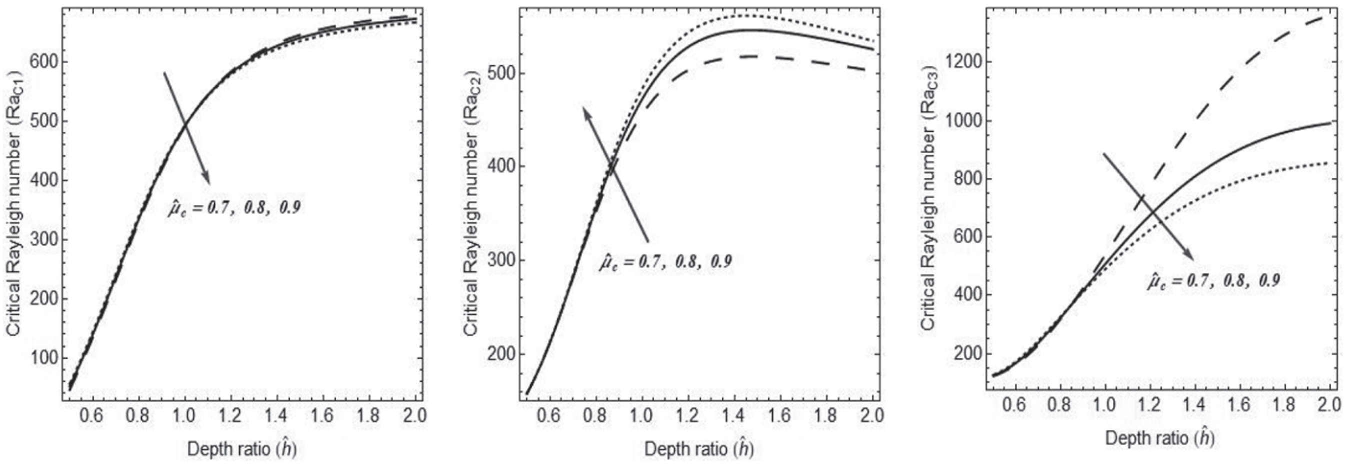

Figure (4a, 4b, 4c): Significance of $\hat{\mu}_C$

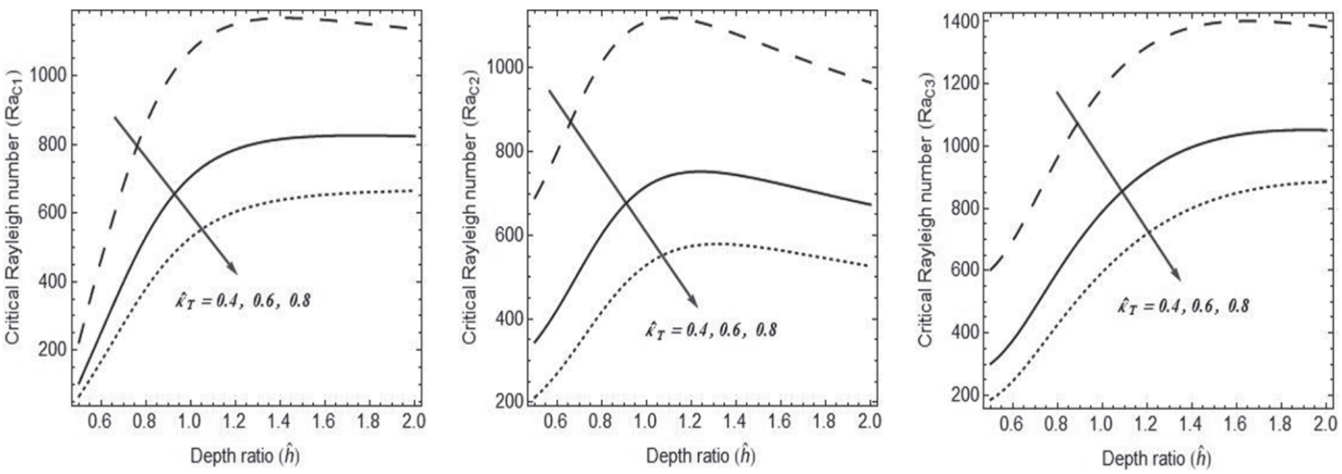

Figure (5a, 5b, 5c): Significance of $\hat{\kappa}_T$

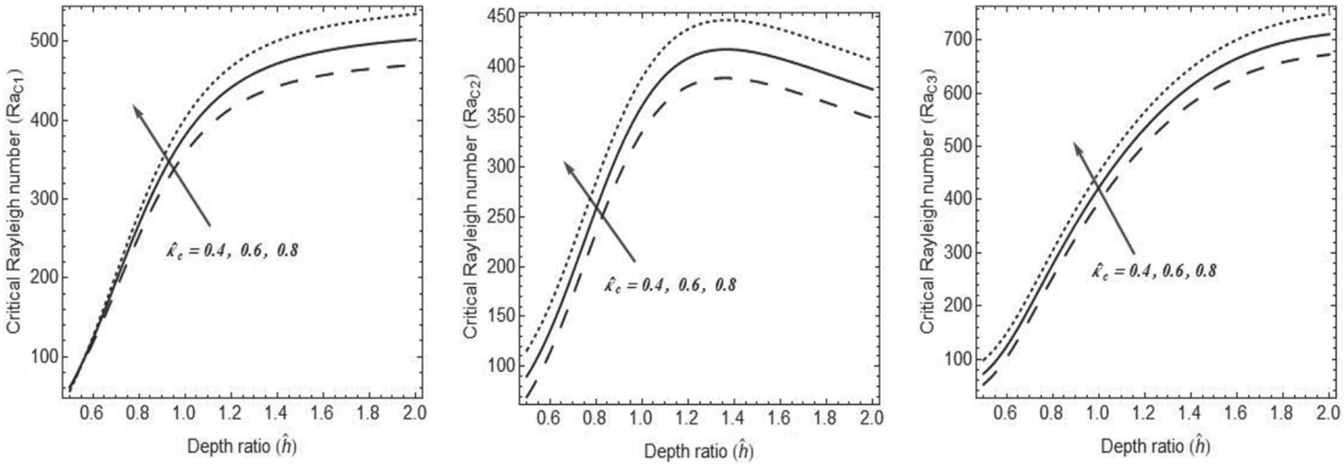

Figure (6a, 6b, 6c): Significance of $\hat{\kappa}_C$

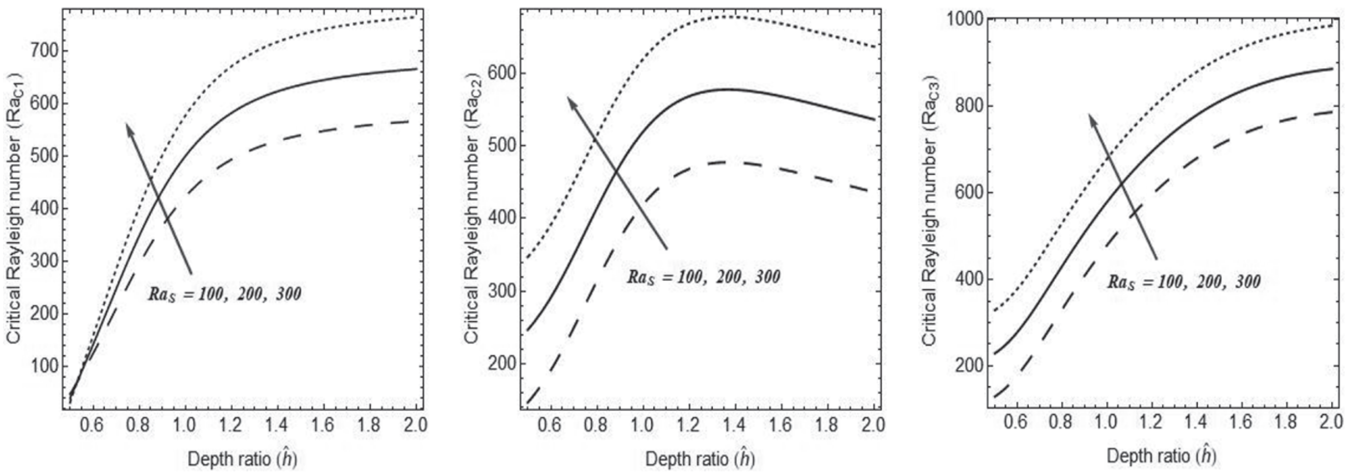

Figure (7a, 7b, 7c): Significance of Ra_S

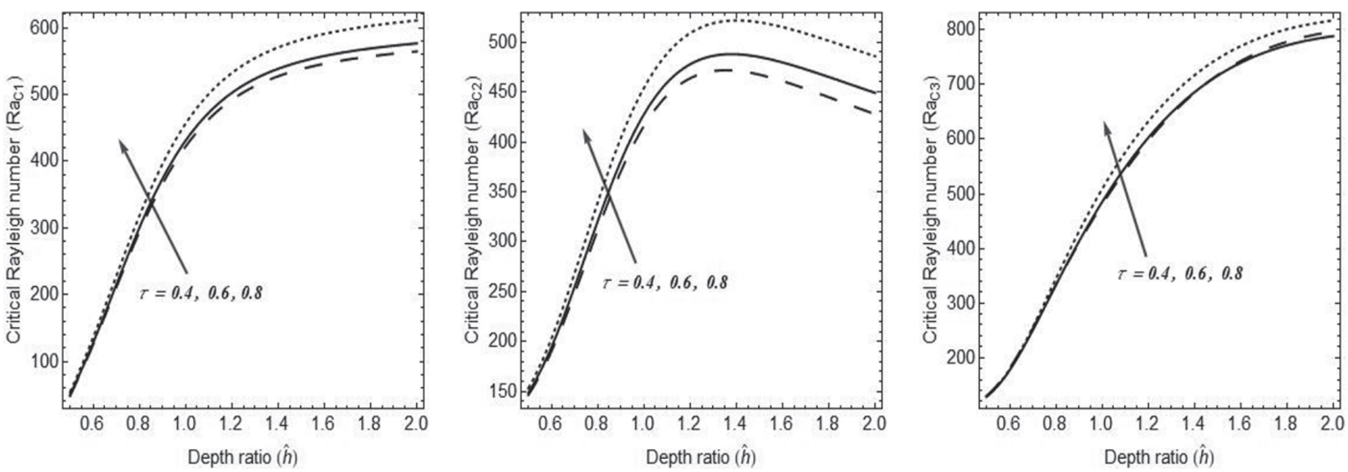

Figure. (8a, 8b, 8c): Significance τ

Conclusions

The impact of LTG, PTG, and ITG on the onset of DDRDC in a composite system with couple stress fluid is investigated using linear stability analysis. The expressions for LTG, PTG, and ITG critical Rayleigh numbers are obtained as functions of various dimensionless parameters, and their effect on system stability are graphically shown.

The conclusions reached are as follows:

- Darcy number and thermal diffusivity ratio both accelerate the onset of DDRDC.
- Couple stress parameter, solute Rayleigh number, ratio of couple stress viscosity, solute diffusivity ratio, and diffusivity ratio of solute to heat all delay the onset of DDRDC.
- The onset of DDRDC is most stable for inverted parabolic temperature gradient.

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