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# Two Component Non-Darcian Benard Marangoni Convection with Uniform and Non-Uniform Temperature Gradients in a Composite Layer with Variable Heat Sources/Sinks

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## *Abstract:*

*In a composite layer that comprises of a porous layer which is sparsely packed and saturated with two component incompressible fluid and above this porous layer lays a layer of the same fluid, with variable heat sources or sinks in both the layers double diffusive non-Darcian Benard Marangoni (DDNBM) convection is investigated. The upper surface of the composite layer has Marangoni effects which depend on temperature and concentration, whereas the lower surface is rigid. The inverted parabolic, parabolic and linear temperature profile is applied to this composite layer, which is surrounded by adiabatic boundaries. The appropriate thermal Marangoni numbers (TMANs) which are the eigen values (EVs) are calculated for all the three temperature gradients. The impact of different parameters on the EVs with respect to depth ratio is examined, thoroughly. The parameters that effect DDNBM convection are found.*

**Keywords:** *Non-Darcy model, Temperature dependent heat source, Temperature gradients, Adiabatic boundaries.*

## 1.0 Introduction

Double diffusive convection is a phenomenon in fluid dynamics that explains a type of convection that is caused by two separate density gradients with differing diffusion rates. The density fluctuations within fluids under the force of gravity drive the convection. Some literature on Double diffusive Marangoni convection/flow and Non-Darcy flow are by Herbert *et al.* (1981) gave a somewhat personal opinion of the significant developments in double-diffusive convection, a subject whose evolution has been the outcome of a close interaction between theoreticians, sea-going oceano-graphers and laboratory experimenters. Applications in astrophysics, geology and engineering have subsequently emerged. Nithiarasu *et al.* (1997) explored convective flow regimes on double diffusive free convection in a porous medium for Darcy and Darcy-Brinkmann cases. Using the Forchheimer-extended Darcy equation, Shivakumara et al. (2006) investigated the effects of quadratic drag and vertical through flow on double diffusive convection in a horizontal porous layer using linear stability theory. Costa *et al.* (2006) considering the transition between stable and oscillatory convection, investigated the double diffusive convection. Umavathi *et al.* (2014) investigated the commencement of thermosolutal convection in a

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porous media saturated with nanofluid for variable viscosity and thermal conductivity. In a Darcy's porous media saturated with nanofluid, Dastmalchi *et al.* (2015) investigated the effects of Brownian motion and thermophoresis on Double diffusive free convection. Sheremet *et al.* (2015) in a porous cavity saturated with nanofluid investigated double diffusive convection using Buongiorno's model. Sara *et al.* (2020) investigated the effect of anisotropic permeability on a Darcy type double-diffusive bidisperse porous media. Manjunatha *et al.* (2020) examined double diffusive Marangoni convection for Darcy model and Thermal Marangoni number was determined by solving the ODE's. For porous media in a vertical channel which was subjected to heat, Yen-Cho Chen (2004) examined the linear stability analysis of mixed convection. Ashok *et al.* (2011) for a porous medium in a vertical pipe, looked into the reports of mixed convection for non-Darcy case. In this problem the effects of uniform and non-uniform temperature profiles on DDNBM convection in a composite layer with variable heat sources or sinks is investigated. For all the three temperature profiles the effects of the Darcy number, wave number, internal Rayleigh numbers, viscosity ratio, solute thermal diffusivity ratios, Solute Marangoni number, solute diffusivity ratio and thermal diffusivity ratio on DDNBM convection are depicted graphically.

## I. Mathematical formulation

Assume a composite layer which consists of a porous layer with depth $d_2$ that is sparsely packed and saturated with single component incompressible fluid and above the porous layer lays a layer of the same fluid with depth $d_1$, with heat sources or sinks $Q_1$ and $Q_2$ which depends on temperature in both layers. The lower layer which is porous layer is rigid, while the upper layer which is fluid layer is free with Marangoni effects that depend on temperature and concentration. The following equations are the governing equations:

$$\left.\begin{aligned}
&\nabla_1.\overrightarrow{q_1} = 0 \quad \ldots(1)\\
&\rho_0\left[\frac{\partial\overrightarrow{q_1}}{\partial t_1} + (\overrightarrow{q_1}.\nabla_1)\overrightarrow{q_1}\right] = -\nabla P_1 + \mu_1{\nabla_1}^2\overrightarrow{q_1} \quad \ldots(2)\\
&\frac{\partial T_1}{\partial t_1} + (\overrightarrow{q_1}.\nabla_1)T_1 = \kappa_1{\nabla_1}^2T_1 + Q_1(T_1 - T_0) \quad \ldots(3)\\
&\frac{\partial C_1}{\partial t_1} + (\overrightarrow{q_1}.\nabla_1)C_1 = \kappa_{C1}{\nabla_1}^2C_1 \quad \ldots(4)
\end{aligned}\right\}\quad \text{For Fluid layer}$$

$$\left.\begin{aligned}
&\nabla_2.\overrightarrow{q_2} = 0 \quad \ldots(5)\\
&\frac{\rho_0}{\emptyset}\left[\frac{\partial\overrightarrow{q_2}}{\partial t_2}\right] = -\nabla_2P_2 - \frac{\mu_1}{K}\overrightarrow{q_2} + \mu_2{\nabla_2}^2\overrightarrow{q_2} \quad \ldots(6)\\
&A\frac{\partial T_2}{\partial t_2} + (\overrightarrow{q_2}.\nabla_2)T_2 = \kappa_2\nabla^2{}_2T_2 + Q_2(T_2 - T_0) \quad \ldots(7)\\
&\phi\frac{\partial C_2}{\partial t_2} + (\overrightarrow{q_2}.\nabla_2)C_2 = \kappa_{C2}{\nabla_2}^2C_2 \quad \ldots(8)
\end{aligned}\right\}\quad \text{For Porous layer}$$

Where, $\vec{q}_1$ is Velocity vector, $T_1$ is the Temperature, $C_1$ is the concentration, $\kappa_1$ is Thermal diffusivity of the fluid, $\Phi$ is the Porosity, $Q_1$ is the heat source/sink for fluid layer, $\rho_0$ is Fluid density, $t_1$ is time, $\mu_1$ is Fluid viscosity, $P_1$ is Pressure, $\kappa_{C1}$ is Solute Thermal diffusivity of the fluid, $K$ is permeability of the porous medium, $\mathrm{A}=\frac{(\rho_0C_p)_2}{(\rho_0C_p)_1}$ ratio of heat capacities, $C_p$ is Specific heat and the quantities with subscript '2' denote the same in the porous layer and '1' denotes the fluid layer.

Both the layers the fluid and the porous layer are at rest in the fundamental state. According to Vanishree and Sumithra (2020), we apply minute disturbances on the fundamental state for both the layers and the obtained equations are non-dimensionalized using proper scale factors. The resulting equations which are dimensionless are then exposed to normal mode analysis, leading to the following EV problem:

In $0 \leq z_1 \leq 1$

For Fluid layer

$$\left(D_1^{\,2} - a_1^{\,2}\right)^2 W_1 = 0 \qquad \dots (9)$$

$$\left[\left(D_1^{\,2} - a_1^{\,2} + R_{I1}\right)\theta_1 + g_1(z_1)\frac{W_1\sqrt{R_{I1}}\,Cos[\sqrt{R_{I1}}\,z_1]}{Sin[\sqrt{R_{I1}}]}\right] = 0 \qquad \dots (10)$$

$$\left[\tau_1\left(D_1^{\,2} - a_1^{\,2}\right)\right] S_1 + W_1 = 0 \qquad \dots (11)$$

In $-1 \leq z_2 \leq 0$

For Porous layer

$$\left[\left(D_2^{\,2} - a_2^{\,2}\right)\hat{\mu}\, Da - 1\right]\left(D_2^{\,2} - a_2^{\,2}\right)W_2 = 0 \qquad \dots (12)$$

$$\left[\left(D_2^{\,2} - a_2^{\,2} + R_{I2}\right)\theta_2 + g_2(z_2)\frac{W_2\sqrt{R_{I2}}\,Cos[\sqrt{R_{I2}}\,z_2]}{Sin[\sqrt{R_{I2}}\,]}\right] = 0 \qquad \dots (13)$$

$$\left[\tau_2\left(D_2^{\,2} - a_2^{\,2}\right)\right] S_2 + W_2 = 0 \qquad \dots (14)$$

Where, $R_{I1} = \frac{Q_1}{\kappa_1} d_1^{\,2}$ is the internal Rayleigh number in fluid layer, $R_{I2} = \frac{Q_2}{\kappa_2} d_2^{\,2}$ is the internal Rayleigh number in porous layer, $\hat{\mu} = \frac{\mu_2}{\mu_1}$ is the viscosity ratio, $Da = \frac{K}{d_2^{\,2}}$ is the Darcy number, $\tau_1 = \frac{\kappa_{C1}}{\kappa_1}$ is the solute thermal diffusivity ratio, $\tau_2 = \frac{\kappa_{C2}}{\kappa_2}$ is the solute thermal diffusivity ratio in porous layer.

Eight velocity equations, four temperature equations and four concentration equations boundary conditions are required to solve the above equations.

## II Boundary conditions

Boundary conditions which are dimensionless and exposed to normal mode analysis are:

### 1. Velocity boundary conditions

$$\left.\begin{aligned}
&W_1(1) = 0,\ \ W_2(-1) = 0,\ \ W_1(0) = \frac{\zeta}{\epsilon_T} W_2(0),\ \ D_1 W_1(0) = \frac{\zeta^2}{\epsilon_T} D_2 W_2(0)\\
&D_2 W_2(-1) = 0\ ,\quad D_1^{\,2} W_1(0) - a_1^{\,2} W_1(0) = \hat{\mu}\frac{\zeta^3}{\epsilon_T} D_2^{\,2} W_2(0) + a_2^{\,2} W_2(0),\\
&D_1^{\,3} W_1(0) - 3a_1^{\,2} D_1 W_1(0) = -\frac{\zeta^4}{Da\,\epsilon_T} D_2 W_2(0) + \frac{\zeta^4}{\epsilon_T}\left[D_2^{\,3} W_2(0) - 3a_2^{\,2} D_2 W_2(0)\right]\ ,\\
&D_1^{\,2} W_1(1) + a_1^{\,2} M\theta_1(1) = 0
\end{aligned}\right\} \qquad \dots (15)$$

**2. Temperature boundary conditions (Adiabatic-Adiabatic)**

$$D_1\theta_1(1) = 0, \theta_1(0) = \theta_2(0)\frac{\epsilon_T}{\zeta},\ D_1\theta_1(0) = D_2\theta_2(0), D_2\theta_2(-1) = 0 \quad \text{... (16)}$$

**3. Concentration boundary conditions**

$$D_1S_1(1) = 0, S_1(0) = S_2(0)\frac{\epsilon_S}{\zeta}, D_1S_1(0) = D_2S_2(0), D_2S_2(-1) = 0 \quad \text{... (17)}$$

Where, $M = \frac{-(T_0-T_u)d_1}{\mu_1 K}\frac{\partial\sigma}{\partial T_1}$ is the thermal Marangoni number (TMAN), $T_1$ is the temperature and $\sigma$ is the surface tension, $\zeta = \frac{d_1}{d_2}$ is the depth ratio, $\epsilon_T = \frac{\kappa_1}{\kappa_2}$ is thermal diffusivity ratio, $\epsilon_S = \frac{\kappa_{C1}}{\kappa_{C2}}$ is solute thermal diffusivity ratio.

## III Solution by Exact method

**1. For vertical velocity distributions $W_1(z_1)$ and $W_2(z_2)$, the velocity equations (9) and (12) are solved exactly and are obtained as,**

$$W_1(z_1) = C_{f1}[(C_{f4}z_1 + C_{f2})\alpha_1 + (C_{f3}z_1 + 1)\alpha_2]$$

$$W_2(z_2) = [C_{p4}\alpha_5 + C_{p3}\alpha_6 + C_{p2}\alpha_3 + C_{p1}\alpha_4]C_{f1}$$

Where,

$$\alpha_1 = sinh[a_1z_1]\ ; \alpha_3 = sinh[a_2z_2]\ ; \alpha_2 = cosh[a_1z_1]\ ; \alpha_5 = sinh[\delta z_2]\ ; \alpha_4 = cosh[a_2z_2]\ ;$$

$$\alpha_6 = cosh[\delta z_2];\quad C_{f2} = \frac{E_9C_{p2}+E_{10}C_{p4}}{E_8};\quad C_{f3} = \frac{a_pC_{p2}+\delta C_{p4}-E_3a_fC_{f2}}{E_3};\ C_{f4} = \frac{E_6C_{p1}+E_7C_{p3}-E_5a_f}{E_5};$$

$$C_{p1} = E_2 - C_{p3}\quad ;\ C_{p2} = \frac{E_{11}C_{p1}+E_{13}C_{p3}-E_{14}C_{p4}}{E_{12}};\ C_{p3} = \frac{-\eta_{15}-\eta_{14}C_{p4}}{\eta_{13}};\ C_{p4} = \frac{\eta_{13}\eta_{18}-\eta_{15}\eta_{16}}{\eta_{16}\eta_{14}-\eta_{13}\eta_{17}};$$

Where,

$$\eta_1 = E_3E_5a_1 - E_1E_3E_5\quad ;\quad \eta_2 = E_3E_5 - E_1E_3E_5a_1\ ; E_{14} = sinh[\delta];$$

$$\eta_4 = E_3E_7E_8 - E_3E_6E_8\ ;\ \eta_6 = E_1E_5E_8\delta + \eta_2E_{10}\ ; \eta_5 = E_1E_5E_8a_2 + \eta_2E_9\ ;$$

$$\eta_7 = E_{11}E_2\ ; \eta_9 = -a_2E_{12}E_2\ ;\quad \eta_8 = E_{13} - E_{11}\ ;\ \eta_{10} = a_2E_{12} - \delta E_{14}\ ;$$

$$\eta_{11} = a_2E_{11}\ ; \eta_{14} = B_{12}\,\eta_6 - E_{14}\,\eta_5\ ;\ \eta_{12} = \delta E_{13}\ ;\ \eta_{13} = E_{12}\,\eta_4 + \eta_5\,\eta_8\ ;$$

$$\eta_{15} = \eta_5\,\eta_7 - \eta_3E_{12}\ ;\ \eta_{17} = \eta_{12}E_{12} - \eta_{11}E_{14}\ ;\ \eta_{16} = \eta_{11}\,\eta_8 + \eta_{10}E_{12}\ ;$$

$$\eta_{18} = \eta_{11}\,\eta_7 + \eta_9E_{12}\ ; E_{11} = cosh[a_2]; E_6 = 2a_2^2\ ; E_7 = \delta^2 + a_2^2\ ;$$

$$E_1 = \frac{cosh[a_1]}{sinh[a_1]} = \frac{E_{16}}{E_{15}};\quad E_2 = \frac{\epsilon_T}{\zeta};\quad E_3 = \frac{\epsilon_T}{\zeta^2};\quad E_5 = \frac{2\,a_1\epsilon_T}{\hat{\mu}\zeta^3}\ ; E_{13} = cosh[\delta]\ ;$$

$$E_8 = -2a_1^3; E_9 = \frac{-\zeta^2a_2}{Da\,\epsilon_T} + \frac{\zeta^4a_2^3}{\epsilon_T} - 3a_2^3\ ; E_{10} = \frac{-\zeta^2\delta}{Da\,\epsilon_T} + \frac{\zeta^4\delta^3}{\epsilon_T} - 3a_2^2\delta;$$

$$E_{12} = sinh[a_2]; \eta_3 = E_8\eta_1 - E_2E_3E_6E_8\ ; E_{15} = \sinh[a_1]\ ; E_{16} = \cosh[a_1]$$

**2. For concentration distributions $S_1(Z_1)$ and $S_2(Z_2)$, the solution equations (11) and (14) are solved exactly and are obtained as,**

$$S_1(z_1) = C_{f1}[C_{f8}\alpha_1 + C_{f7}\alpha_2] - \frac{C_{f1}}{\tau_1}\left[\frac{C_{f2}\,z_1}{2a_1}\alpha_2 + \frac{z_f}{2a_f}\alpha_1 + \left[\frac{1}{8{a_1}^3}\alpha_2 - \frac{z_1}{4a_1^2}\alpha_1 + \frac{{z_1}^2}{4a_1}\alpha_2\right]C_{f4}\right.$$

$$\left. + \left[\frac{1}{8{a_1}^3}\alpha_1 - \frac{z_1}{4a_1^2}\alpha_2 + \frac{{z_1}^2}{4a_1}\alpha_1\right]C_{f3}\right]$$

$$S_2(z_2) = [C_{p8}\,\alpha_3 + C_{p7}\,\alpha_4]C_{f1} - C_{f1}\left[\frac{(z_2C_{p2}\,\alpha_4)}{2a_2\tau_2} + \frac{(z_2C_{p1}\,\alpha_3)}{2a_2\tau_2} - \frac{C_{p4}\,\alpha_5}{\tau_2(a_2^2-\delta^2)} - \frac{C_{p3}\,\alpha_6}{\tau_2(a_2^2-\delta^2)}\right]$$

Where

$$C_{f7} = E_3 + C_{p7}E_2\ ;\ C_{f8} = \frac{E_1 - C_{f7}\,a_1\,E_{15}}{E_{16}}\ ;\ C_{p7} = \frac{E_8}{E_9}\ ;\ C_{p8} = \frac{C_{f8}a_1 - E_4}{a_2}$$

**3. For Temperature distributions $\theta_1(z_1)$ and $\theta_2(z_2)$, the temperature equations (10) and (13) are solved exactly for three temperature profiles:**

### 3.1 TMAN for Linear Temperature Profile (LTP):

The LTP of the form $g_1(z_1) = 1$ and $g_2(z_2)$=1, is introduced in (10) and (13) and solving we obtain temperature distributions $\theta_1(z_1)$ and $\theta_2(z_2)$ using (16):

$$\theta_1(z_1) = C_{f1}[\,f_1(z_1) + C_{f6}\,\alpha_7 + C_{f5}\,\alpha_8]$$

$$\theta_2(z_2) = [f_2(z_2) + C_{p6}\,\alpha_9 + C_{p5}\,\alpha_{10}]\,C_{f1}$$

Where $\alpha_7 = sinh(b_1z_1); \alpha_8 = \cosh(b_1z_1)\ ; \alpha_9 = sinh(b_2z_2);\ \alpha_{10} = \cosh(b_2z_2);$

$$\alpha_{12} = \cos[\sqrt{R_{I1}}z_1]\ ;\ \alpha_{11} = \sin[\sqrt{R_{I1}}z_1]\ ;$$

$$\alpha_{13} = \sin[\sqrt{R_{I2}}z_2]\ ;\ \alpha_{14} = \cos[\sqrt{R_{I2}}z_2]\ ;$$

$$\alpha_{15} = \sinh[b_1]; \alpha_{16} = \cosh[b_1]\ ; \alpha_{18} = \cosh[b_2]\ ; \alpha_{17} = \sinh[b_2]\ ;$$

$$\alpha_{20} = \cos[\sqrt{R_{I1}}]\ ;\ \alpha_{19} = \sin[\sqrt{R_{I1}}\ ]; \alpha_{21} = \sin[\sqrt{R_{I2}}\ ]\ ; \alpha_{22} = cos[\sqrt{R_{I2}}];$$

$$b_1 = \sqrt{{a_1}^2 - R_{I1}}, b_2 = \sqrt{{a_2}^2 - R_{I2}}\ ;$$

$$f_1(z_1) = S_{f1}[f1 + f2 + f4 + f3]$$

$$f_2(z_2) = S_{p1}[p1 + p4 + p2 + p3]$$

$$S_{f1} = -\frac{\sqrt{R_{I1}}C_1}{\alpha_{19}}\ ; \quad S_{p1} = -\frac{\sqrt{R_{I2}}C_1}{\alpha_{21}}$$

$$f1 = \frac{1}{2a\sqrt{R_{I1}}}(\alpha_1\,\alpha_{11}\,); \quad f2 = \frac{C_{f2}}{2a\sqrt{R_{I1}}}(\alpha_2\,\alpha_{11})\ ;$$

$$f3 = \frac{C_{f3}}{2a\sqrt{R_{I1}}}\left[z_1\,\alpha_1\,\alpha_{11} + \frac{\alpha_{12}\alpha_1}{\sqrt{R_{I1}}} - \frac{\alpha_2\,\alpha_{11}}{a_1}\right];$$

$$f4 = \frac{C_{f4}}{2a\sqrt{R_{I1}}}\left[z_1\,\alpha_2\,\alpha_{11} + \frac{\alpha_2\,\alpha_{12}}{\sqrt{R_{I1}}} - \frac{\alpha_1\,\alpha_{11}}{a_1}\right];$$

$$p1 = \frac{C_{p1}}{2a_2\sqrt{R_{I2}}}\alpha_3\,\alpha_{13}\,; \quad p2 = \frac{C_{p2}}{2a_2\sqrt{R_{I2}}}\alpha_4\,\alpha_{13};$$

$$p3 = \frac{C_{p3}}{c^2+4\delta^2 R_{I2}}\left[\alpha_{13}\,\left(2\delta\sqrt{R_{I2}}\right)\alpha_5 + c\,\alpha_{14}\,\alpha_6\right];$$

$$p4 = \frac{C_{p4}}{c^2+4\delta^2 R_{I2}}\left[\alpha_{13}\left(2\delta\sqrt{R_{I2}}\right)\alpha_6 + c\,\alpha_{14}\,\alpha_5\right];$$

$$C_{f5} = \frac{\epsilon_T}{\zeta}\left[C_{p5} - P_3\right] + P_2\,; \quad C_6 = \frac{1}{b_1}\left[C_{p6}b_2 - P_5 + P_4\right];$$

$$C_{p5} = \frac{P_7\,\alpha_{18} - P_6\,\alpha_{16}}{b_1\alpha_{15}\alpha_{18}\frac{\epsilon_T}{\zeta} + b_2\alpha_{17}\alpha_{16}}\,; \quad C_{p6} = \frac{P_6 + C_{p5}\,b_2\alpha_{17}}{b_2\alpha_{18}};$$

$$P_7 = P_1 + P_5\alpha_{16} - \alpha_{16}P_4 + \alpha_{15}\left(\frac{\epsilon_T}{\zeta}P_3 - P_2\right)b_1;$$

$$\begin{aligned} P_6 = \frac{\sqrt{R_{I2}}}{\alpha_{21}}\Bigg\{ & \frac{C_{p1}}{2\,a_2\sqrt{R_{I2}}}\left(-E_{12}\left(\sqrt{R_{I2}}\right)\alpha_{22} - \mathrm{E}_{11}\,(a_2)\;\alpha_{21}\right) \\ & + \frac{C_{p2}}{2\,a_2\sqrt{R_{I2}}}\left(\sqrt{R_{I2}}E_{11}\alpha_{22} + a_2E_{12}\alpha_{21}\right) \\ & + \frac{C_{p3}}{c^2+4\delta^2R_{I2}}\,\Big(-2\delta R_{I2}\alpha_{22}\mathrm{E}_{14} - \alpha_{21}(2\delta^2\sqrt{R_{I2}}\,)\mathrm{E}_{13} - c\delta\,\mathrm{E}_{14}\,\alpha_{22} \\ & + c\sqrt{R_{I2}}\,\mathrm{E}_{13}\,\alpha_{21}\Big) \\ & + \frac{C_{p4}}{c^2+4\delta^2R_{I2}}\,\Big(\alpha_{22}\,(2\delta R_{I2})\,\mathrm{E}_{13} + \alpha_{21}(2\delta^2\sqrt{R_{I2}}\,)\,\mathrm{E}_{14} + \alpha_{22}\;(c\delta)\,\mathrm{E}_{13} \\ & - \alpha_{21}\;(c\sqrt{R_{I2}})\mathrm{E}_{14}\,\Big)\Bigg\} \end{aligned}$$

$$P_5 = \frac{\sqrt{R_{I2}}}{\alpha_{21}}\left\{\frac{C_{p2}}{2a_2} + \frac{C_{p4}}{c^2+4\delta^2R_{I2}}\left[2\delta R_{I2} + c\delta\right]\right\}$$

$$P_4 = A_1\left[C_{f2}\sqrt{R_{I1}} + C_{f3}\,A_2\right]$$

$$A_1 = \frac{1}{2\,a_1\,\alpha_{19}}$$

$$A_2 = \left(\frac{a_1}{\sqrt{R_{I1}}} - \frac{\sqrt{R_{I1}}}{a_1}\right)$$

$$P_3 = \frac{c\sqrt{R_{I2}}\,C_{p3}}{(c^2+4\delta^2R_{I2})\alpha_{19}}$$

$$P_2 = \frac{C_{f4}}{2\,a\,\sqrt{R_{I1}}\alpha_{19}}$$

$$P_1 = A_1\big[\alpha_{19}\, a_1 E_{16} + \alpha_{20}\big(\sqrt{R_{I1}}\,\big)E_{15} + \alpha_{19}(C_{f2}a_1)E_{15} + \alpha_{20}(C_{f2}\sqrt{R_{I1}}\,)E_{16} \\ + C_{f4}\big(-E_{16}\,\alpha_{19} + \sqrt{R_{I1}}\,\alpha_{20}E_{16} + E_{15}\,A_2\,\alpha_{20} + a_1\,E_{15}\,\alpha_{19}\big) \\ + C_{f3}\big(-\alpha_{19}E_{15} + \sqrt{R_{I1}}\,E_{15}\,\alpha_{20} + \alpha_{20}\,A_2\,E_{16} + a_1\,E_{16}\,\alpha_{19}\big)\,\big]$$

From the last velocity boundary conditions in (15), the TMAN for LTP is as follows:

$$M_L = \frac{-Q_1 + {a_1}^2 M_s Q_3}{{a_1}^2 Q_2}$$

Where

$$Q_1 = \left(2C_{f4}a_1 + {a_1}^2 + C_{f3}{a_1}^2\right)E_{16} + \left(2C_{f3}a_1 + C_{f2}{a_1}^2 + C_{f4}{a_1}^2\right)E_{15}$$

$$Q_2 = C_{f6}\alpha_{15} + C_{f5}\alpha_{16} - \frac{1}{2a_1\alpha_{19}}\left[C_{f3}\left(\frac{E_{15}\,\alpha_{20}}{\sqrt{R_{I1}}} + \alpha_{19}E_{15} - \frac{E_{16}\,\alpha_{19}}{a_1}\right) + C_{f2}E_{16}\,\alpha_{19} + C_{f4}\left(\frac{\alpha_{20}E_{16}}{\sqrt{R_{I1}}} + E_{16}\,\alpha_{19} - \frac{\alpha_{19}E_{15}}{a_1}\right) + \alpha_{19}E_{15}\right]$$

$$Q_3 = C_{f8}E_{15} + C_{f7}E_{16} - \frac{1}{2\tau_1}\left[\frac{1}{a_1}E_{15} + \frac{C_{f3}}{2a_1}\left(\frac{1}{2{a_1}^2}E_{15} + E_{15} - \frac{1}{a_1}E_{16}\right) + \frac{C_{f2}}{a_1}E_{16} + \frac{C_{f4}}{2a_1}\left(\frac{1}{2{a_1}^2}E_{16} + E_{16} - \frac{1}{a_1}E_{15}\right)\right]$$

## 3.2 TMAN for Parabolic Temperature Profile (PTP):

The PTP of the form $g_1(z_1)$ and $g_2(z_2) = 2z_2$, is introduced in (10) and (13) and solving we obtain temperature distributions $\theta_1(z_1)$ and $\theta_2(z_2)$ using (16):

$$\theta_1(z_1) = C_1[f_2(z_1) + C_{f6}\alpha_7 + C_{f5}\alpha_8]$$

$$\theta_p(z_p) = C_1[f_{2p}(z_2) + C_{p6}\alpha_9 + C_{p5}\alpha_{10}]$$

Where

$$f_2(z_1) = S_{f2}[f1 + f2 + f3 + f4]$$

$$f_{2p}(z_2) = S_{p2}[p1 + p2 + p3 + p4]$$

$$S_{f2} = \frac{-2\sqrt{R_{I1}}}{\alpha_{19}} \quad ; \quad S_{p2} = \frac{-2\sqrt{R_{I2}}}{\alpha_{21}}$$

$$f1 = \frac{1}{2\ a_1\ \sqrt{R_{I1}}}\left[z_1\ \alpha_{11}\alpha_1 + \frac{\alpha_{12}\ \alpha_1}{\sqrt{R_{I1}}} - \frac{\alpha_{11}\ \alpha_2}{a_1}\right]$$

$$f2 = \frac{1}{2\ a\ \sqrt{R_{I1}}}\left[z_1\ \alpha_{11}\alpha_2 + \frac{\alpha_{12}\ \alpha_2}{\sqrt{R_{I1}}} - \frac{\alpha_{11}\ \alpha_1}{a_1}\right]$$

$$f3 = \frac{C_{f3}}{2a\sqrt{R_{I1}}}\left[-I_{32}\alpha_2 + I_{31}\alpha_1\right]$$

$$I_{31} = \frac{2z_1}{\sqrt{R_{I1}}}\alpha_{12} - \frac{2}{R_{I1}}\alpha_{11} + {z_1}^2\alpha_{11} + \frac{2}{{a_1}^2}\alpha_{11}$$

$$I_{32} = \frac{3}{a_1\sqrt{R_{I1}}}\alpha_{12} + \frac{2z_1}{a_1}\alpha_{11}$$

$$f4 = \frac{C_{f4}}{2a_1\sqrt{R_{I1}}}\left[I_{41}\alpha_2 - I_{42}\alpha_1\right]$$

$$I_{41} = \frac{2z_1}{\sqrt{R_{I1}}}\alpha_{12} - \frac{2}{R_{I1}}\alpha_{11} + {z_1}^2\alpha_{11} + \frac{2}{{a_1}^2}\alpha_{11}$$

$$I_{42} = \frac{3}{a_1\sqrt{R_{I1}}}\alpha_{12} + \frac{2z_1}{a_1}\alpha_{11}$$

$$p1 = \frac{C_{p1}}{2a_2\sqrt{R_{I2}}}\left[\frac{\alpha_{14}\ \alpha_3}{\sqrt{R_{I2}}} + z_2\ \alpha_{13}\ \alpha_3 - \frac{\alpha_{13}\ \alpha_4}{a_2}\right]$$

$$p2 = \frac{C_{p2}}{2a_2\sqrt{R_{I2}}}\left[\frac{\alpha_{14}\ \alpha_4}{\sqrt{R_{ip}}} + z_2\ \alpha_{13}\ \alpha_4 - \frac{\alpha_{13}\ \alpha_3}{a_2}\right]$$

$$p3 = \frac{C_{p3}}{2\delta\sqrt{R_{I2}}}\left[I_{p31} - I_{p32}\right]$$

$$P_4 = \frac{2\sqrt{R_{I2}}}{\alpha_{21}}\left[\frac{C_{p1}}{2a_2\sqrt{R_{I2}}}P_{41} + \frac{C_{p2}}{2a_2\sqrt{R_{I2}}}P_{42} + \frac{C_{p3}}{2\delta\sqrt{R_{I2}}}P_{43} + \frac{C_{p4}}{2\delta\sqrt{R_{I2}}}P_{44}\right]$$

$$P_{41}=a_2\alpha_{21}\mathrm{E}_{11}-\alpha_{21}\mathrm{E}_{12}+\alpha_{22}\mathrm{E}_{11}\left(\frac{a_2}{\sqrt{R_{I2}}}-\frac{\sqrt{R_{I2}}}{a_2}\right)+\sqrt{R_{I2}}\alpha_{22}\mathrm{E}_{12}$$

$$P_{42}=-a_2\alpha_{21}\mathrm{E}_{12}+\alpha_{21}\mathrm{E}_{11}-\alpha_{22}\mathrm{E}_{12}\left(\frac{a_2}{\sqrt{R_{I2}}}-\frac{\sqrt{R_{I2}}}{a_2}\right)-\sqrt{R_{I2}}\alpha_{22}\mathrm{E}_{11}$$

$$P_{43}=P_{43b}+P_{43a}$$

$$P_{43a}=\left(\frac{c_p}{2\sqrt{R_{I2}}}+\sqrt{R_{I2}}-\frac{c_p^2}{4\sqrt{R_{I2}}\,\delta^2}\right)\alpha_{22}\mathrm{E}_{14}-\left(\frac{c_p}{2\delta}-\delta+\frac{c_p^2}{4R_{I2}\delta}\right)\alpha_{21}\mathrm{E}_{13}$$

$$P_{43b}=\left(\frac{\delta}{\sqrt{R_{I2}}}+\frac{c_p}{2\sqrt{R_{I2}}\delta}-\frac{2c_p}{\sqrt{R_{I2}}\delta}-\frac{\sqrt{R_{I2}}}{\delta}\right)\alpha_{22}\mathrm{E}_{13}+\left(\frac{c_p}{\delta^2}-\frac{c_p^2}{4R_{I2}\delta^2}-1-\frac{c_p}{R_{I2}}\right)\alpha_{21}\mathrm{E}_{14}$$

$$P_{44}=P_{44b}+P_{44a}$$

$$P_{44a}=\alpha_{22}\mathrm{E}_{13}\left(\frac{-c_p}{2\sqrt{R_{I2}}}-\sqrt{R_{I2}}+\frac{c_p^2}{4\sqrt{R_{I2}}\,\delta^2}\right)+\alpha_{21}\mathrm{E}_{14}\left(\frac{c_p}{2\delta}-\delta+\frac{c_p^2}{4R_{I2}\delta}\right)$$

$$P_{44b}=-\alpha_{22}\mathrm{E}_{13}\left(\frac{c_p}{2\sqrt{R_{I2}}\delta}+\frac{\delta}{\sqrt{R_{I2}}}-\frac{\sqrt{R_{I2}}}{\delta}-\frac{2c_p}{\sqrt{R_{I2}}\delta}\right)-\alpha_{21}\mathrm{E}_{14}\left(\frac{c_p}{\delta^2}-1-\frac{c_p^2}{4R_{I2}\delta^2}-\frac{c_p}{R_{I2}}\right)$$

$$P_3=P_{31}-P_{32}$$

$$P_{31}=\frac{2\sqrt{R_{I1}}}{\alpha_{21}}\left[\frac{1}{2a_1\sqrt{R_{I1}}}\left(\frac{a_1}{\sqrt{R_{I1}}}-\frac{\sqrt{R_{I1}}}{a_1}\right)+\frac{C_{f4}}{2a_1\sqrt{R_{I1}}}\left(\frac{2\sqrt{R_{I1}}}{{a_1}^2}-\frac{3}{\sqrt{R_{I1}}}\right)\right]$$

$$P_{32}=\frac{2\sqrt{R_{I2}}}{\alpha_{21}}\left[\frac{C_{p1}}{2a_2\sqrt{R_{I2}}}\left(\frac{a_2}{\sqrt{R_{I2}}}-\frac{\sqrt{R_{I2}}}{a_2}\right)+\frac{C_{p3}}{2\delta\sqrt{R_{I2}}}\left(\frac{c_p}{2\sqrt{R_{I2}}\delta}+\frac{\delta}{\sqrt{R_{I2}}}-\frac{\sqrt{R_{I2}}}{\delta}-\frac{2c_p}{\sqrt{R_{I2}}\delta}\right)\right]$$

$$P_2=P_{21}-P_{22}$$

$$P_{21}=\frac{2\sqrt{R_{I1}}}{\alpha_{19}}\left[\frac{C_{f2}}{2\,a_1R_{I1}}-\frac{3C_{f3}}{2{a_1}^2R_{I1}}\right]$$

$$P_{22}=\frac{2\sqrt{R_{I2}}}{\alpha_{21}}\left[\frac{C_{p2}}{2a_2R_{I2}}+\frac{C_{p4}}{2\delta\sqrt{R_{I2}}}\left(\frac{1}{\sqrt{R_{I2}}}-\frac{c_p}{\sqrt{R_{I2}}\delta^2}\right)\right]$$

$$P_1=\frac{2\sqrt{R_{I1}}}{\alpha_{19}}\left[\frac{1}{2a_1\sqrt{R_{I1}}}P_{11}+\frac{C_{f2}}{2a_1\sqrt{R_{I1}}}P_{12}+\frac{C_{f3}}{2a_1\sqrt{R_{I1}}}P_{13}+\frac{C_{f4}}{2a_1\sqrt{R_{I1}}}P_{14}\right]$$

$$P_{11} = \sqrt{R_{I1}}\alpha_{20}E_{15} + a_1\,\alpha_{19}E_{16} - \alpha_{19}E_{15} + \left(\frac{a_1}{\sqrt{R_{I1}}} - \frac{\sqrt{R_{I1}}}{a_1}\right)\alpha_{20}E_{16}$$

$$P_{12} = \sqrt{R_{I1}}\alpha_{20}E_{16} + a_1\,\alpha_{19}E_{15} - \alpha_{19}E_{16} + \left(\frac{a_1}{\sqrt{R_{I1}}} - \frac{\sqrt{R_{I1}}}{a_1}\right)\alpha_{20}E_{15}$$

$$P_{13} = \left(\frac{a_1}{\sqrt{R_{I1}}} - \frac{\sqrt{R_{I1}}}{a_1}\right)2\alpha_{20}E_{16} + \left(a_1 - \frac{2a_1}{R_{I1}} + \frac{3}{a_1}\right)\alpha_{19}E_{16} - 2\alpha_{19}E_{15} + \alpha_{20}\,E_{15}\left(\sqrt{R_{I1}} + \frac{2\sqrt{R_{I1}}}{{a_1}^2} - \frac{3}{\sqrt{R_{I1}}}\right)$$

$$P_{14} = \left(\frac{a_1}{\sqrt{R_{I1}}} - \frac{\sqrt{R_{I1}}}{a_1}\right)2\alpha_{20}\,E_{15} + \left(a_1 - \frac{2a_1}{R_{I1}} + \frac{3}{a_1}\right)\alpha_{19}E_{15} - 2\alpha_{19}E_{16} + \alpha_{20}\,E_{16}\left(\sqrt{R_{I1}} + \frac{2\sqrt{R_{I1}}}{{a_1}^2} - \frac{3}{\sqrt{R_{I1}}}\right)$$

From the last velocity boundary conditions in (15), the TMAN for PTP is as follows:

$$M_P = \frac{-Q_1 + {a_1}^2 M_s Q_3}{{a_1}^2 Q_2}$$

Where

$$Q_1 = E_{16}\left({a_1}^2 + C_{f3}{a_1}^2 + 2C_{f4}a_1\right) + E_{15}\left(C_{f2}{a_1}^2 + C_{f4}{a_1}^2 + 2C_{f3}a_1\right)$$

$$Q_2 = C_{f6}\alpha_{15} + C_{f5}\alpha_{16} - \frac{2\sqrt{R_{I1}}}{\alpha_{19}}\left[\frac{1}{2a_1\sqrt{R_{I1}}}Q_{21} + \frac{C_{f2}}{2a_1\sqrt{R_{I1}}}Q_{22} + \frac{C_{f3}}{2a_1\sqrt{R_{I1}}}Q_{23} + \frac{C_{f4}}{2a_1\sqrt{R_{I1}}}Q_{24}\right]$$

$$Q_{21} = \frac{\alpha_{20}E_{15}}{\sqrt{R_{I1}}} + \alpha_{19}E_{15} - \frac{E_{16}\,\alpha_{19}}{a_1}$$

$$Q_{22} = \frac{\alpha_{20}E_{16}}{\sqrt{R_{I1}}} + \alpha_{19}E_{16} - \frac{E_{15}\,\alpha_{19}}{a_1}$$

$$Q_{23} = E_{15}\left(\frac{2}{\sqrt{R_{I1}}}\alpha_{20} - \frac{2}{R_{I1}}\alpha_{19} + \alpha_{19} + \frac{2}{{a_1}^2}\alpha_{19}\right) - E_{16}\left(\frac{2}{a_1}\alpha_{19} + \frac{3}{a_1\sqrt{R_{I1}}}\alpha_{20}\right)$$

$$Q_{24} = E_{16}\left(\frac{2}{\sqrt{R_{I1}}}\alpha_{20} - \frac{2}{R_{I1}}\alpha_{19} + \alpha_{19} + \frac{2}{{a_1}^2}\alpha_{19}\right) - E_{15}\left(\frac{2}{a_1}\alpha_{19} + \frac{3}{a_1\sqrt{R_{I1}}}\alpha_{20}\right)$$

$$Q_3 = C_{f7}E_{16} + C_{f8}E_{15} - \frac{1}{\tau_1}\left[\frac{C_{f2}}{2a_1}E_{16} + \frac{1}{2a_1}E_{15} + \frac{C_{f3}}{4a_1}\left(\frac{1}{2{a_1}^2}E_{15} + E_{15} - E_{16}\right) + \frac{C_{f4}}{4a_1}\left(\frac{1}{2{a_1}^2}E_{16} + E_{16} - \frac{1}{a_1}E_{15}\right)\right]$$

### 3.3 TMAN for Inverted Parabolic Temperature Profile (ITP):

The ITP of the form $g_1(z_1) = 2(1–z_1)$ and $g_2(z_2) = 2(1–z_2)$, is introduced in (10) and (13) and solving we obtain temperature distributions $\theta_1(z_1)$ and $\theta_2(z_2)$ using (16):

$$\theta_1(z_1) = C_1[C_{f5}\alpha_8 + C_{f6}\alpha_7 + f_3(z_1)]$$

$$\theta_2(z_2) = C_1[C_{p5}\alpha_{10} + C_{p6}\alpha_9 + f_{3p}(z_2)]$$

Where

$$f_3(z_1) = S_3\left[\frac{1}{2a_1\sqrt{R_{I1}}}f1 + \frac{C_{f2}}{2a_1\sqrt{R_{I1}}}f2 + \frac{C_{f3}}{2a_1\sqrt{R_{I1}}}f3 + \frac{C_{f4}}{2a_1\sqrt{R_{I1}}}f4\right]$$

$$f_{p3}(z_2) = S_{p3}\left[\frac{C_{p1}}{2a_2\sqrt{R_{I2}}}p1 + \frac{C_{p2}}{2a_2\sqrt{R_{I2}}}p2 + p3 + p4\right]$$

$$S_3 = \frac{-2\sqrt{R_{I1}}}{\alpha_{19}};\ S_{p3} = \frac{-2\sqrt{R_{I2}}}{\sin[\sqrt{R_{I2}}]}$$

$$f1 = \frac{\alpha_{11}\alpha_2}{a_f} + \alpha_1\alpha_{11}\left(1 - z_f\right) - \frac{\alpha_{12}\alpha_1}{\sqrt{R_{if}}}$$

$$f2 = \frac{\alpha_1\alpha_{11}}{a_f} + \alpha_2\alpha_{11}\left(1 - z_f\right) - \frac{\alpha_{12}\alpha_2}{\sqrt{R_{if}}}$$

$$f3 = I_{32} + I_{31}$$

$$I_{31} = \frac{\alpha_{12}\alpha_1}{\sqrt{R_{if}}} + z_f\,\alpha_1\ \alpha_{11} - \frac{\alpha_{11}\alpha_2}{a_1}$$

$$I_{32} = -\left(\frac{2z_1}{\sqrt{R_{I1}}}\alpha_{12} - \frac{2}{\sqrt{R_{I1}}}\alpha_{11} + {z_1}^2\,\alpha_{11} + \frac{2}{{a_1}^2}\ \alpha_{11}\right)\alpha_1 + \left(\frac{3}{a_1\sqrt{R_{I1}}}\alpha_{12} + \frac{2z_1}{a_1}\alpha_{11}\right)\alpha_2$$

$$I_4 = I_{42} + I_{41}$$

$$I_{41} = \frac{\alpha_{12}\alpha_2}{\sqrt{R_{I1}}} - \frac{\alpha_{11}\alpha_1}{a_1} + z_1\,\alpha_2\alpha_{11}$$

$$I_{42} = -\left(\frac{2z_1}{\sqrt{R_{I1}}}\alpha_{12} - \frac{2}{\sqrt{R_{I1}}}\alpha_{11} + {z_1}^2\,\alpha_{11} + \frac{2}{{a_1}^2}\ \alpha_{11}\right)\alpha_2 + \left(\frac{3}{a_1\sqrt{R_{I1}}}\alpha_{12} + \frac{2z_1}{a_1}\alpha_{11}\right)\alpha_1$$

$$f_{3p}(z_2) = p1 + p2 + p3 + p4$$

$$p1 = -\frac{\alpha_{14}\alpha_3}{\sqrt{R_{I2}}} + \mathrm{E}_{12}\alpha_{13}\,(1 - z_2) + \frac{\alpha_{13}\;\alpha_4}{a_2}$$

$$p2 = -\frac{\alpha_{14}\alpha_4}{\sqrt{R_{I2}}} + \mathrm{E}_{11}\alpha_{13}\,(1 - z_2) + \frac{\alpha_{13}\;\alpha_3}{a_2}$$

$$p3 = \frac{C_{p3}}{c^2 + 4\delta^2 R_{I2}} I_{p31} + \frac{C_{p3}}{2\delta\sqrt{R_{I2}}} I_{p32}$$

$$I_{p31} = c_p\;\alpha_{14}\alpha_6 + \;2\delta\sqrt{R_{I2}}\;\alpha_{13}\;\alpha_5$$

$$I_{p32} = -\left(\frac{1}{\sqrt{R_{I2}}} - \frac{c_p}{\sqrt{R_{I2}}\,\delta^2}\right)\alpha_{14}\alpha_5 - \frac{c_p z_2}{2\sqrt{R_{I2}}\delta}\alpha_{14}\alpha_6 - (z_2)\left(z_2 - \frac{c_p^2\, z_2}{4R_{I2}\,\delta^2}\right)\alpha_{13}\;\alpha_5 + \left(\frac{1}{\delta} + \frac{c_p}{R_{I2}\delta}\right)\;\alpha_{13}\;\alpha_6$$

$$p4 = \frac{C_{p4}}{c^2 + 4\delta^2 R_{I2}} I_{p41} + \frac{C_{p4}}{2\delta\sqrt{R_{I2}}} I_{p42}$$

$$I_{p41} = c_p\;\alpha_{14}\alpha_5 + \;2\delta\sqrt{R_{I2}}\;\alpha_{13}\;\alpha_6$$

$$I_{p42} = \left(\frac{1}{\delta} + \frac{c_p}{R_{I2}\delta}\right)\alpha_{13}\alpha_5 - \frac{c_p z_2}{2\sqrt{R_{I2}}\delta}\alpha_{14}\alpha_5 - \left(\frac{1}{\sqrt{R_{I2}}} - \frac{c_p}{\sqrt{R_{I2}}\,\delta^2}\right)\alpha_{14}\alpha_6 - \left(z_2 - \frac{c_p^2\, z_2}{4R_{I2}\delta^2}\right)\alpha_{13}\alpha_6$$

$$P_1 = \frac{2\sqrt{R_{I1}}}{\alpha_{19}}\,[P_{11} + P_{12} + P_{13} + P_{14}] - \frac{2\sqrt{R_{I1}}}{\alpha_{19}}\,[P_{15} + P_{16} + P_{17} + P_{18}]$$

$$P_{11} = \frac{1}{2\sqrt{R_{I1}}} E_{16}\;\alpha_{19} + \frac{1}{2a_f} E_{15}\,\alpha_{20}$$

$$P_{12} = \frac{C_{f2}}{2\sqrt{R_{I1}}} E_{15}\,\alpha_{19} + \frac{C_{f2}}{2a_f} E_{16}\alpha_{20}$$

$$P_{13} = \frac{C_{f3}}{2a_1\sqrt{R_{I1}}}\left[\sqrt{R_{I1}}\,E_{15}\alpha_{20} + a_1\,E_{16}\,\alpha_{19} + \left(\frac{a_1}{\sqrt{R_{I1}}} - \frac{\sqrt{R_{I1}}}{a_1}\right)\alpha_{20}E_{16} - E_{15}\;\alpha_{19}\right]$$

$$P_{14} = \frac{C_{f4}}{2a_1\sqrt{R_{I1}}}\left[\sqrt{R_{I1}}\,E_{15}\,\alpha_{20} + a_1\,E_{16}\,\alpha_{19} + \left(\frac{a_1}{\sqrt{R_{I1}}} - \frac{\sqrt{R_{I1}}}{a_1}\right)\alpha_{20}E_{16} - E_{15}\;\alpha_{19}\right]$$

$$P_{15} = \frac{1}{2a_1\sqrt{R_{I1}}}\left[\sqrt{R_{I1}}\alpha_{20}E_{15} + a_1\,\alpha_{19}\,E_{16} + \left(\frac{a_1}{\sqrt{R_{I1}}} - \frac{\sqrt{R_{I1}}}{a_1}\right)\alpha_{20}E_{16} - E_{15}\;\alpha_{19}\right]$$

$$P_{16} = \frac{C_{f2}}{2a_1\sqrt{R_{I1}}}\left[\sqrt{R_{I1}}\alpha_{20}E_{16} + a_1\alpha_{19}E_{15} + \left(\frac{a_1}{\sqrt{R_{I1}}} - \frac{\sqrt{R_{I1}}}{a_1}\right)E_{15}\,\alpha_{20} - E_{16}\alpha_{19}\right]$$

$$P_{17} = \frac{C_{f3}}{2a_1\sqrt{R_{I1}}}\Bigg[\left(\frac{2a_1}{\sqrt{R_{I1}}} - \frac{2\sqrt{R_{I1}}}{a_1}\right)\alpha_{20}E_{16} + \left(\sqrt{R_{I1}} + \frac{2\sqrt{R_{I1}}}{{a_1}^2} - \frac{3}{\sqrt{R_{I1}}}\right)\ \alpha_{20}E_{15} - 2\alpha_{19}E_{15} + \left(a_1 - \frac{2a_1}{R_{I1}} + \frac{3}{a_1}\right)\alpha_{19}E_{16}\Bigg]$$

$$P_{18} = \frac{C_{f4}}{2a_1\sqrt{R_{I1}}}\Bigg[\left(\frac{2a_1}{\sqrt{R_{I1}}} - \frac{2\sqrt{R_{I1}}}{a_1}\right)\alpha_{20}E_{15}\ \ + \left(\sqrt{R_{I1}} + \frac{2\sqrt{R_{I1}}}{{a_1}^2} - \frac{3}{\sqrt{R_{I1}}}\right)\ \alpha_{20}E_{16} - 2\alpha_{19}E_{16} + \left(a_1 - \frac{2a_1}{R_{I1}} + \frac{3}{a_1}\right)\alpha_{19}\ E_{15}\Bigg]$$

$$P_2 = \frac{2\sqrt{R_{I1}}}{\alpha_{19}}\left[\frac{3C_{f3}}{2{a_1}^2R_{I1}} + \frac{C_{f4}}{2\ a_1\ R_{I1}} - \frac{C_{f2}}{2\ a_1\ R_{I1}}\right] + \frac{\epsilon}{\zeta}\frac{2\sqrt{R_{I2}}}{\alpha_{21}}\left(\frac{C_{p2}}{2a_2R_{I2}} - \frac{C_{p3\ .c_p}}{c_p^2 + 4\delta^2R_{I2}} + \left(\frac{1}{\sqrt{R_{I2}}} - \frac{c_p}{\sqrt{R_{I2}}\delta^2}\right)\frac{C_{p4}}{2\delta\sqrt{R_{I2}}}\right)$$

$$P_3 = P_{31} - P_{32}$$

$$P_{31} = \frac{2\sqrt{R_{I1}}}{\alpha_{19}}\Bigg[\left(\frac{a_1}{\sqrt{R_{I1}}} - \frac{\sqrt{R_{I1}}}{a_1}\right)\frac{C_{f3}}{2a_1\sqrt{R_{I1}}} - \left(\frac{a_1}{\sqrt{R_{I1}}} - \frac{\sqrt{R_{I1}}}{a_1}\right)\frac{1}{2a_1\sqrt{R_{I1}}} - \left(\frac{2\sqrt{R_{I1}}}{{a_1}^2} - \frac{3}{\sqrt{R_{I1}}}\right)\frac{C_{f4}}{2a_1\sqrt{R_{I1}}} + \frac{C_{f2}}{2a_1}\Bigg]$$

$$P_{32} = \frac{2\sqrt{R_{I2}}}{\alpha_{21}}\Bigg[\left(2\delta R_{I2} + c_p\delta\right)\frac{C_{p4}}{{c_p}^2 + 4\delta^2R_{I2}} - \left(\frac{a_2}{\sqrt{R_{I2}}} - \frac{\sqrt{R_{I2}}}{a_2}\right)\frac{C_{p1}}{2a_2\sqrt{R_{I2}}} - \left(\frac{\delta}{\sqrt{R_{I2}}} - \frac{\sqrt{R_{I2}}}{\delta} - \frac{3c_p}{2\sqrt{R_{I2}}\delta}\right)\frac{C_{p3}}{2\delta\sqrt{R_{I2}}} + \frac{C_{p2}}{2a_p}\Bigg]$$

$$P_4 = \frac{2\sqrt{R_{I2}}}{\alpha_{21}}\left[P_{41} + P_{42} + P_{43} + P_{44}\right] - \frac{2\sqrt{R_{I2}}}{\alpha_{21}}\left[P_{45} + P_{46} + P_{47} + P_{48}\right]$$

$$P_{41} = \left[-a_2\mathrm{E}_{11}\alpha_{21} - \sqrt{R_{I2}}\mathrm{E}_{12}\alpha_{22}\right]\frac{C_{p1}}{2a_2\sqrt{R_{I2}}}$$

$$P_{42} = \left[a_2\mathrm{E}_{12}\alpha_{21} + \sqrt{R_{I2}}\mathrm{E}_{11}\alpha_{22}\right]\frac{C_{p2}}{2a_2\sqrt{R_{I2}}}$$

$$P_{43} = \frac{C_{p3}}{c^2 + 4\delta^2R_{I2}}\left(c_p\sqrt{R_{I2}}\ \alpha_{21}\mathrm{E}_{13} - 2\delta^2\sqrt{R_{I2}}\alpha_{21}\mathrm{E}_{13} - \mathrm{E}_{14}c_p\delta\ \alpha_{22} - 2\delta R_{I2}\alpha_{22}\mathrm{E}_{14}\right)$$

$$P_{44} = \frac{C_{p4}}{{c_p}^2 + 4\delta^2 R_{I2}}\left(-c_p\sqrt{R_{I2}}\alpha_{21}\mathrm{E}_{14} + 2\delta^2\sqrt{R_{I2}}\alpha_{21}\mathrm{E}_{14} + \mathrm{E}_{13}c_p\delta\,\alpha_{22} + 2\delta R_{I2}\alpha_{22}\,\mathrm{E}_{13}\right)$$

$$P_{45} = \frac{C_{p1}}{2a_2\sqrt{R_{I2}}}\left[\sqrt{R_{I2}}\alpha_{22}\mathrm{E}_{12} - \mathrm{E}_{12}\alpha_{21} + \left(\frac{a_2}{\sqrt{R_{I2}}} - \frac{\sqrt{R_{I2}}}{a_2}\right)\alpha_{22}\mathrm{E}_{11} + a_2\alpha_{21}\mathrm{E}_{11}\right]$$

$$P_{46} = \frac{C_{p2}}{2a_2\sqrt{R_{I2}}}\left[-\sqrt{R_{I2}}\alpha_{22}\mathrm{E}_{11} + \mathrm{E}_{11}\alpha_{21} - \left(\frac{a_2}{\sqrt{R_{I2}}} - \frac{\sqrt{R_{I2}}}{a_2}\right)\alpha_{22}\mathrm{E}_{12} - a_2\alpha_{21}\mathrm{E}_{12}\right]$$

$$P_{47} = \frac{C_{p3}}{2\delta\sqrt{R_{I2}}}\left[P_{47a} + P_{47b}\right]$$

$$P_{47a} = \alpha_{22}\mathrm{E}_{13}\left(\frac{\delta}{\sqrt{R_{I2}}} - \frac{\sqrt{R_{I2}}}{\delta} - \frac{3c_p}{2\sqrt{R_{I2}}\delta}\right) + \alpha_{21}\mathrm{E}_{13}\left(\delta - \frac{c_p}{2\delta} - \frac{c_p^2}{4R_{I2}\delta}\right)$$

$$P_{47b} = \alpha_{22}\mathrm{E}_{14}\left(\frac{c_p}{2\sqrt{R_{I2}}} + \sqrt{R_{I2}} - \frac{c_p^2}{4\sqrt{R_{I2}}\delta^2}\right) + \alpha_{21}\mathrm{E}_{14}\left(\frac{c_p}{\delta^2} - \frac{c_p^2}{4R_{I2}\delta^2} - 1 - \frac{c_p}{R_{I2}}\right)$$

$$P_{48} = \frac{C_{p4}}{2\delta\sqrt{R_{I2}}}\left[-P_{48b} + P_{48a}\right]$$

$$P_{48a} = -\alpha_{22}\mathrm{E}_{14}\left(\frac{\delta}{\sqrt{R_{I2}}} - \frac{\sqrt{R_{I2}}}{\delta} - \frac{3c_p}{2\sqrt{R_{I2}}\delta}\right) - \left(\delta - \frac{c_p}{2\delta} - \frac{c_p^2}{4R_{I2}\delta}\right)\alpha_{21}\mathrm{E}_{14}$$

$$P_{48b} = \alpha_{22}\mathrm{E}_{13}\left(\frac{c_p}{2\sqrt{R_{I2}}} + \sqrt{R_{I2}} - \frac{c_p^2}{4\sqrt{R_{I2}}\delta^2}\right) + \left(\frac{c_p}{\delta^2} - \frac{c_p^2}{4R_{I2}\delta^2} - 1 - \frac{c_p}{R_{I2}}\right)\alpha_{21}\mathrm{E}_{13}$$

From the last velocity boundary conditions in (15), the TMAN for ITP is as follows:

$$M_I = \frac{-Q_1 + {a_1}^2 M_s Q_3}{{a_1}^2 Q_2}$$

Where

$$Q_1 = \left({a_1}^2 + C_{f3}{a_1}^2 + 2C_{f4}a_1\right)E_{16} + \left(C_{f2}{a_1}^2 + C_{f4}{a_1}^2 + 2C_{f3}a_1\right)E_{15}$$

$$Q_2 = C_{f5}E_{16} + C_{f6}E_{15} - \frac{2\sqrt{R_{I1}}}{\alpha_{19}}\left[\frac{1}{2a_1\sqrt{R_{I1}}}Q_{21} + \frac{C_{f2}}{2a_1\sqrt{R_{I1}}}Q_{22} + \frac{C_{f3}}{2a_1\sqrt{R_{I1}}}Q_{23} + \frac{C_{f4}}{2a_1\sqrt{R_{I1}}}Q_{24}\right]$$

$$Q_{21} = -\frac{E_{15}\alpha_{20}}{\sqrt{R_{I1}}} + \frac{E_{16}\,\alpha_{19}}{a_1}$$

$$Q_{22} = -\frac{E_{16}\alpha_{20}}{\sqrt{R_{I1}}} + \frac{E_{15}\,\alpha_{19}}{a_1}$$

$$Q_{23} = \left(\frac{2}{R_{I1}} - \frac{2}{{a_1}^2}\right)E_{15}\,\alpha_{19} + \frac{E_{16}\alpha_{19}}{a_1} + \frac{3\,E_{16}\,\alpha_{20}}{a_1\sqrt{R_{I1}}} - \frac{E_{15}\alpha_{20}}{\sqrt{R_{I1}}}$$

$$Q_{24} = \left(\frac{2}{R_{I1}} - \frac{2}{a_1{}^2}\right) E_{16}\,\alpha_{19} + \frac{E_{15}\alpha_{19}}{a_1} + \frac{3\,E_{15}\,\alpha_{20}}{a_1\sqrt{R_{I1}}} - \frac{E_{16}\alpha_{20}}{\sqrt{R_{I1}}}$$

$$Q_3 = C_{f7}E_{16} + C_{f8}E_{15} - \frac{1}{\tau_1}\left[\frac{1}{2a_1}E_{15} + \frac{C_{f2}}{2a_1}E_{16} + \frac{C_{f3}}{4a_1}\left(E_{15} - \frac{1}{a_f}E_{16} + \frac{1}{2a_f{}^2}E_{15}\right) + \frac{C_{f4}}{4a_1}\left(\mathrm{E}_{16} - \frac{1}{a_1}E_{15} + \frac{1}{2a_1{}^2}E_{16}\right)\right]$$

## IV Result and Discussions

The three TMNs $M_L$, $M_P$ and $M_I$ versus $\varsigma$ for the LTP, PTP and ITP respectively are obtained for the parameters horizontal wave number $a_1$, Darcy number $Da$, thermal diffusivity ratio $\in_T$, solute thermal diffusivity ratio $\tau_1$, solute thermal diffusivity ratio in porous medium $\tau_2$, solute Marangoni number $M_S$, viscosity ratio $\hat{\mu}$, solute diffusivity ratio $\in_S$, internal Rayleigh numbers $R_{I2}$ and $R_{I1}$ for porous and fluid layers respectively. For all three temperature profiles, the effects of the specified parameters on the TMAN, $M$ vs $\varsigma$ are visually illustrated in the given figures. The curve patterns for LTP and ITP are same, however the PTP has a slight difference. That is, when the value of $\varsigma$ increases, the TMAN increases for parabolic profiles, while when the value of $\varsigma$ increases for inverted parabolic and linear curves, the TMAN initially increases and subsequently drops for higher values of $\varsigma$..

Effect of the $a_f$ on the TMANs $M_L$, $M_P$ and $M_I$ are depicted in the Fig.1(a), 1(b) and 1(c) respectively. It is observed that, for PTP the curves are diverging and for LTP and ITP the curves are converging. Also, by fixing $\varsigma$, one can observe that the increase in the value of $a_1$, raises the values of the TMANs $M_L$, $M_P$ and $M_I$ i.e., the increase in the value of $a_1$ stabilizes the composite layer system for LTP, PTP and ITP, hence the DDNBM convection is postponed.

The impact of '$Da$' on the TMAN are shown in Fig.2(a), 2(b) & 2(c) for LTP, PTP and ITP respectively for $Da$=1,10,100. The curves are diverging as the value of '$Da$' increases for all the three profiles, indicating that it is effective for the composite layer dominated by fluid layer. As $Da$ increases, the TMAN $M_L$, $M_P$ decreases in a composite layer dominated by porous layer and increases for the composite layer dominated by fluid layer. The same effect of '$Da$' for ITP is to decrease '$M_I$' in both the extent of layers. This indicates the system is destabilized for ITP and stabilized for LTP and PTP for composite layer dominated by fluid layer.

The impact of $\in_S$ on the TMAN are shown in Fig. 3(a), 3(b) & 3(c) for LTP, PTP and ITP respectively for $\in_S$

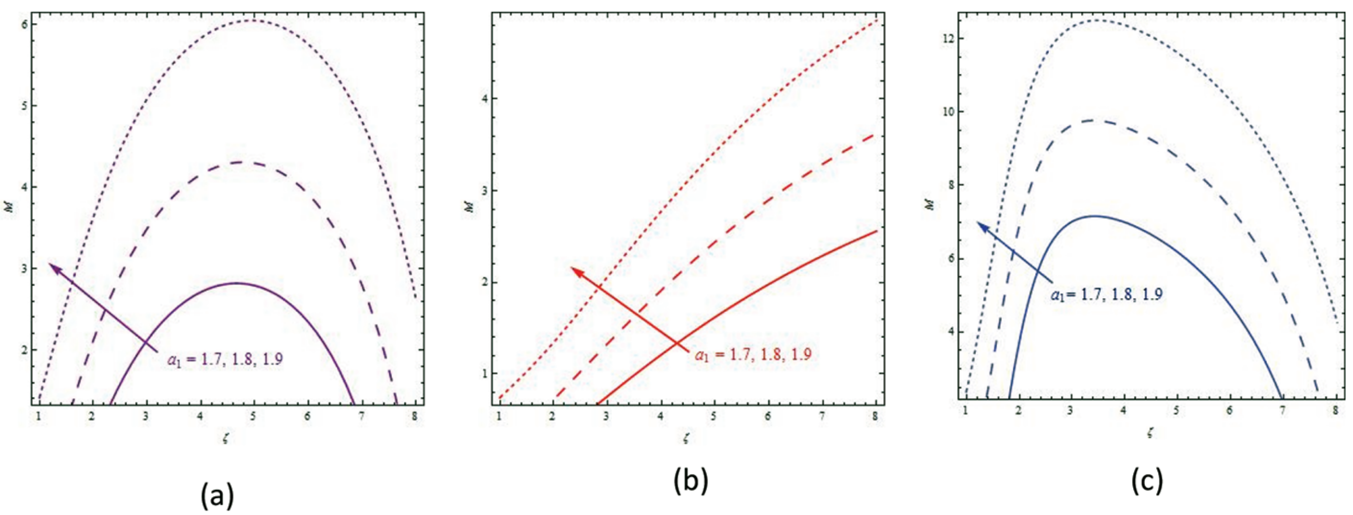


**Figure 1.** Wave number '$a_1$' on LTP, PTP and ITP

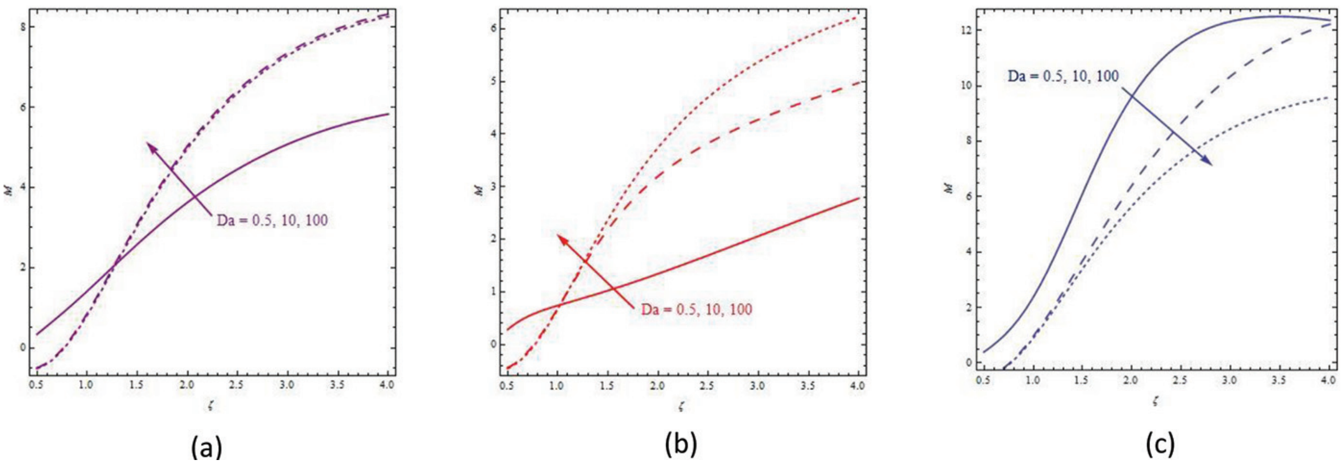


**Figure 2.** Darcy number 'Da' on LTP, PTP and ITP

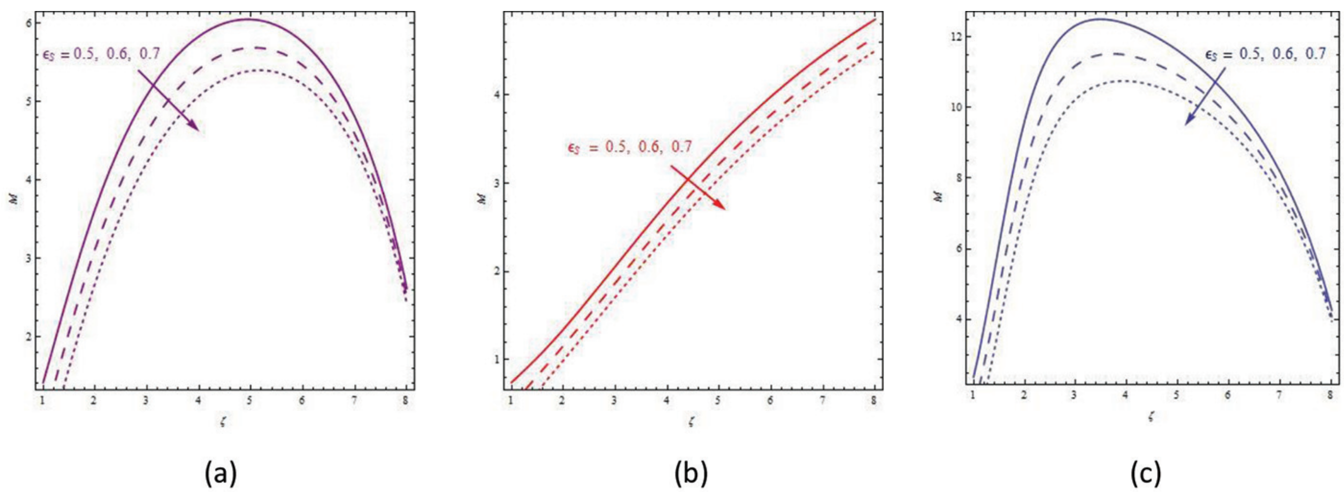


**Figure 3.** Solute diffusivity ratio '$\varepsilon_s$' on LTP, PTP and ITP

= 0.5, 0.6, 0.7. For the lower values of $\varsigma$, the curves are converging for LTP and ITP. The increase in the value of $\in_S$ decreases all the three TMANs $M_L$, $M_P$ and $M_I$. This indicates the system can be destabilized for all the profiles LTP, PTP and ITP by increasing this parameter, hence DDNBM convection can be proponed.

The influence of $\in_T$ on the TMAN are shown in Fig. 4(a), 4(b) & 4(c) for LTP, PTP and ITP respectively for $\in_T$ = 0.1, 0.2, 0.3. Here similar effects are noted for the three profiles and this parameter is effective for the composite layer dominated by fluid layers. The increase in the value of $\in_T$ decreases the TMAN for LTP and PTP, the same effect, decreases $M_I$ for the ITP for fluid layer dominant composite layers, where as for the composite layer dominated by porous layer, the TMAN decreases for the ITP.

The effects of $M_S$ on the TMAN are shown in Fig 5(a), 5(b) & 5(c) for $M_S$ = 25, 26, 27. Here, similar effect is observed for all the profiles, LTP, PTP and ITP i.e., the TMAN decreases with increase in $M_S$ which shows that the composite layer system is destabilizing for all the profiles. In case of PTP the curves appear to diverge whereas the curves converge for LTP and ITP proving its effectiveness for lower values of $\varsigma$.

In Fig 6a, 6b, 6c the effects of $\hat{\mu}$ = 0.1, 0.2, 0.3 are shown. Here, similar effect is observed for all the three profiles, LTP, PTP and ITP i.e., the TMAN increases with increase in $\hat{\mu}$ which shows that the composite layer system is stabilizing for all the profiles and DDNBM convection is postponed. The curves converge for LTP and ITP and are found diverging for PTP.

The effects of $\tau_1$ on the TMAN are shown in Fig.7(a), 7(b) & 7(c) for the values of $\tau_1$ = 0.65, 0.7, 0.75. Here, similar effect is observed for all the three profiles, LTP, PTP and ITP i.e., increase in $\tau_1$ increases TMAN which shows that the composite layer system is stabilizing

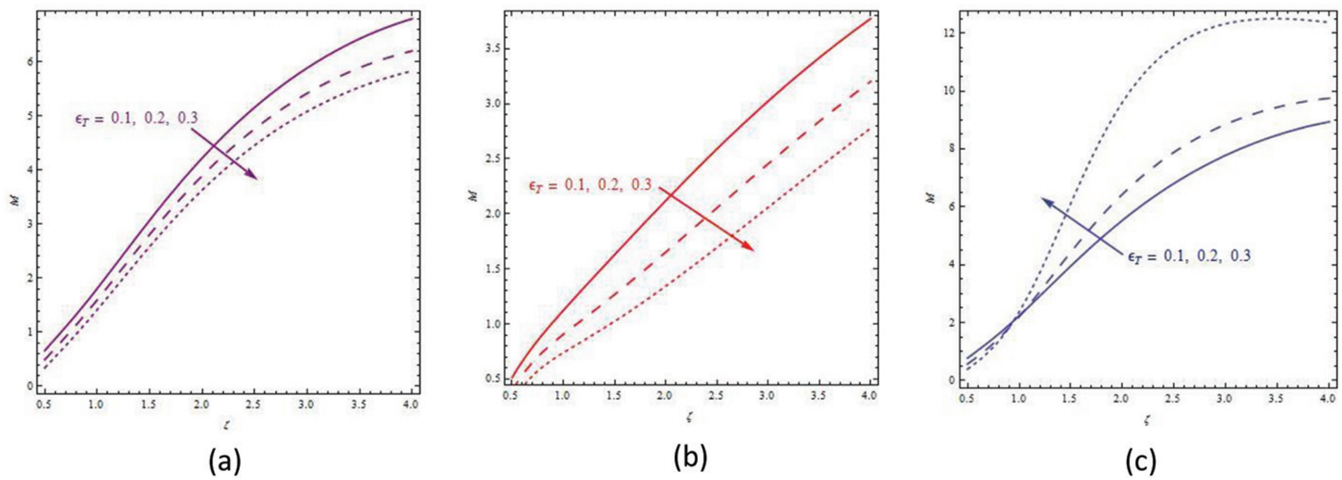


**Figure 4.** Thermal diffusivity ratio '$\varepsilon_T$' on LTP, PTP and ITP

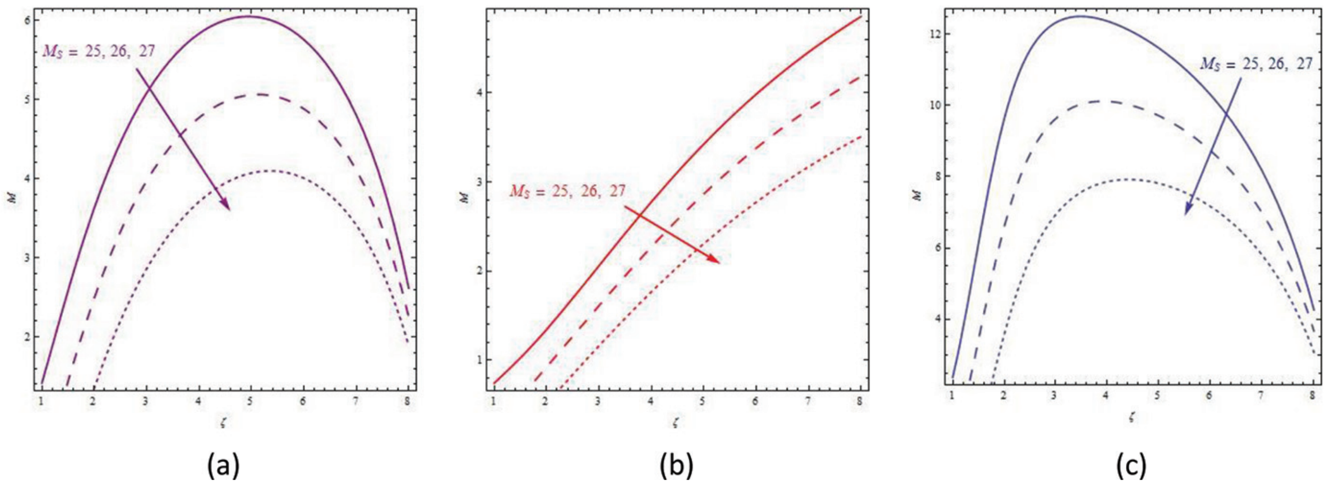


**Figure 5.** Solute Marangoni number '$M_s$' on LTP, PTP and ITP

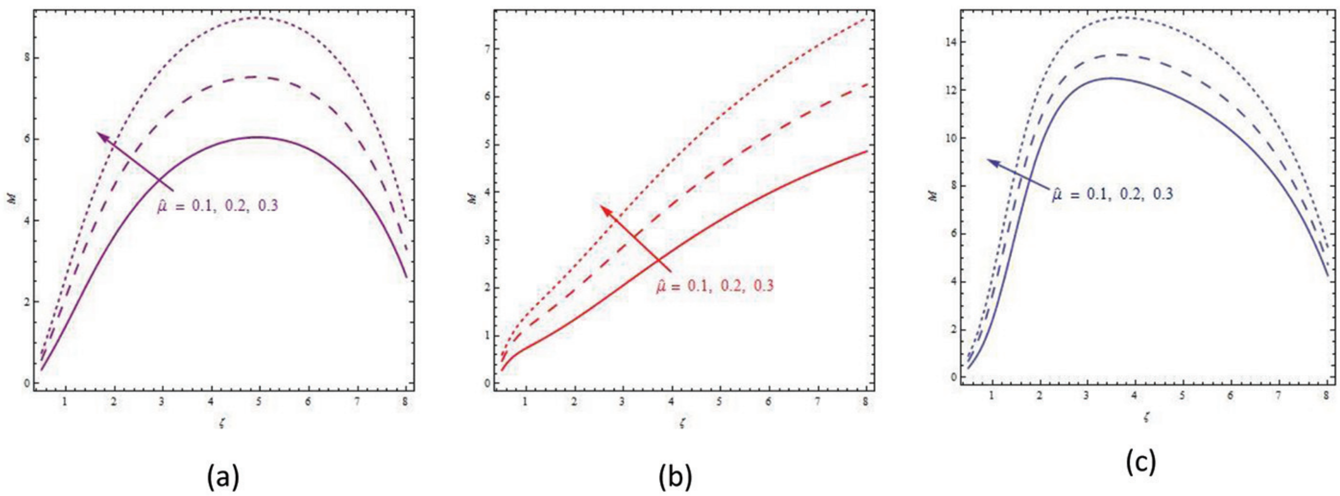


**Figure 6.** Viscosity ratio '$\hat{\mu}$' on LTP, PTP and ITP

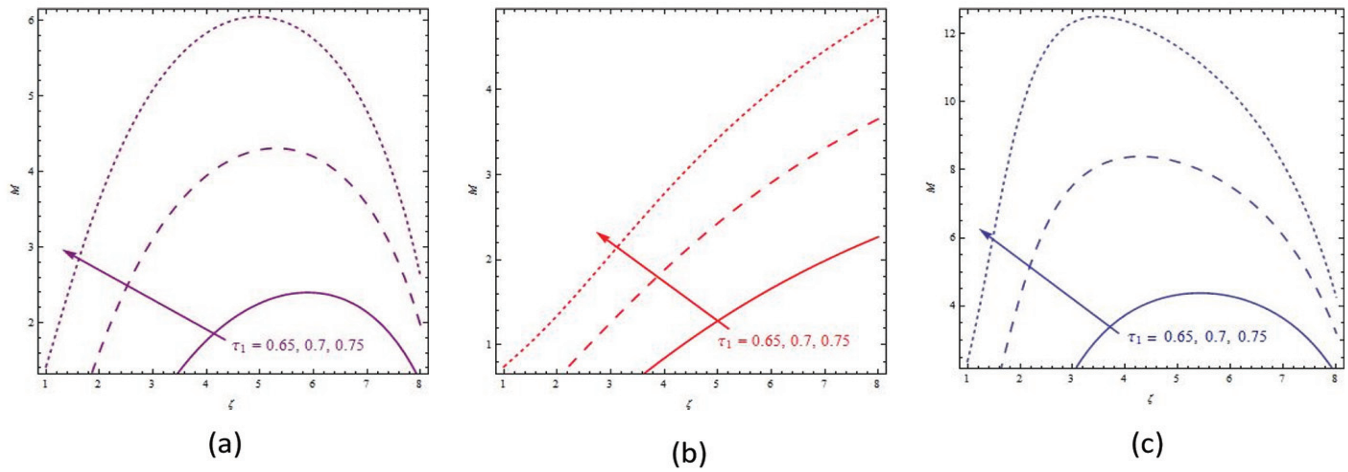


**Figure 7.** Solute thermal diffusivity ratio for fluid layer '$\tau_1$' on LTP, PTP and ITP

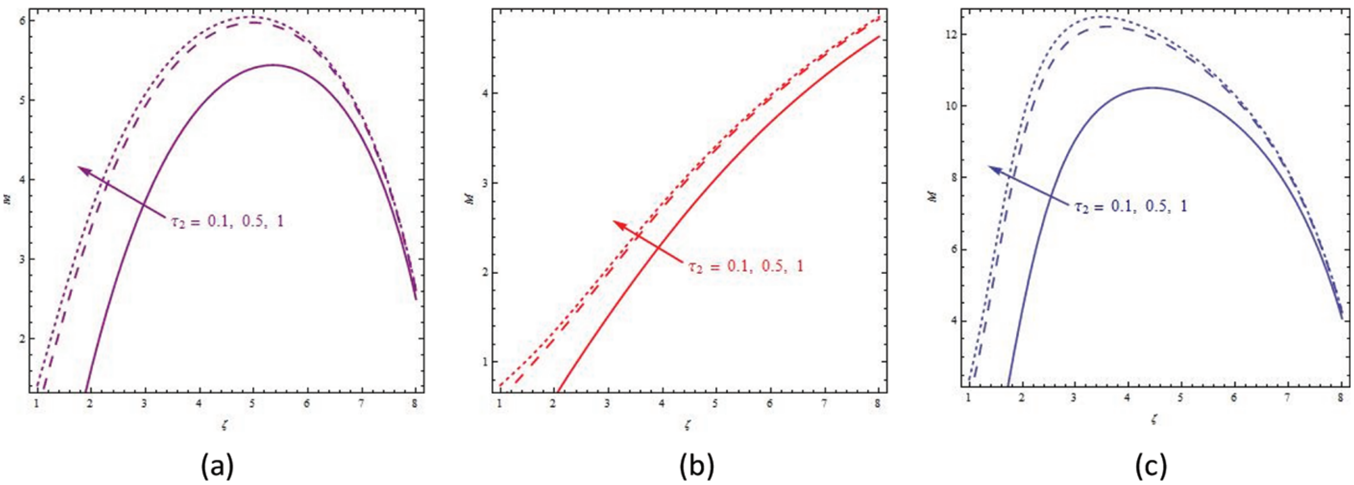


**Figure 8.** Solute thermal diffusivity ratio for porous layer '$\tau_2$' on LTP, PTP and ITP

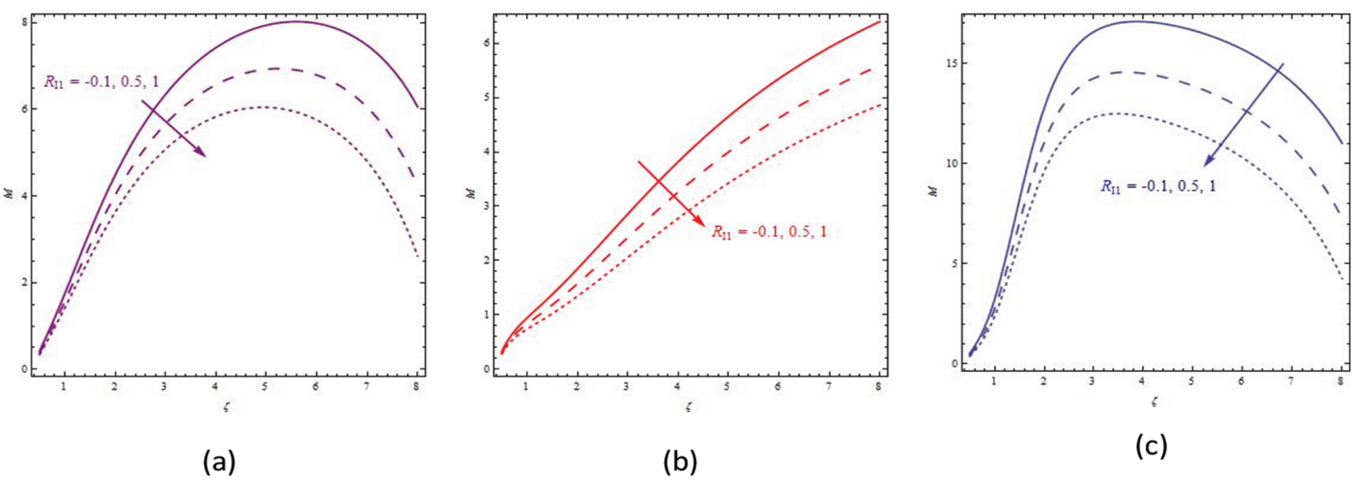


**Figure 9.** Internal Rayleigh number for fluid layer '$R_{I1}$' on LTP, PTP and ITP

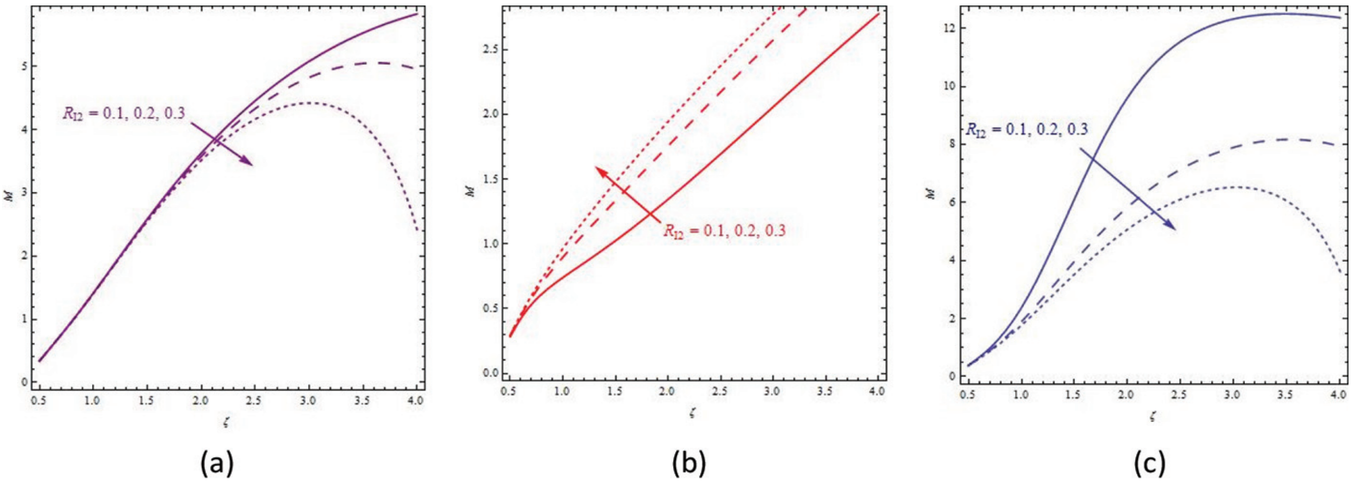


**Figure 10.** Internal Rayleigh number for porous layer '*Rim*' on LTP, PTP and ITP

for all the profiles, hence DDNBM convection can be postponed. The curves converge for LTP and ITP and are found be diverging for PTP.

The effects of $\tau_2$ on the TMAN are shown in Fig 8(a), 8(b) & 8(c) for the values of $\tau_2$ = 0.1, 0.5, 1. Here, similar effect is observed for increase in $\tau_2$ increases $M_L$, $M_P$ and $M_I$, which shows that the composite layer system is stabilizing for all the profiles, so DDNBM convection is postponed. The curves are converging for all the three profiles indicating its effectiveness for lower values of $\varsigma$.

In Fig. 9(a), 9(b), 9(c) the effects of $R_{I1}$ = –0.1, 0.5, 1 (sink to source) on $M_L$, $M_P$ and $M_I$ are shown. The curves are found to be diverging and also the as $R_{I1}$ increases TMAN decreases for all the three profiles, which is physically reasonable. This shows the effect of $R_{I1}$, is to destabilize the system and hence its effect is to prepone DDNBM convection.

The effects of $R_{I2}$ on the TMAN are shown in Fig.10(a), 10(b) & 10(c) for LTP, PTP and ITP respectively for $R_{I2}$ = 0.1, 0.2, 0.3. The curves are found diverging drastically for LTP, PTP and ITP. The increase in the value of $R_{I2}$ increases the TMAN for the PTP and decreases the TMAN for the LTP and ITP. Hence the increase in the value of $R_{I2}$ stabilizes the composite layer system for PTP and destabilizes for the other two profiles.

# V. Conclusions

From the investigation considered, the following conclusions can be drawn for set of parameters considered in this study,

1. The most stable profile is inverted parabolic, which can be used to control DDNBM convection, whereas the most unstable profile is the parabolic, which can be used to augment DDNBM convection.
2. For Parabolic profile larger values of $\varsigma$, the parameters $a$, $Da$, $\in_T$, $Ms$, $\hat{\mu}$, $\tau_1$, $R_{I1}$ and $R_{I2}$ are significant. For lower values of $\varsigma$, the $\hat{\mu}$ and $\tau_1$ are effective.
3. In the Linear profile for larger values of $\varsigma$, the parameters $a$, $Da$, $\in_T$, $Ms$, $R_{I1}$ and $R_{I2}$ are effective. For lower $\varsigma$, $\in_S$, $Ms$, $\tau_1$, $\tau_2$ are significant.
4. In the Inverted parabolic profile for larger values of $\varsigma$, the parameters $a$, $Da$, $\in_T$, $Ms$, $\hat{\mu}$, $R_{I1}$ and $R_{I2}$ are appropriate. For lower $\varsigma$, the $Ms$, $\hat{\mu}$, $\in_S$, $\tau_1$, $\tau_2$ are effective.

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