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Outlook of density maximum on the onset of Forchheimer-Bénard convection with throughflow

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Abstract

The vertical throughflow effect is investigated on the onset of porous convection by considering a cubic density-temperature relationship and using the Forchheimer-Darcy model. The stability eigenvalue problem is explained numerically using the Galerkin technique. Contrary to the linear density-temperature relationship, the direction of throughflow alters the onset of convection. The throughflow dependent Péclet number is found to stabilize the fluid motion against convection and the upflow is found to be either stabilizing or destabilizing than the downflow depending on the values of thermal condition parameters λ_1 and λ_2. A destabilizing effect on the onset is observed with increasing λ_1 and λ_2. The Darcy number Da and the Forchheimer drag co-efficient, cb instability characteristics have been investigated and depicted graphically.

Keywords: *Convection, maximum density, porous medium, vertical throughflow, Galerkin technique.*

1.0 Introduction

The study of convection in porous media which is heated from below is a classical problem and apparently independent investigations were carried out by Horton and Rogers [4] and Lapwood [8]. This convective instability problem is referred to as the Darcy-Bénard problem and it has been studied widely in the last few decades by considering various additional effects depending on the varied state of affairs. The gargantuan works available on this topic is aptly acknowledged in the books (Straughan [15], Nield and Bejan [10], Nield and Simmons [13]). One such extension work that has been carried out is the vertical throughflow effect on the Darcy-Bénard problem because of its application in in-situ coal gasification and also in controlling the convective instability by altering the vertical throughflow (Sutton [19], Homsy and Sherwood [3], Jones and Persichetti [5], Nield [11], Khalili and Shivakumara [6], Zhao et al. [25], Shivakumara and Sureshkumar [14], Brevdo and Ruderman [2], Barletta et al. [1]). In all these studies, a direct relationship between the density and temperature is considered and the effect of throughflow was shown to be stabilizing (destabilizing) if the porous layer bounding surfaces are of the same (different) kind. Besides, if the porous layer is heated internally then the throughflow is found to instill instability even in the case of symmetric boundaries (Khalili and Shivakumara [6]).

The density of most of the fluids encountered in geophysical and industrial problems may not be strictly decreasing/increasing function of temperature and for example, at a specific temperature, water reaches a maximum temperature and then starts falling down with further rise in its temperature. Such a property is known as density maximum which can be envisaged through nonlinear variation of density with temperature. The importance for considering the property of density maximum in the studies of buoyancy-driven convection has been discussed by Straughan [17]. The onset of convective instability in a porous medium has been studied widely considering the density maximum property. Along with this, the combined result of density maximum and a uniform vertical throughflow on the onset of convection in a layer of Darcy porous medium has been discussed by Wu et al. [23] using finite-difference as well as finite-element methods. Recently, Capone et al. [26] analyzed the effect of vertical throughflow on the onset of double diffusive magnetoconvection in a horizontal porous layer. The critical Rayleigh numbers for the onset of steady or oscillatory convection are obtained using a single-term Galerkin method.

Nonetheless, the effect of inertia becomes important in the existence of vertical throughflow and such effects will not be taken care by the Darcy law as it is valid for small velocities. In such cases, usage of a non-Darcian model is necessary to understand the problem better. The suitable inertia expression in the momentum equation is to consider the Forchheimer drag term which is quadratic in velocity for naturally occurring porous media. The main purpose of this paper is to reconsider the work of Wu et al. [23] by deriving the correct stability equation and to account for the inertial effect through Forchheimer-extended Darcy equation. This results in an eigenvalue problem which is resolved mathematically by employing the Galerkin technique.

2.0 Mathematical Formulation

In Figure 1, the physical model and the coordinates are shown. It contains an incompressible fluid-saturated porous layer which is horizontal with thickness d under the influence of a constant vertical throughflow W0 and the gravitational acceleration $\vec{g}$. The porous layer bottom and top surfaces are held at constant temperatures $T_L (>T_U)$ and T_U, respectively. A system of Cartesian coordinate (x, y, z) is considered representing the bottom of the porous layer at the origin and the z-axis showing vertically upward while axes x and y are in the horizontal plane. The porous medium is in local thermal equilibrium and the density is considered to be a maximum in the interior of the porous layer.

According to Yen [24], the relation between the density-temperature for water at the temperature limit 0 to 30°C is represented in the following form

$$\rho = \rho_{\max}\left\{1-\beta_1\left(T-T_{\max}\right)^2-\beta_2\left(T-T_{\max}\right)^3\right\} \quad \text{... (1)}$$

where ρ represents the density of fluid, T is the temperature, T_{max} is the maximum temperature, $\beta_1 = 7.94\times10^{-6}\,°C^{-2}$ and $\beta_2 = -6.56\times10^{-8}\,°C^{-3}$ are the coefficients of thermal expansion and ρ_{max} is the density at T=T_{max}. If the cubic term in Eq. (1) is dropped then we get a parabolic equation for ρ which is valid for the temperature range 0-8°C (Moore and Weiss [9]).

The conservation equations are:

$$\nabla.\vec{q} = 0\,, \quad \text{... (2)}$$

$$\frac{\rho_{\max}}{\varepsilon}\frac{\partial\vec{q}}{\partial t}+\frac{\rho_{\max}c_b}{\sqrt{K}}\left|\vec{q}\right|\vec{q} = -\left(\nabla p+\frac{\mu}{K}\vec{q}\right)+\rho\vec{g}\,, \quad \text{... (3)}$$

$$A\frac{\partial T}{\partial t}+\left(\vec{q}\cdot\nabla\right)T = \kappa\nabla^2 T\,, \quad \text{... (4)}$$

where $\vec{q}$ = (u, v, w) - velocity, p - pressure, μ - fluid viscosity, c_b - Forchheimer drag coefficient, ε - porosity of the porous medium, κ - effective thermal diffusivity, A - ratio of heat capacities, K - permeability and $\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ - Laplacian operator.

The boundary conditions are

$\vec{q}.\hat{k} = W_0$ at $z = 0,\ d$; $T = T_L$ at z = 0

and T = T_U at z = d, ... (5)

where, $\hat{k}$ represents the unit vector in the vertical z-direction.

The basic state is given by

$$\vec{q} = \vec{q}_b = W_0\hat{k}, \qquad T = T_b(z), \qquad p = p_b(z), \quad \text{... (6)}$$

where the subscript b signifies the basic state. With this, Eqs. (2) - (4) now become

$$\nabla.(W_0\hat{k}) = 0 \quad \text{... (7)}$$

$$\left[\frac{\rho_{\max}c_b}{\sqrt{K}}\left|\vec{q}_b\right|(W_0\hat{k})\right] = -\nabla p_b+\rho_{\max}\left\{1-\beta_1\left(T_b-T_{\max}\right)^2 -\beta_2\left(T_b-T_{\max}\right)^3\right\}\vec{g}-\frac{\mu}{K}(W_0\hat{k}) \quad \text{... (8)}$$

$$W_0\frac{dT_b}{dz} = \kappa\frac{d^2T_b}{dz^2} \quad \text{... (9)}$$

Solving Eq. (9) using the corresponding boundary conditions, we get

$$T_b = T_U+(T_L-T_U)\frac{(e^{W_0 d/\kappa}-e^{W_0 z/\kappa})}{(e^{W_0 d/\kappa}-1)} = T_U+\Delta T\,\theta_b \quad \text{... (10)}$$

where $\Delta T = T_L - T_U$ and $\theta_b = \frac{e^{W_0 d/\kappa}-e^{W_0 z/\kappa}}{(e^{W_0 d/\kappa}-1)}$. In the absence of vertical throughflow, T_b is found to be $T_b = T_U+\Delta T(1-z/d)$. Thus the vertical throughflow effect is to change the basic temperature field to nonlinear from the linear variation in the vertical z

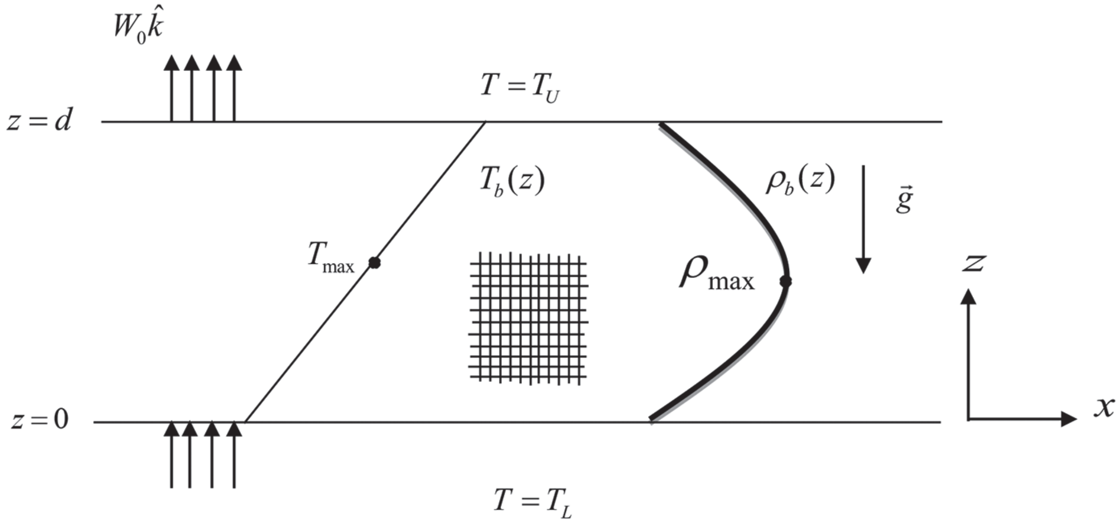

Figure 1: Physical configuration

direction which in turn has an important implication on the onset.

To study the stability of base flow, the basic state is perturbed as below

$$p = p_b + p', \quad \vec{q} = q_b + \vec{q}', \quad T = T_b + \theta', \qquad \text{... (11)}$$

where the perturbed terms p', $\vec{q}'$ and θ' represent pressure, velocity and temperature, respectively. Substituting Eq. (11) into Eqs. (2)-(4), using Eqs.(7)-(9) and linearizing the resulting equations yield (after neglecting the primes),

$$\nabla.\vec{q} = 0, \qquad \text{... (12)}$$

$$\left[\frac{\rho_{max}}{\varepsilon}\frac{\partial \vec{q}}{\partial t} + \frac{\rho_{max} c_b}{\sqrt{K}}|\vec{q}_b|\vec{q}\right] = -\nabla p + \rho_{max}\left\{2\beta_1\left(T_b - T_{max}\right)\right.$$

$$\left. + 3\beta_2\left(T_b - T_{max}\right)^2\right\}\theta g\hat{k} - \frac{\mu}{K}\vec{q} \qquad \text{... (13)}$$

$$A\frac{\partial \theta}{\partial t} + \left(\vec{q}_b\cdot\nabla\right)\theta + (\vec{q}\cdot\nabla)\theta_b = \kappa\nabla^2\theta. \qquad \text{... (14)}$$

It is also seen that

$$T_b - T_{max} = \Delta T(B + \Phi), \qquad \text{... (15)}$$

where $B = \dfrac{T_L - T_{max}}{\Delta T}$ and $\Phi = \theta_b - 1$.

Introducing the non-dimensional quantities as mentioned below

$$\vec{q}^* = \frac{\vec{q}}{\left(\kappa/d\right)},\ p^* = \left(\frac{K}{\kappa\mu}\right)p,\ \theta^* = \frac{\theta}{\Delta T},\ t^* = \frac{t}{\left(d^2/\kappa\right)},\quad \nabla = \frac{1}{d}\nabla^*, \qquad \text{... (16)}$$

and substituting Eq. (16) into Eqs. (12) - (14) we get (after neglecting the asterisks)

$$\nabla.\vec{q} = 0, \qquad \text{... (17)}$$

$$\left[\frac{Da}{Pr\varepsilon}\frac{\partial}{\partial t} + \frac{c_b\sqrt{Da}|Q|}{Pr} + 1\right]\vec{q} = -\nabla p + R_D(1 - \lambda_1\Phi + \lambda_2\Phi^2)\theta\hat{k}, \qquad \text{... (18)}$$

$$A\frac{\partial \theta}{\partial t} + Q\frac{\partial \theta}{\partial z} + \frac{d\theta_b}{dz} = \nabla^2\theta, \qquad (19)$$

where

$Q = \dfrac{W_0 d}{\kappa}$ - Péclet number,

$Pr = \nu/\kappa$ - Prandtl number,

$Da = K/d^2$ - Darcy number,

$R_D = 2\beta_1(\mathrm{B}\Delta T)\left[1 + \dfrac{3\beta_2}{2\beta_1}(\mathrm{B}\Delta T)\right](\mathrm{g}\,\Delta TKd)/\nu\kappa$ - modified Darcy-Rayleigh number,

$\lambda_1 = -\dfrac{1}{B}\left\{\dfrac{1 + (3\beta_2/\beta_1)\mathrm{B}\Delta T}{1 + (3\beta_2/2\beta_1)\mathrm{B}\Delta T}\right\}, \lambda_2 = \dfrac{1}{B}\left\{\dfrac{(3\beta_2/2\beta_1)\Delta T}{1 + (3\beta_2/2\beta_1)\mathrm{B}\Delta T}\right\}$

- thermal condition parameters,

$\nu = \mu/\rho_{max}$ - kinematic viscosity and

$\theta_b = (e^Q - e^{Qz})/(e^Q - 1)$.

By operating curl twice to eliminate the pressure term from Eq. (18) and keeping the z component of the obtained equation, we get

$$\left[\frac{Da}{Pr\varepsilon}\frac{\partial}{\partial t} + \frac{c_b\sqrt{Da}|Q|}{Pr} + 1\right]\nabla^2 w = -R_D(1 - \lambda_1\Phi + \lambda_2\Phi^2)\nabla_h^2\theta, \qquad \text{...(20)}$$

where $\nabla_h^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ represents the horizontal Laplacian operator. The boundary conditions now become

$$w = \theta = 0 \quad \text{at} \quad z = 0, 1. \qquad (21)$$

After ensuring the principle of exchange of stability is valid, a normal mode analysis is assumed in the following form.

$$(w,\theta)(x,y,z)=(W,\Theta)(z)\ \exp\left[i\left(\ell x+my\right)\right], \quad \text{... (22)}$$

where l and m represent the wave numbers in the x and y directions, respectively. Substituting Eq. (22) into Eqs. (19) and (20), we arrive at the following stability equations

$$\left\{\frac{c_b\sqrt{Da}\,|Q|}{Pr}+1\right\}(D^2-a^2)W(z) \quad \text{... (23)}$$

$$=a^2R_D(1-\lambda_1\Phi+\lambda_2\Phi^2)\Theta(z)$$

$$(D^2-a^2)\Theta(z)=Q\,D\Theta(z)+D\theta_b\,W(z)\,, \quad \text{... (24)}$$

where $D=d/dz$ and $a=\sqrt{l^2+m^2}$ is the total horizontal wave number.

The boundary conditions are

W = Q = 0

$$W=\Theta=0 \ \text{ at } z=0,1. \quad \text{... (25)}$$

3. Method of Solution

Equations (23) - (25) form an eigenvalue problem with R_D as the eigenvalue and solved by applying the Galerkin technique. Accordingly, the dependent variables are expanded as below

$$W=\sum_{i=1}^{N}A_iW_i(z),\quad \Theta=\sum_{i=1}^{N}B_i\Theta_i(z) \quad \text{... (26)}$$

where A_i and B_i are constants, W_i and Θ_i are the basis functions selected so that they fulfill the respective boundary conditions. Equation (26) is replaced back in Eqs. (23) and (24) and then these equations are respectively multiplied by $W_j(z)$ and $\Theta_i(z)$, and integrated by parts w.r.t z between in the form z = 0 and 1 to arrive at a system of linear homogeneous algebraic equations in the form.

$$A_iC_{ji}+B_iD_{ji}=0,$$
$$A_iE_{ji}+B_iF_{ji}=0, \quad \text{... (27)}$$

where

Table 1: Convergence process of the Galerkin method for two values of λ_1 and λ_2 when Q = 1 and c_b = 0

λ_2	Order	λ = –3.5		λ_1 = –0.5	
		a_c	R_{Dc}	a_c	R_{Dc}
1.4	1	3.142	348.0258	3.142	38.0562
	2	4.02	133.6332	3.205	37.4455
	3	4.207	128.5258	3.208	37.3790
	4	4.222	128.2208	3.21	37.3871
	5	4.224	128.2142	3.21	37.3856
	6	4.224	128.2149	3.21	37.3881
	7	4.224	128.2180	3.21	37.3879
	8	4.224	128.2195	3.21	37.3887
–1.4	1	3.142	63.5531	3.142	74.5413
	2	3.678	42.8605	3.208	79.5746
	3	3.795	41.7249	3.213	79.4145
	4	3.809	41.6110	3.214	79.4606
	5	3.811	41.5989	3.214	79.4594
	6	3.812	41.5972	3.214	79.4666
	7	3.812	41.5970	3.214	79.4660
	8	3.812	41.5973	3.214	79.4683

$$C_{ij} = \left(\frac{c_b \sqrt{Da}\,|Q|}{Pr} + 1 \right) \{ \langle DW_j DW_i \rangle + a^2 \langle W_j W_i \rangle \},$$

$$D_{ij} = -a^2 R_D \left\langle \left(1 - \lambda_1 \Phi + \lambda_2 \Phi^2\right) W_j \Theta_i \right\rangle,$$

$$E_{ij} = -\langle D\theta_b \Theta_j W_i \rangle,$$

$$F_{ij} = \left[\langle D\Theta_j D\Theta_i(z) \rangle + a^2 \langle \Theta_i \Theta_j \rangle + Q \langle \Theta_j D\Theta_i \rangle \right],$$

and $\langle \ldots \rangle = \int_0^1 (\cdots)\, dz$. The suitable trial functions satisfying the boundary are chosen as given below.

$$W_i = \Theta_i = (1-z) z^i \qquad \text{... (28)}$$

4.0 Results and Discussion

The consequence of non-monotone density change with temperature on thermal convective instability in a porous layer under the influence of a uniform vertical throughflow is investigated. To describe the flow in a porous medium, the Darcy-Forchheimer equation is employed. The eigenvalue problem is numerically solved using the Galerkin technique. It is observed that the converging results are obtained by using six terms in the Galerkin method. To achieve this various orders of approximations in the Galerkin method is displayed in Table.1 for different values of governing parameters. It is observed that, the results converge as the number of terms in the Galerkin expansion increase. In Table 2, the results obtained by the Galerkin expansion are compared to Wu et al. [23] for different λ_1 and λ_2 values when Q=1 and c_b=0 . Although there is an agreement between the results for some ranges of λ_1 and λ_2 values, our computed values are not agreeing with those of Wu et al. [23] at higher values of λ_1 and this may be due to the error in their stability equations. Also, the finite difference method used by Wu et al. [23] fails to give convergent results for some ranges of λ_1 and λ_2. However, the present numerical solution procedure gives convergent results for all the values of λ_1 and λ_2 considered. Besides, there appears a threshold value of $|\lambda_1|$ below which the system always gets destabilized $-1.4 < \lambda_2 < 1.4$. That is, the modified critical Darcy-Rayleigh number R_{Dc} goes on decreasing in this range. However, this threshold value of $|\lambda_1|$, the system shows some stabilizing effect. Initially up to a few values of λ_2 and after which the system gets destabilized. The range of λ_2 till that the system gets stabilized increases with increasing value of $|\lambda_1|$. For $\lambda_1 = -3$, the system shows only stabilizing effect for all λ_2 in $-1.4 < \lambda_2 < 1.4$.

Table 2: Comparison of (R_{Dc}, a_c) with Wu et al. [23] for various values of λ_2 and λ_1 when Q = 1 and c_b = 0

λ_1	λ_2	a_c	R_{Dc}	Wu et al. [23]
-1.5	-1.4	3.275	181.4333	174.96
	-1.2	3.896	165.1968	165.21
	-1.0	3.757	155.1174	155.13
	-0.8	3.62	144.7821	144.79
	-0.6	3.494	134.36	134.35
	-0.4	3.388	124.1007	124.11
	-0.2	3.305	114.274	114.28
	0.0	3.245	105.0982	105.1
	0.2	3.204	96.70213	96.71
	0.4	3.179	89.12756	89.13
	0.6	3.165	82.35361	82.36
	0.8	3.158	76.32308	76.33
	1.0	3.158	70.96252	70.97
	1.2	3.161	66.19496	66.20
	1.4	3.167	61.94687	61.95
-2	-1.4	4.807	103.4288	-
	-1.2	4.923	115.8302	-
	-1.0	5.056	131.2733	-
	-0.8	5.21	150.9541	-
	-0.6	5.391	176.7622	-
	-0.4	4.038	194.4818	194.50
	-0.2	3.879	182.4423	182.45
	0.0	3.713	169.8125	169.82
	0.2	3.556	156.7576	156.77
	0.4	3.42	143.6432	143.65
	0.6	3.317	130.9519	130.96
	0.8	3.245	119.1118	119.12
	1.0	3.199	108.3798	108.38
	1.2	3.173	98.8371	98.84
	1.4	3.161	90.4433	90.45

The variation of critical Darcy-Rayleigh number R_{Dc} as a function of λ_2 is shown in Figs.2 and 3 for various values of λ_1 for $Q = \pm 1$ when $c_b = 0$. It is noted that R_{Dc} increases with increasing λ_2 initially, and attains different maximum value for each considered parametric values and eventually decreases with further increase in λ_2. Moreover, decrease in λ is to increase λ_2 at which R_{Dc} attains its maximum value.

Table 3: Comparison of (RDc, a_c) with Wu et al. [23] for various values of λ_2 and λ_1 when Q = 1 and c_b = 0

λ_1 / λ_2	-2.5			-3.0		
	ac	RDc	Wu et al. 23	ac	RDc	Wu et al. 23
-1.4	4.285	70.8502	-	3.985	52.7329	-
-1.2	4.342	76.9432	-	4.011	56.1612	-
-1.0	4.407	84.1207	-	4.04	60.0548	-
-0.8	4.481	92.6866	-	4.073	64.5132	-
-0.6	4.568	103.0660	-	4.11	69.6654	-
-0.4	4.668	115.8713	-	4.153	75.6825	-
-0.2	4.784	132.0161	-	4.203	82.7952	-
0.0	4.92	152.9219	-	4.26	91.3221	-
0.2	5.081	180.9213	-	4.328	101.7149	-
0.4	5.274	220.1092	220.09	4.407	114.6355	-
0.6	4.051	218.6511	218.67	4.501	131.0908	-
0.8	3.854	203.3928	203.41	4.613	152.6881	-
1.0	3.652	187.0338	187.05	4.747	182.1532	-
1.2	3.473	169.9938	170	4.91	224.4921	-
1.4	3.336	153.1211	153.13	4.293	266.9716	268.3

The maximum value is higher for Q=–1 when compared to Q=1. Figure 3 indicates that the rise in the value of Q hinders the onset of convection only up to certain value of λ_2 and after that the trend gets reversed.

In Figs. 4-7, the variation of R_{Dc} as a function of $|Q|$ is presented for different values of λ_1, λ_2, D_a and C_b. Figure 4 shows that the variation of R_{Dc} against $|Q|$

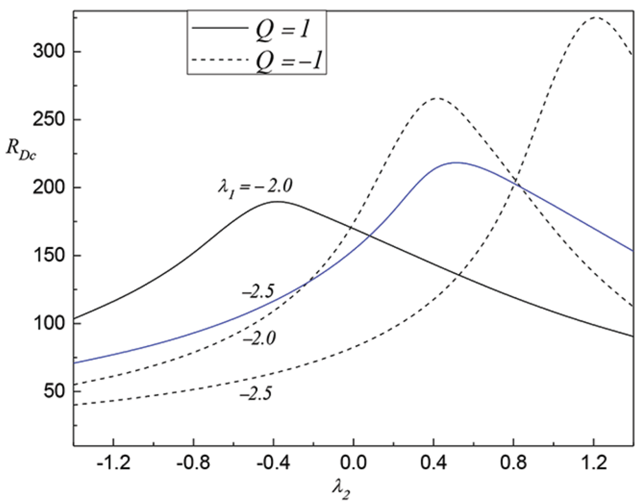

Figure 2: Variation of R_{Dc} against λ_2 for two values of Q and λ_1.

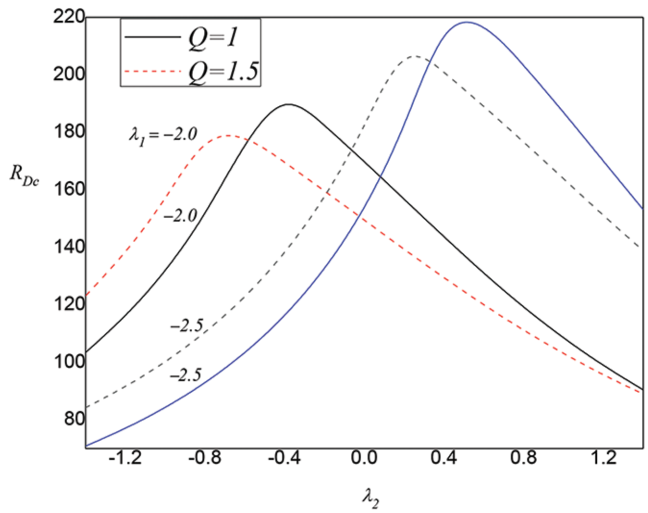

Figure 3: Variation of R_{Dc} against λ_2 for two values of Q and λ_1.

for different values of λ_1 for fixed value of λ_2=1, D_a = 0.01, C_b = 0.55 and Pr = 7. It is noted that R_{Dc} values for Q < 0 are higher than those of Q > 0 indicating the downward flow has more stabilizing effect against convection than the upward flow. Moreover, increase in the $|\lambda_1|$ is to increase R_{Dc} and hence its effect is to suppress the onset of convection. Figures 5 and 6

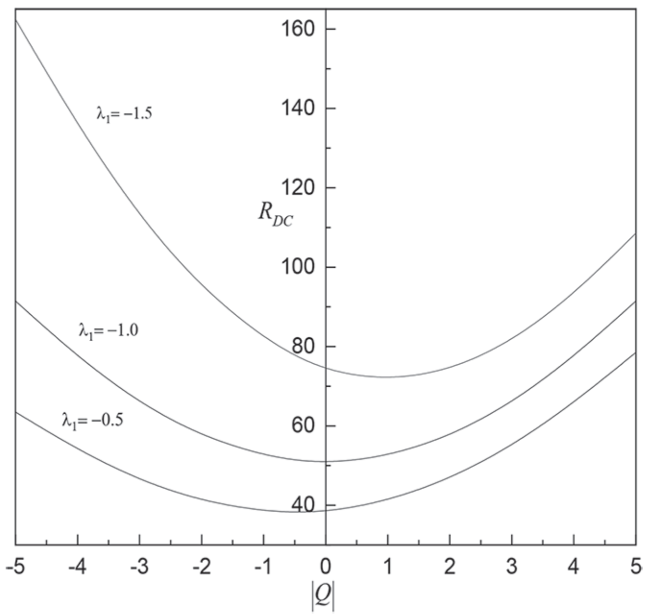

Figure 4: Variation of R_{Dc} against Q for different values of λ_1 when $\lambda_2 = 1$, $Da = 0.01$, $C_b = 0.55$ and Pr=7

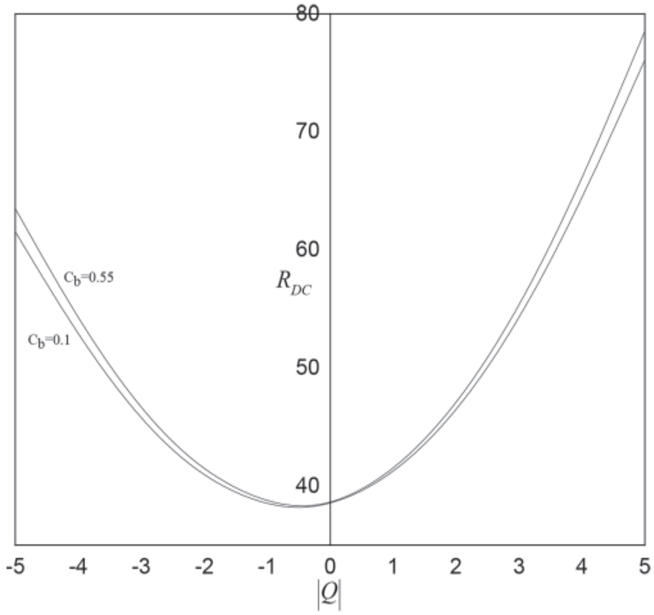

Figure 6: Variation of RDc against Q for different values of C_b when $\lambda_1 = -0.5$, $\lambda_2 = 1$, $Da = 0.01$ and Pr = 7

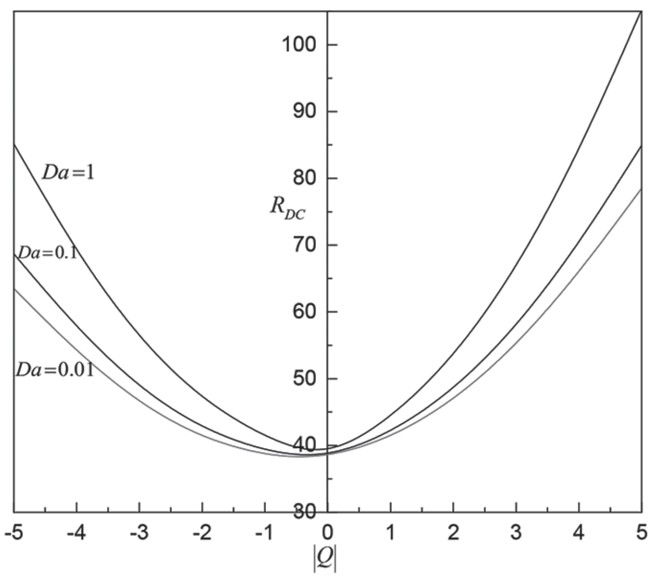

Figure 5: Variation of R_{Dc} against Q for different values of Da when $\lambda_1 = -0.5$, $\lambda_2 = 1$ $C_b = 0.55$ and Pr = 7

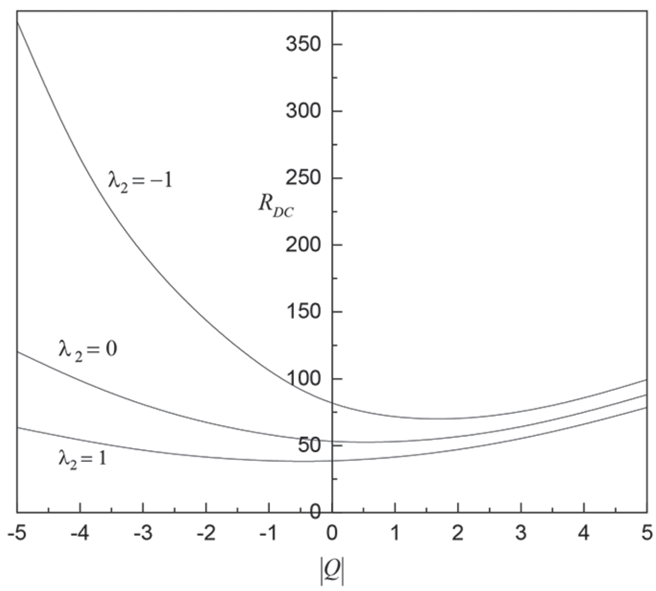

Figure 7: Variation of RDc against Q for different values of λ_2 when $\lambda_1 = -0.5$, $Da = 0.01$, $C_b = 0.55$ and Pr = 7

respectively show that decrease in Da (with $\lambda_1 = -0.5$, $\lambda_2 = 1$, $C_b = 0.55$, Pr = 7) and C_b (with $\lambda_1 = -0.5$, $\lambda_2 = 1$ Da = 0.01, Pr = 7) is to speed up the onset of convection. Figure 7 shows that the system is more destabilizing for positive value of λ_2 when compared to the negative value with $\lambda_1 = -0.5$, $Da = 0.01$, $C_b = 0.55$ and Pr = 7.

5.0 Conclusions

The simultaneous influence of vertical throughflow and a density maximum property on the onset of porous convection is numerically investigated. The Darcy-Forchheimer model is used to describe the flow in a porous medium and a cubic density-temperature relationship is employed to account for the density maximum property. The direction of throughflow alters the stability characteristics of the system contrary to the case of linear density-temperature relationship. The throughflow has a stabilizing effect against convection, in general. The upflow (i.e. antigravity throughflow) is found to be more stabilizing/destabilizing than downflow depending on the magnitude of thermal condition parameters. The effect of Forchheimer drag coefficient is to delay the onset of convection. It is delineated that the effect of density maximum along with the magnitude of vertical throughflow can be conveniently used to either suppress or augment of convection.

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