

# Dynamic Analysis of Flexible two Link Robotic Arm Considering Joint Stiffness

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## Abstract

The robotic manipulator is a machine that can perform a variety of tasks according to specifications without the need of human interference. In order to model and control such devices, vibration analysis of flexible manipulators has become a critical field of study. The finite element approach has been used to analyze single and double connection flexible manipulators made of advanced composite material in the current study. For the modelling and study of the versatile composite manipulators, a three-noded beam feature is used. The effects of the taper angles of tapered flexible composite manipulators on the final product effector movement and vibration has been considered. In present work CATIA V5 is used to model the flexible single link robotic arm and static structural analysis to find stress and total deformation, dynamic analysis is carried out to find different modes, corresponding frequency, and life estimation of flexible single and double link robotic arm and also material optimization is done using different material composition for structural steel, aluminium and CFRP composite material using ANSYS WORKBENCH 18.2 (Finite Element Approach) for life estimation and evaluation using analytical approach.

**Keywords:** Flexible manipulators, Boundary conditions, Assumed modes, Initial modes

## 1.0 Introduction

Robot manipulators are designed to increase the productivity and to help humans in tedious and hazardous work environment. The manipulator arms are made of heavy and stiff materials to achieve high precision on end effector motion. However, the heavy manipulator arms are required to have bulky actuators for robot manipulation in workspace. In addition, the heavy manipulators have higher mass, consume more power and have limited operation speed with respect to operating payload. In order to build power efficient robot manipulators and to increase the operation speed, the focus is switched towards development of light weight manipulators. The control of light weight manipulators is complex based on the nature of flexibility in the system, i.e.

flexible joints and flexible links. Among them, the most difficult task is to control the flexible link schemers because of link flexibility, under actuation and non-minimal phase nature. Under actuation is due to finite number of actuators to control infinite degrees of freedom that arise due to link flexibility. Non-minimum phase nature occurs because of non-collocation of actuators and sensors [1]. Despite of various advantages, the flexible link schemers have less progress at the industrial level. There is a need to bring the advantages of flexible link schemers to more general industrial applications by eliminating the difficulties surrounded to it such as modelling link flexibility, under actuation and non-minimum phase nature in control design. The dynamic modelling that includes the flexible link is considered as an important step in model based control design and to achieve

better performance. The following research work is dedicated to identify and develop a systematic approach for the modelling dynamically and model based control of spatial flexible manipulators. In earlier studies, Venkatesha B K et al. [14-15] studied the numerical analysis of damage tolerance design. Fatigue crack growth rate and stress intensity factor range was estimated with of Paris law of damage crack growth.

## 2.0 Objectives

Many model based controllers were developed in the past for trajectory tracking and vibrations suppression. However, these are limited to numerical or experimental studies on planar flexible link manipulators. The current research will be focused to develop a systematic approach for the modelling dynamically and model based control design of spatial flexible manipulators. The research activity on flexible manipulators is compiled into two sections [2]. The first section will be focused on dynamic modelling of spatial flexible manipulators while the second section will be focused on control design for tracking of trajectory and vibration suppression. In present work CATIA V5 is used to modelling flexible single link robotic arm and static structural to find stress and total deformation, dynamic analysis to find different modes, corresponding frequency and life estimation of flexible single link robotic arm using ANSYS WORKBENCH 19.2.

## 3.0 Methodology

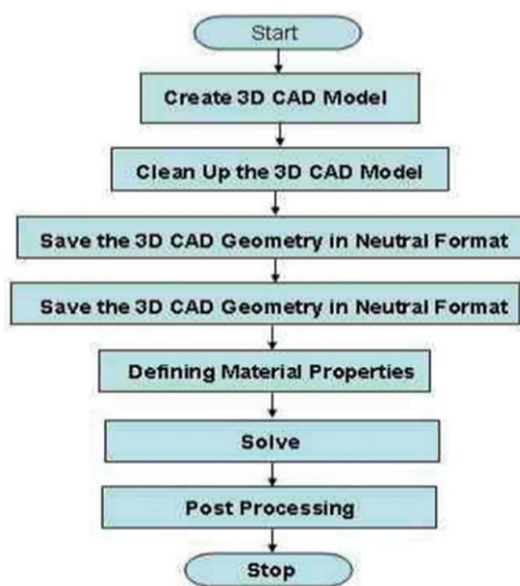


Figure 1: Methodology

Geometric Modelling: The 3D drawing of model is created using CATIA V5. Finite Element Method (FEM): The CAD model is imported to ANSYS and meshed using tetra elements to obtain FE model using Suitable Boundary Conditions with fixed and load boundary conditions are applied to FE model and sensitive analysis is carried out using FE software (ANSYS) [3].

### 3.1 Dynamic Modelling

A systematic approach for the dynamic modelling of flexible manipulator is considered. Rigid links, flexible links, and flexible joints are considered in the dynamic formulation. The kinematics of flexible links are derived using the floating reference formulation. The flexible links are deformable due to bending and torsion. The elastic deformations of flexible link due to bending are defined using the Euler-Bernoulli beam formulation. The inclusion of the dynamics due to link flexibility makes the robot manipulator a continuous system and requires infinite Degrees of Freedom (DOF) to estimate the dynamic parameters. It is not feasible to include infinite DOF in the dynamic model from the numerical simulations and control design point of view. Thus, the FEM is used to discretize the flexible link to get the finite dimensional dynamic model. With the help of virtual work principle, equations of motions are derived in an absolute coordinate system for the general purpose implementation. Then, the equations in absolute coordinates is converted into relative or independent coordinates using the recursive kinematic formulations. The dynamic models that are derived using virtual work principle and finite element method consider the coupling effect of rigid body motion and elastic deformations of flexible link [4].

A general purpose multi-body code has been developed based on systematic approach that is presented for dynamic formulation of flexible manipulators. The input to the multi-body code is physical parameters of the flexible manipulator and the output is finite dimensional dynamic model of flexible manipulator.

### 3.2 Kinematic Description

In this section, the kinematic equations that describe the position, velocity and acceleration of an arbitrary point is presented. A set of coordinate systems i.e. global coordinate system, body-fixed coordinate system, and floating coordinate system are used to derive the kinematic equations. The body-fixed coordinate system is used to define the translation and rotation of rigid link, where as a floating coordinate system is used to define the translation and rotation of flexible link. The general displacements of a point is described using Chasles theorem. It defines an arbitrary displacement as a sum of the translation of a point and a rotation along the axis of rotation.

### 3.2.1 Kinematics of Rigid Link

The representation of an arbitrary point  $O_i$  on the rigid link  $i$  is described in Figure 2. The kinematic equation defines a rigid link  $i$  arbitrary displacement that is derived using the body-fixed coordinate system. The body coordinate system  $X_i Y_i Z_i$  is attached to the rigid link  $i$  to identify the linear and angular position of the rigid link  $i$  in space. The configuration of each point on the rigid link  $i$  in space can be described using linear and angular positions of the body coordinate system  $X_i Y_i Z_i$  that are defined w.r.t the global coordinate system  $XYZ$ .

### 3.3 Multi Body Code Structure

A general purpose multi-body code has been developed in MATLAB to get the dynamic model for numerical simulation and model based control design purpose. The structure of the multi-body code is represented in Figure 2. The input to the multi-body code consists of physical parameters of body, joint, and actuators. The dynamic model is non-linear and configuration dependent. Hence, to get the dynamic parameters such as matrixes (Coriolis, inertia, centrifugal, stiffness and damping) the position and velocities of manipulator links are necessary. In the multi-body code the damping matrix is defined using Rayleigh damping. Overall the input file format consists of body definition, joint definition, actuator definition, position vector and velocity vector of the manipulator.

### 3.4 Body Definition

Explain in Table 1

Table 1: Input Fields [5]

Field	Input Format	Description
Name	String	Input to assign name to the body
Body	Number	1 = Rigid link / 2 = Flexible link
Element	Number	Number of finite element to discretize the link(1 - for rigid link, n - for flexible link)
Density	Number	Density of the link ( $\text{Kg/m}^3$ )
Young's Modulus	Number	Young's Modulus of the link (MPa)
Shear Modulus	Number	Shear Modulus of the link (MPa)
Moment Inertia	Number	Area Moment of Inertia ( $\text{m}^4$ )
Polar Inertia	Number	Moment of Inertia ( $\text{m}^4$ )
Length	Number	Length of the link (m)
Area	Number	Cross-section area ( $\text{m}^2$ )
Alpha	Number	Rayleigh Damping constant
Beta	Number	Rayleigh Damping constant

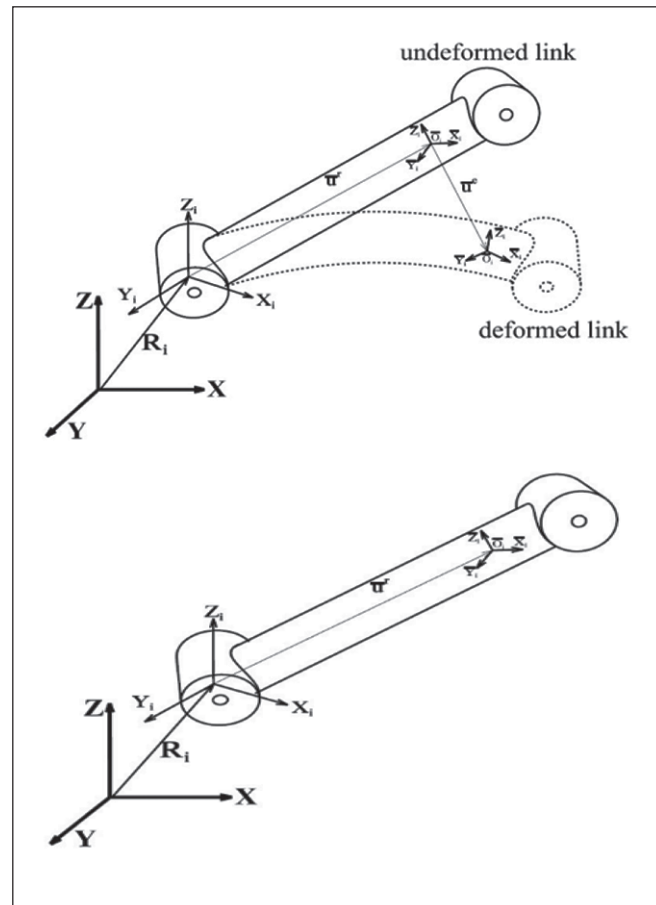


Figure 2: Representation of an arbitrary point on a rigid link

**Table 2: The input fields of joint structure**

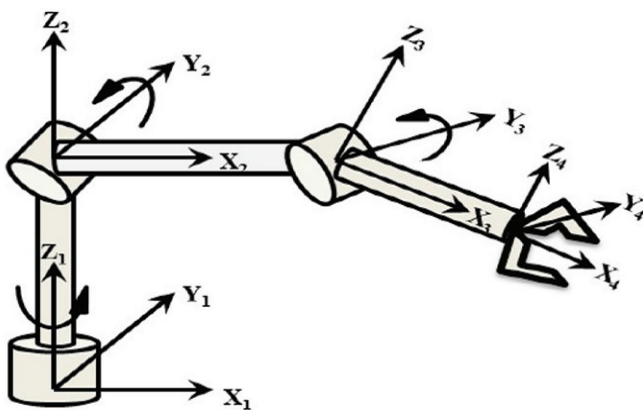
Field	Input Format	Description
Name	String	Input To Assign Name To The Joint
Joint	Number	1 = Rigid Joint ; 2 = Flexible Joint;
Inertia	Number	Inertia Of The Motor
Stiff	Number	Stiffness Of The Joint
Damp	Number	Damping Of The Joint
Axis	Number	1 = X-Axis; 2 = Y-Axis; 3 = Z-Axis;
Body1	Number	Link Number
Body2	Number	Link Number
Body1_Frame	Vector	Joint Location W.R.T Body 1
Body2_Frame	Vector	Joint Location W.R.T Body 2

### 3.5 Joint Definition

The joint definition is declared as a MATLAB structure called joint structure. The input fields to the joint structure is shown in Table 2. The user can choose the rigid joint or flexible joint modelling in the structure input field “joint”. In the joint structure, inertia, damping, and stiffness properties are considered to model dynamics of flexible joint. The flexible joint is necessary for the modelling of flexible joint manipulator such as KUKA-DLR lightweight manipulator [7].

### 3.6 Study of Flexibility Effects on Spatial Manipulator

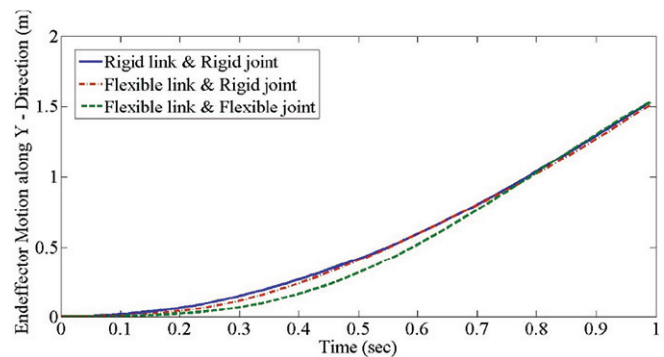
A spatial RRR manipulator shown in Figure 3, is considered to demonstrate the effect of link and joint flexibility on manipulator dynamics. The physical parameters of a RRR spatial manipulator is presented in Table 2. Uniform cross-section and material properties are assumed on each link [8].



**Figure 3:** Spatial RRR flexible manipulator

### 3.7 Simulation Results

A constant torque of 400 Nm, is applied at each manipulator joint for each case to compare manipulator end effector  $X_4Y_4Z_4$  motion. The end effector  $X_4Y_4Z_4$  motion in global coordinate system along the X, Y and Z direction. The elastic displacements of manipulator end effector  $X_4Y_4Z_4$  along the X, Y and Z direction [6].



**Figure 4:** End effector  $X_4Y_4Z_4$  trajectory response along X-direction

### 4.0 Finite Element Approach

Measured information analysis is used to investigate theoretical models, which are best described as dynamic behaviour of the structure under test. Even though this phrase is regularly used incorrectly to describe the entire modal test, this portion of the modal test is called experimental modal analysis. The first phase is called information analysis, and it continues in two steps. Determining parameters of the selected model to be fitted. Recurrence reaction capacities, similarly termed modal parameters extraction, are complete when the measured

capabilities are bent to meet the theoretical expressions.

The FEM tool is most useful tool that is capable of accurately reacting to several challenges encountered in construction testing. Using finite element approach results in concerns like static, warm, and warm trade, a flow of fluid, depletion concerns, and an electric and other field. With this approach, the examination zone is partitioned into finite components. Direct at an unlimited field of centres are limited to a limiting number of focus interests by adding cut-off centres. Finite components are manifested in the person's obsessions. The interests link to the various aspects as focal points. Components from throughout the world are brought together and the basic principles of headway and concordance are applied to nearby elements. A remarkable roadmap is located to the general procedure of direct logarithmic conditions, as long as a motivating motive is placed behind the achievement of restraint states of the real blue structure. If we have an overall plan of these situations, we will know what the whole of the continuum looks like. The technique should be considered into account the unpredictability of changing pieces in order to use a greater number of modest element [9].

### 4.1 Geometry Model

ANSYS meshing is a general-purpose, intelligent, automated high-performance product. It produces the most appropriate mesh for accurate, efficient multi-physics solutions. A single mouse click generates a mesh ideally suited for a specific analysis for all sections in a model. These fine-tuning options are accessible to the skilled user who

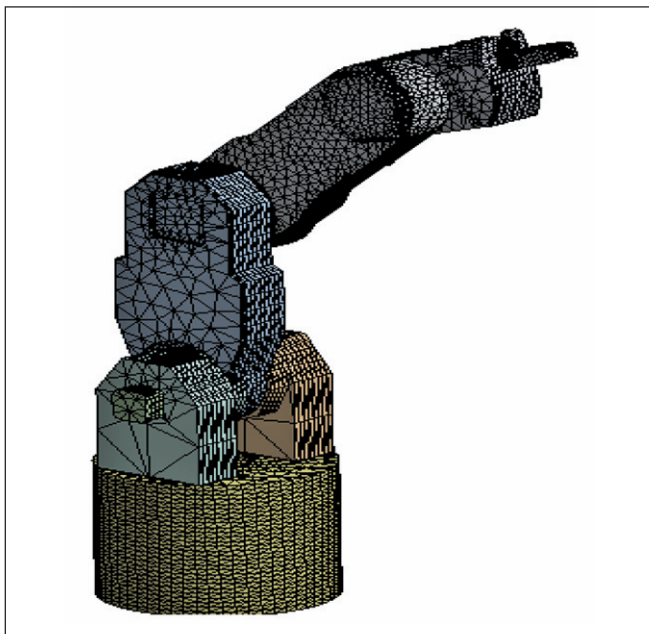


Figure 5: Isometric 3D geometry flexible link robotic arm

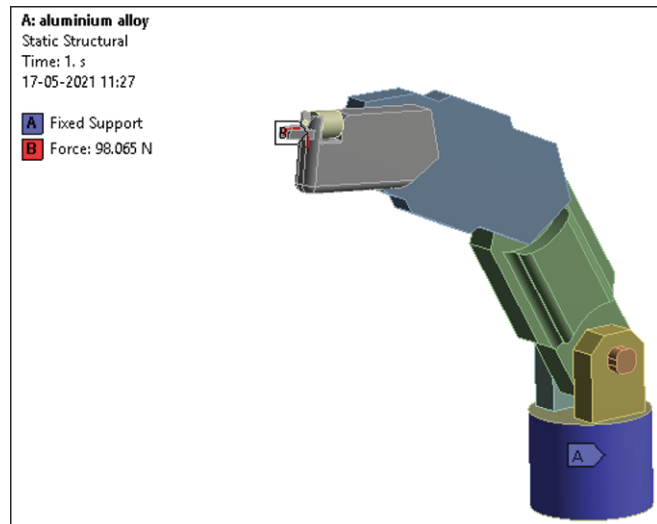
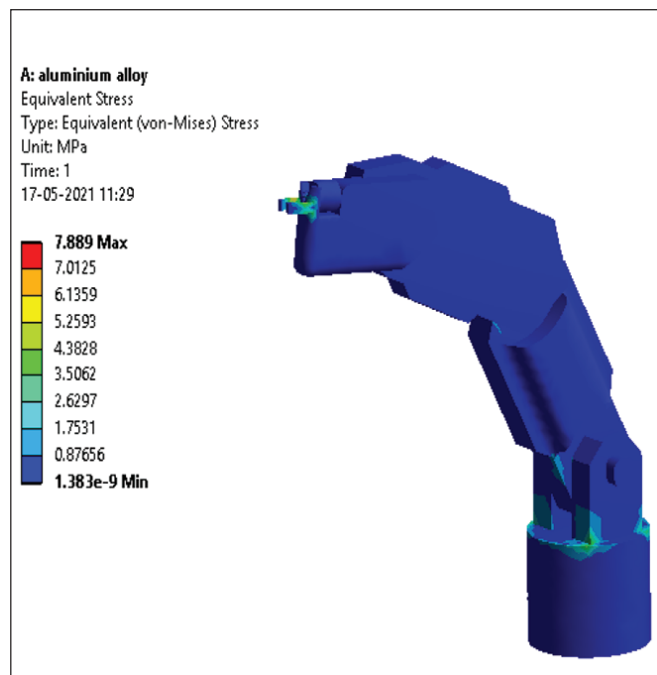


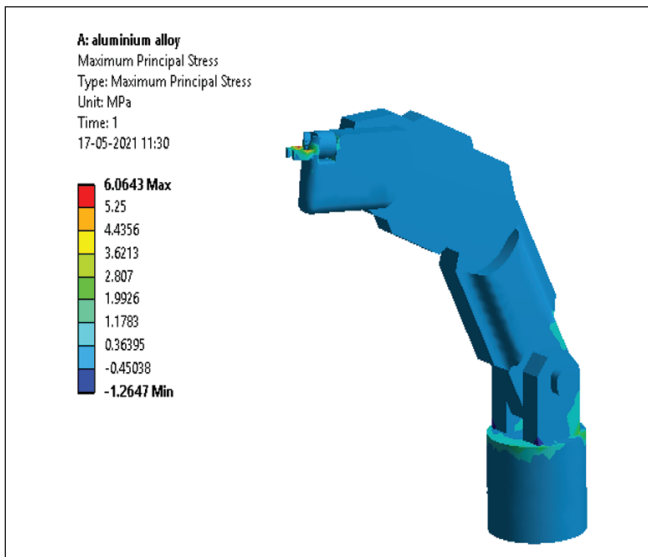
Figure 6: Load of 98.65 applied on gripper of robotic arm and other end fixed

wants full control over how the mesh is generated. The power of parallel processing is automatically used to reduce the time for mesh generation [10,11].

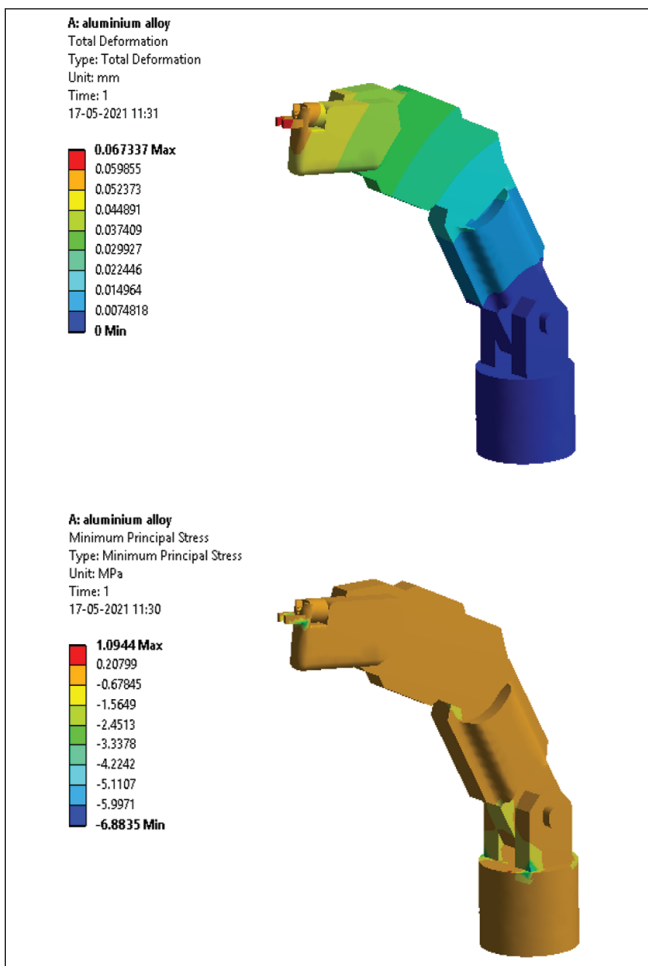
### 4.2 Results and Discussion

1. Minimum principal stress
2. Equivalent stress
3. Total deformation
4. Minimum principal stress





**Figure 7:** Maximum Principal Stress is 6.065 MPa and Equivalent stress is 7.889 MPa for applied load condition

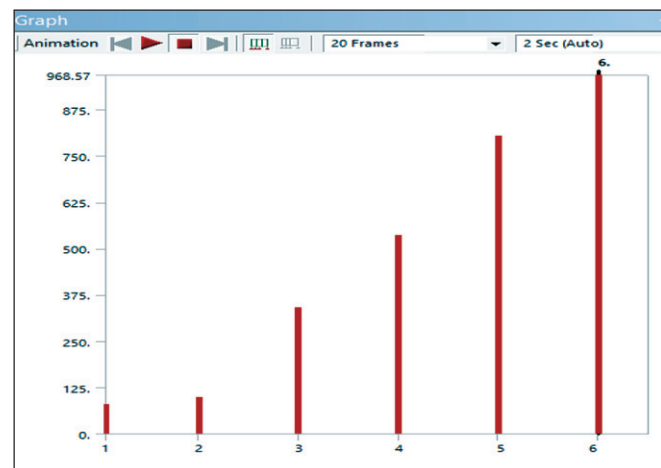


**Figure 8:** Total Deformation is 0.067337MPa and Minimum Principal Stress is 1.0944MPa for applied load condition

## 5.0 Dynamic Analysis (Model Analysis)

In order to get a theoretical model of the dynamic behaviour of the structure under test, we must first find the observed recurrence response functions, and then break them down into more manageable pieces. The experimental modal analysis section of this modal test is termed, in fact that this term is frequently wrongly used for the test. This accomplishment of this information analysis continues in two stages:

1. Identifying the proper model (with thick or structural damping). This option is reduced by programming and being used for the modal analysis. Many of software bundles work with one kind of damping and give no result to the client.
2. Determining model fitting parameters, at this stage, the measured recurrent reaction capabilities are fit to the theoretical expressions to complete the process of modal parameters extraction [12-13].



**Figure 9:** Above Figure Shows Initial Six Modes and Corresponding Frequencies

Tabular Data		
	Mode	<input checked="" type="checkbox"/> Frequency [Hz]
1	1.	81.011
2	2.	98.57
3	3.	340.06
4	4.	535.72
5	5.	805.02
6	6.	968.57

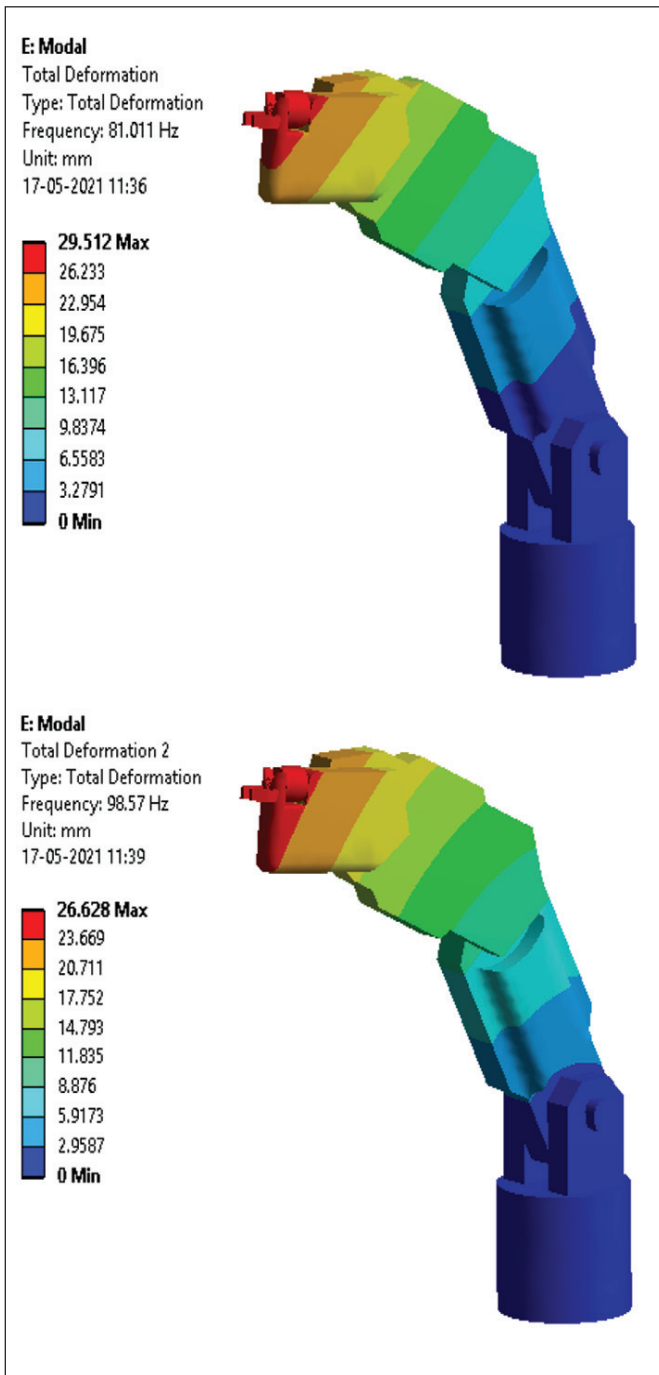


Figure 10: 1 and 2 mode and its corresponding natural frequency

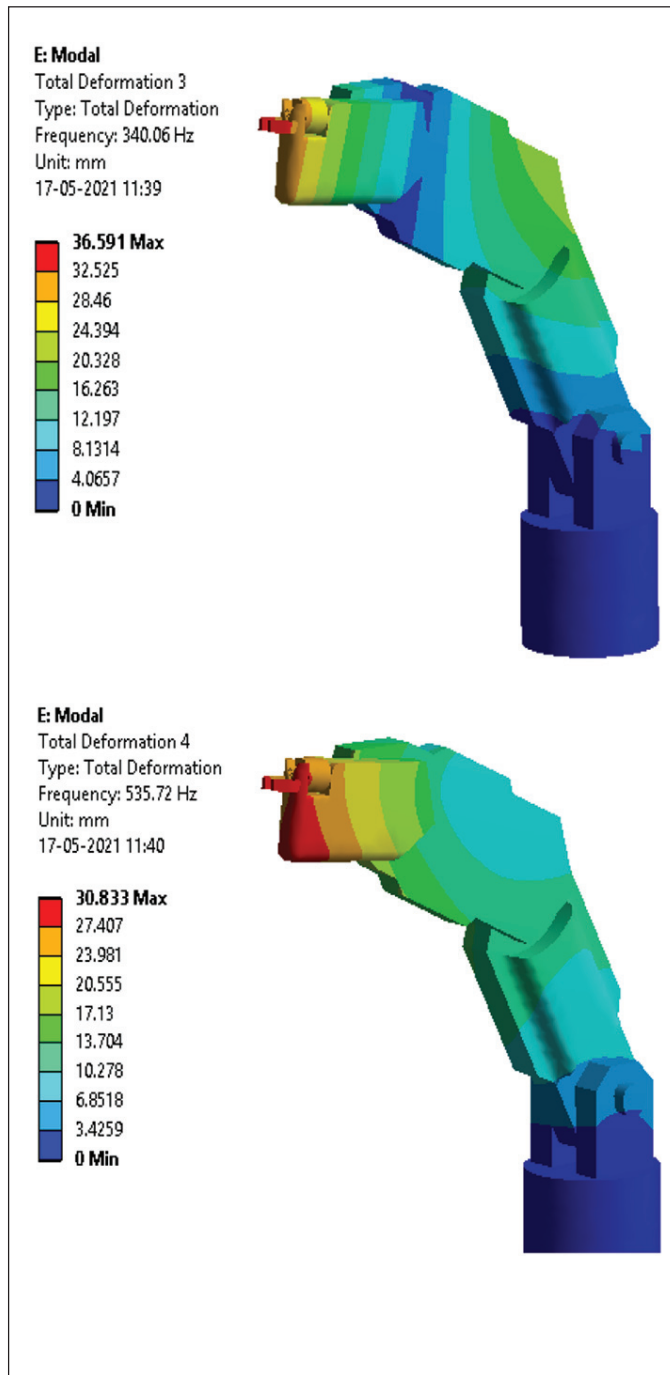


Figure 11: 3 and 4 mode and its corresponding natural frequency

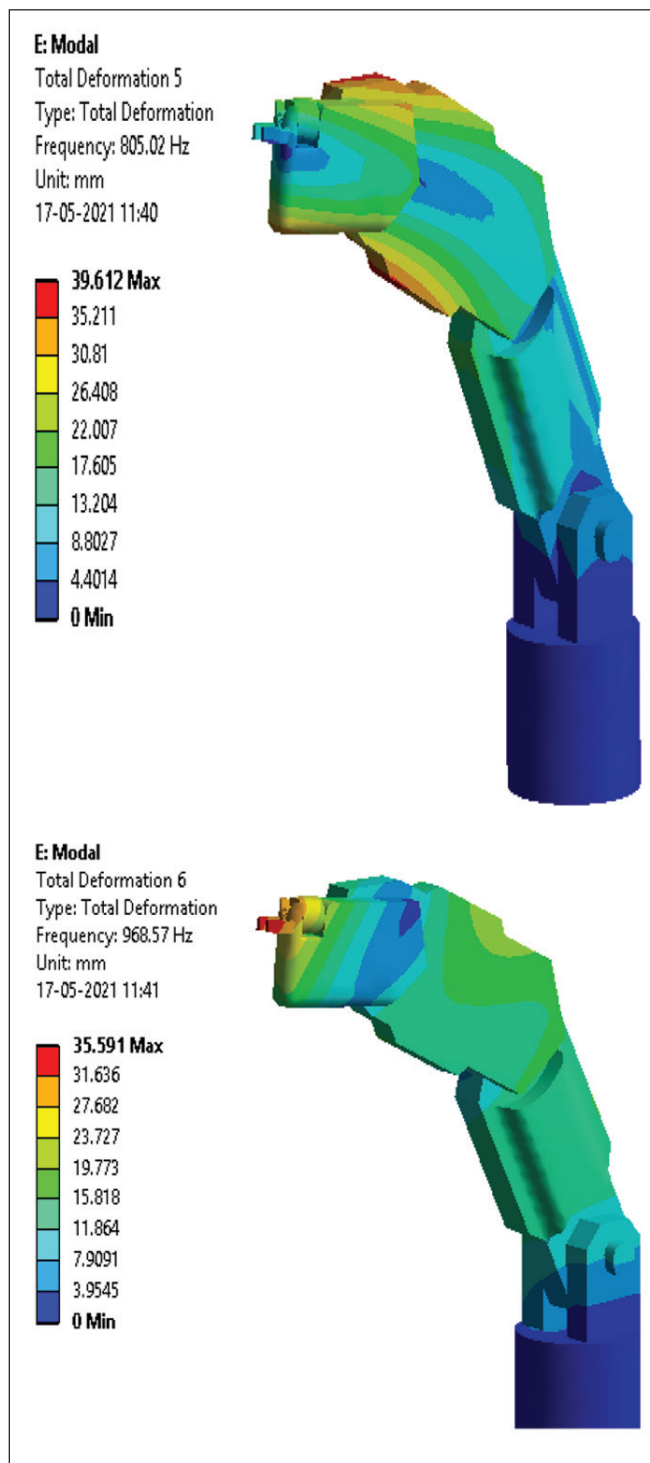


Figure 12: 5 and 6 mode and its corresponding natural frequency

## 6.0 Conclusions

Linear analysis is performed to determine stress and total deformation for materials like aluminium alloy, structural steel and CFRP material. The stresses obtained from the analysis are 12.9 Mpa for structural steel, 7.88 Mpa for aluminium, 5.6 Mpa for CFRP. Total deformation obtained from the analysis is 0.0883 mm for structural steel, 0.0673 mm for aluminium and 0.0321 mm for CFRP material. CFRP material is found to be having less equivalent stress comparatively. CFRP is also found to have less weight compared to other materials used in analysis. Due to minimum weight and stress, performance of the robot i.e., arm is expected to improve. Dynamic analysis is performed to determine six initial modes and corresponding natural frequency. The results obtained from dynamic analysis are 81.011Hz for 1st mode, 98.57Hz for 2nd mode, 340.06Hz for 3rd mode, 535.72Hz for 4th mode, 805.02Hz for 5th mode and 968.57Hz for 6th mode. Life estimated for the robotic arm is found to be  $10^6$  cycle based on the validation with analysis.

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