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Effect of Magnetic Interaction Parameter and the Electrical Conductivity of the Fluid on the Stability of Hydromagnetic Channel Flow

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Abstract

The influence of magnetic interaction parameter and conductivity of the fluid on the stability against small perturbations on the streamlined base flow between two infinitely long parallel fixed plates is studied numerically. By normal mode analysis, the disturbance equations are reduced to Orr-Sommerfeld-type. Using the energy method, sufficient conditions for stability are derived by using the nature of the growth rate and sufficiently small values of the Reynold numbers. The disturbance equations are then solved using the Galerkin method corresponding to the base functions as Legendre-polynomials. Critical values for the Reynolds number, wave number, and speed of the wave are computed for various ranges of the magnetic interaction parameter and the magnetic Reynolds number . The curves of neutral stability are presented for different values of the nondimensional parameters that appeared in this study. The stability analysis is also discussed with the help of the plots of the rate of growth of disturbances for several values of the electrical conductivity and the magnetic interaction parameter. It is observed that both the fluid conductivity and the magnetic interaction parameter have direct control over fluctuations in the system. The results of this study are accurate and are comparable with the existing literature in the absence of a parallel magnetic field.

Keywords: *Conductivity, Eigenvalue Problem, Energy Method, Galerkin Method, Hydromagnetic, Instability*

1.0 Introduction

In the recent times hydrodynamic and hydromagnetic stability analysis with some constraint is one of the very important area to do the research because of its vast applications in the field of engineering. One can find the applications in the field of active flow control and its impact on many real-world problems. Equally thrilling applications can also be found from petroleum engineering, industrial process control, and even in the field of biomedical engineering.

The analysis of the growth rate of magnetohydrodynamic shear flows is an important and classical type of problem in fluid mechanics, several authors[1-5] studied extensively this topic. The stability analysis of the flow of MHD fluid between infinite parallel walls is investigated theoretically by Lock[5]. He took the problem of stability when a constant magnetic field is introduced normally to the parallel plates externally. Before solving the problem, he further simplified the resulting stability equation by considering Prandtl number is less for conducting liquids. He derived the relationship between the *Ha* and Re_c , showed that Re_c becomes greater than 10^6 when *Ha* is greater than 20. Later, Hains[6] examined the similar type of problem to find the impact of an externally applied continual magnetic field strength on the onset

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of instability of a conducting fluid flow between parallel walls. A similar type of problem considered by Lock was reconsidered by Potter and Kutchey[7], they showed that by increasing the Prandtl number we can control the instability of the fluid.

Orszag[8] calculated an exact solution for the Orr-Sommerfeld eigenvalue problem by the Chebyshev collocation method. This solution is the benchmark for most research problems even today. The problem of Potter and Kutchey was reconsidered by Takashima[9] with the appropriate boundary condition on both velocity and magnetic stream and examined the validity of Locks simplification. He determined accurately how the critical Reynolds number varies directly with reference to the Hartmann number. Basavaraj[10-14] investigated the influence of the porosity of the porous media and uniform vertical magnetic field on the fluid using the energy method. A variety of models have been used to explain the Newtonian/non-Newtonian behaviour of fluids and their applications[15-24].

As per the authors knowledge, not much work has been carried out on the effect of a magnetic field in a perpendicular direction on the stability of the hydrodynamic horizontal shear flow. Using the linear stability analysis, the magnetohydrodynamic instability of the constant streamline flow corresponding to a parallel magnetic field is studied by Aruna, Basavaraj *et al.*[25] studied the influence of temperature on the fluid flow for different viscosity for cellular convection of finite amplitude. Basavaraj *et al.*[26] discussed the influence of the both permeability of the porous media and the magnetic field parallelly on the stability of the modified plane Poiseuille flow. Girinath Reddy *et al.*[13] investigated the disturbances that occurred due to variable fluid characteristics on double-diffusive mixed convection for accelerating surface with chemical reaction. Basavaraj[27] inspected the problem of stability of parallel flow in a saturated porous medium when a magnetic field is applied parallel by the spectral Chebyshev method.

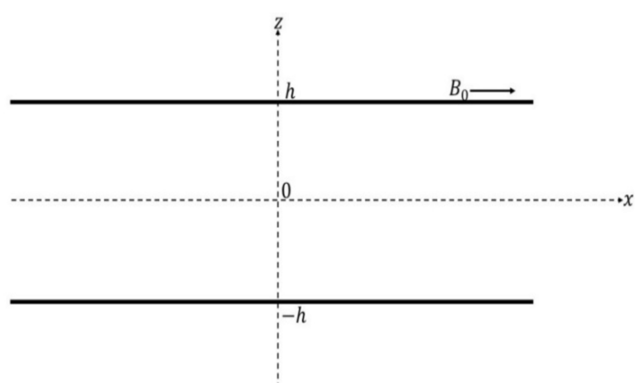

Figure 1. Physical layout of the problem.

2.0 Mathematical Formulation

Present study includes flow between infinite two parallel nonconducting fixed plates $z = h$ and $z = -h$ which are at a distance *2h* apart. The flow in this channel is electrically conducting. The rectangular coordinate system is considered with the origin O in the middle between the plates, and the flow along the *x*-axis and z-axis is vertical to the plates. A consistent magnetic field B_0 is externally applied along the fluid as shown in Figure 1.

The equations representing the electrically conducting flow between infinite, two non-conducting plates with a uniform magnetic field and with usual MHD approximations are given by adding the additional force term $\vec{J}\times\vec{B}$ in the Navier-Stokes equation, where the current density and magnetic induction are given by $\vec{J}$ and $\vec{B}$. This force term is considered when an applied electric field is absent, by $\vec{J} = \sigma\left(\vec{q}\times\vec{B}\right)$, where σ is the electrical conductivity and $\vec{q}$ is the velocity of the fluid. Hence the equations representing the system in the non-dimensional form are:

$$\frac{\partial\vec{B}}{\partial t} + (\vec{V}\cdot\nabla)\vec{B} = (\vec{B}\cdot\nabla)\vec{V} + \frac{1}{R_m}\nabla^2\vec{B}, \tag{1}$$

$$\frac{\partial\vec{V}}{\partial t} + \left(\vec{V}\cdot\nabla\right)\vec{V} = -\nabla P + \frac{N}{R_m}(\vec{B}\cdot\nabla)\vec{B} + \frac{1}{Re}\nabla^2\vec{V}, \tag{2}$$

$$\nabla.\vec{V} = 0, \quad \nabla.\vec{B} = 0 \,. \tag{3}$$

Where $\vec{V}$ - the velocity vector, $\vec{B}$ - the magnetic field, $R_m = 4\pi\mu h u_0\sigma$ - the magnetic Reynolds number, $Re = u_0 h/\nu$ - Reynolds number, $\nu = \mu_f/\rho$ - kinematic viscosity, $N = \sigma B_0^2 h/\rho u_0$ - magnetic interaction parameter, ρ - density of the fluid, μ_f - fluid viscosity, B_0 -reference magnetic field, u_0 - reference velocity, $P = p + (\vec{B}\cdot\vec{B})/8\pi\mu$ -total pressure and h is the half channel width, μ is the magnetic permeability, σ is the electrical conductivity and t is the time. It is to be noted that here that the quantity $A^2 R_m = \sigma B_0^2 h / \rho u_0 = N$ is referred to as the magnetic interaction parameter, where A^2 is the Alfven number, when this parameter is negligibly small, equation (2) is identical to the Orr-Sommerfeld equation for ordinary Newtonian fluids.

2.1 Linear Stability Analysis

To study the linear stability of the system, the flow field variables are split into a basic state and small disturbances as

$$\vec{V}=\vec{V_b}+\vec{v}, \quad \vec{B}=\vec{B_b}+\vec{b}, \tag{4}$$

Where

$$\left.\begin{aligned} \vec{V_b} &= (U,0), & \vec{B_b} &= (B_0,\, 0) \\ \vec{v} &= (v_x, v_z), & \vec{b} &= (b_x,\, b_z) \end{aligned}\right\} \tag{5}$$

Further the two-dimensional disturbances take the variable separable form

$$(\vec{v},\, \vec{b}, P) = \{\phi,\, \psi,\, P'\}(z)\, e^{i(\alpha x - \alpha c t)}, \tag{6}$$

The assumed form of the disturbances implies a spatially periodic wave where α is a wave number in dimensionless form, which is a positive real number (α >0) in the x-direction. Where ϕ the velocity stream function, ψ is the stream function related to magnetic field, $U = 1 - z^2$ is the basic velocity, $c = c_r + ic_i$ represents complex wave speed, phase velocity is given by c_r and c_i corresponds to rate of change of growth. The flow is treated as linearly stable if $c_i < 0$, flow is linearly unstable if $c_i > 0$, and the flow is neutrally stable for $c_i = 0$. the constant horizontal magnetic field (B_0) applied externally is assumed as one. The pressure term is eliminated from the governing equation (2) and using the procedure followed by Stuart[1] yields the following disturbance equations.

$$(U(z)-c)\left(D^2-\alpha^2\right)\phi - \left(D^2U(z)\right)\phi + \frac{i}{\alpha Re}\left(D^2-\alpha^2\right)^2\phi - \frac{N}{R_m}\left(D^2-\alpha^2\right)\psi = 0 \tag{7}$$

$$\phi - \left(U(z)-c\right)\psi - \frac{i}{\alpha R_m}\left(D^2-\alpha^2\right)\psi = 0 \tag{8}$$

If the magnetic field is absent (i.e., $N = 0$), equation (7) represents ordinary Orr-Somerfield equation. The homogeneous system of equations (7) and (8), and by considering velocity is zero at the boundaries given by,

$$\phi = D\phi = \psi = 0 \quad \text{at} \quad z = \pm 1, \tag{9}$$

Using equation (8) in equation (7) we have the following homogeneous equation

$$\begin{aligned} &(U(z)-c)\left(D^2-\alpha^2\right)\phi - \left(D^2U(z)\right)\phi + \\ &\frac{i}{\alpha Re}\left(D^2-\alpha^2\right)^2\phi + i\frac{N}{R_m}\alpha\phi \\ &-iA^2\alpha R_m\left(U(z)-c\right)\psi = 0 \end{aligned} \tag{10}$$

2.2 Energy Analysis

The analytical energy method is employed to get the suitable conditions for sufficiency in terms of the growth rate and Reynolds number. Following Drazin and Ried (2004), the conditions are derived. For this multiply the entire homogeneous equation (10) by $\bar{\phi}$, the complex conjugate of ϕ, and by integrating the simplified equation with respect to z from -1 to 1, using the conditions (9), we have the following

$$\begin{aligned} &\left(I_2^2 + 2\alpha^2 I_1^2 + \alpha^4 I_0^2\right) + \alpha^2 Re\frac{N}{R_m}I_0^2 = -i\,\alpha Re\, Q + \\ &i\,c\,\alpha Re\left[\left(I_1^2 + \alpha^2 I_0^2\right) - i\,\alpha\frac{N}{R_m}\left\{\begin{matrix}(\phi_r\psi_r + \phi_i\psi_i) \\ +i(\phi_r\psi_i - \phi_i\psi_r)\end{matrix}\right\}\right] \end{aligned} \tag{11}$$

where

$$I_n^2 = \int_{-1}^{1}\left|D^n\phi\right|^2 dz \qquad (n = 0 \text{ to } 2) \tag{12}$$

$$\begin{aligned} Q &= \int_{-1}^{1}\left[U(z)\left|D\phi\right|^2 + \left(D^2U(z) + \alpha^2U(z)\right)\left|\phi\right|^2\right]dz + \\ &\int_{-1}^{1}\bar{\phi}\left(DU(z)\right)\left(D\phi\right)dz - i\alpha\frac{N}{R_m}\int_{-1}^{1}U(z)\left\{\begin{matrix}(\phi_r\psi_r + \phi_i\psi_i) + \\ i(\phi_r\psi_i - \phi_i\psi_r)\end{matrix}\right\}dz \\ &= Q_r + iQ_i \end{aligned} \tag{13}$$

$$Q_r = \mathrm{Re}(Q) = \int_{-1}^{1}\left[\begin{matrix} U(z)\,|D\phi|^2 + \\ \left(D^2+\alpha^2\right)U(z)\end{matrix}\right]|\phi|^2\,dz +$$

$$\int_{-1}^{1} DU(z)\left(\phi_r D\phi_r + \phi_i D\phi_i\right)dz + \alpha\frac{N}{R_m}\int_{-1}^{1} u_b\left(\phi_r\psi_i - \phi_i\psi_r\right)dz \tag{14}$$

$$Q_i = \mathrm{Im}(Q) = \int_{-1}^{1} DU(z)\left(\phi_r\, D\phi_i - \phi_i\, D\phi_r\right)dz - \alpha\frac{N}{R_m}\int_{-1}^{1} U(z)\left(\phi_r\psi_r + \phi_i\psi_i\right)dz. \tag{15}$$

Separating both real part and imaginary part of equation (13) and equating to zero then, we get

$$c_r = \frac{Q_r}{\left(I_1^2+\alpha^2 I_0^2\right)+\alpha^2\dfrac{N}{R_m}Re\left(\phi_r\psi_r+\phi_i\psi_i\right)}, \tag{16}$$

$$c_i = \frac{1}{M}\left[Q_i - \frac{1}{\alpha ReM}\left\{\left(I_2^2+2\alpha^2 I_1^2+\alpha^4 I_0^2\right)+\left(\alpha^2 Re\frac{N}{R_m}\right)I_0^2\right\}\right] \tag{17}$$

where

$$M = \left(I_1^2+\alpha^2 I_0^2\right) - \alpha\frac{N}{R_m}\left(\psi_i\phi_r - \phi_i\psi_r\right).$$

Equation (17) is known as energy equation for basic flow in the direction of propagation of disturbances in two-dimension. We write Equation (15) in the form,

$$\mathrm{Im}(Q) = \frac{i}{2}\int_{-1}^{1}\left\{DU(z)\left(\phi\, D\bar{\phi} - \bar{\phi}\,D\phi\right)D(U(z))\right\}dz -$$

$$i\alpha A^2 Rm\int_{-1}^{1} U(z)\psi\bar{\phi}\,dz \tag{18}$$

Observe that

$$|\mathrm{Im}(Q)| \le \left\{\int_{-1}^{1}|\phi||D\phi||DU(z)|\,dy - i\alpha\frac{N}{R_m}\int_{-1}^{1}|U(z)||\psi||\phi|\,dz\right\}$$

and using Schwarz's inequality, we get $|\mathrm{Im}(Q)| \le M'$

where

$$M' = I_1 I_0 q - \alpha\frac{N}{R_m}I_0 q',\ q' = \max_{-1<z<1}|U(z)| \text{ and}$$

$$q = \max_{-1<z<1}|DU(z)|.$$

This gives the upper bound for c_i of the form

$$c_i \le \frac{1}{\left(I_1^2+\alpha^2 I_0^2\right)+\alpha\dfrac{N}{R_m}I_0^2}\left[M' - \frac{1}{\alpha Re}\left\{\left(I_2^2+2\alpha^2 I_1^2+\alpha^4 I_0^2\right)+\left(\alpha^2 Re\frac{N}{R_m}\right)I_0^2\right\}\right] \tag{19}$$

From above equation we have sufficient condition for stability is

$$Re < \frac{1}{\alpha M'}\left\{\left(I_2^2+2\alpha^2 I_1^2+\alpha^4 I_0^2\right)+\left(\alpha^2 Re\frac{N}{R_m}\right)I_0^2\right\}. \tag{20}$$

2.3 Numerical Solution to the Problem

The linear system of homogeneous differential equations (7) and (8) along with the boundary conditions (9) formulate the generalized eigenvalue problem. Solution of this generalized eigen value problem is carried out by Galerkin method with basis function as Legendre polynomials, $P_n(z)$. Accordingly, $\phi(z)$ and $\psi(z)$ are described with the Legendre polynomials as

$$\phi(z) = \sum_{n=0}^{M} A_n\chi_n(z),\ \psi(z) = \sum_{n=0}^{M} B_n\zeta_n(z) \tag{21}$$

with the corresponding base polynomials $\chi_n(z) = (1-z^2)^2\, P_n(z)$, $\zeta_n(z) = (1-z^2)\, P_n(z)$. Here the degree of the Legendre polynomial is represented n and, A_n & B_n are being constant coefficients. Where $\psi(z)$ and $\phi(z)$ satisfies the boundary conditions. Equations (7) and (8) using the Equation (21) and the net error is necessary to be perpendicular to $\chi_m(z)$ and $\zeta_m(z)$ meant for m = 0, 1, 2,....... M, this implies

$$\frac{1}{Re}\sum_{n=0}^{M} A_n\int_{-1}^{1}\left(\xi''\xi'' + 2\alpha^2\xi'\xi' + \alpha^4\xi\,\xi\right)dz + i\alpha\sum_{n=0}^{M} A_n\int_{-1}^{1}\left(\begin{matrix} D^2U(z)\xi_n\xi_m + \alpha^2 U(z)\xi_n\xi_m \\ -U(z)\xi_n''\xi_m\end{matrix}\right)dz$$

$$-i\alpha\frac{N}{R_m}\sum_{n=0}^{M} B_n\left(\zeta_n'\xi_m' + \alpha^2\zeta_n\xi_m\right)dz = -i\alpha c\sum_{n=0}^{M} A_n\int_{-1}^{1}\left(\xi_n'\xi_m' + \alpha^2\xi_n\xi_m\right)dz \tag{22}$$

$$i\alpha\sum_{n=0}^{M} A_n \int_{-1}^{1} \chi_n \zeta_m dz - \frac{1}{R_m}\sum_{n=0}^{M} B_n \int_{-1}^{1} \left(\zeta_n^{'}\zeta_m^{'} + \alpha^2 \zeta_n \zeta_m\right) dz - i\alpha\sum_{n=0}^{M} B_n \int_{-1}^{1} U(z)\zeta_n \zeta_m dz = -i\alpha c\sum_{n=0}^{M} B_n \int_{-1}^{1} \zeta_n \zeta_m dz \qquad (23)$$

Equations (22) and (23) reduces to following system of linear equations:

$$\Omega_1 X = c\,\Omega_2 X \qquad (24)$$

where Ω_1 and Ω_2 are the 2(N+1)th order complex matrices, the eigenvalue and corresponding eigenvectors in discrete form are represented by c and X. When both R_m and N are fixed, the values of c gives a non-trivial

Table 1. Eigenvalue convergence

M	N=0.2, *Rm* =0.1, *Re* =8000, & α=1	N=0.6, *Rm* = 0.3, *Re* =30000 & α=0.8
	$c = c_r + ic_i$	$c = c_r + ic_i$
5	0.33768358258214176+0.0067332251723915*i*	0.817483614433903 -0.004259465778156 *i*
1 0	0.2425018654901316 +0.0130781122069553 *i*	0.184067718123462+0.011379812679697 *i*
2 0	0.2385557709405035 +0.0059548912540639 *i*	0.166632513656469+0.002764796155647 *i*
3 0	0.2375828537277001+0.0032274953460328 *i*	0.170165997107799 +0.000437130388416 *i*
4 0	0.2374735460747337+0.0031638101715215 *i*	0.1696087730670 +0.00032858773455456 *i*
5 0	0.2374734716391618+0.0031626179970966 *i*	0.169623734717128 +0.000358684872481 *i*
6 0	0.2374734758663112+0.0031626196396392 *i*	0.16962473086253144+0.00035842938683*i*
7 0	0.2374734766220434+0.0031626194474267 *i*	0.169624782389621+0.000358409668283 *i*
8 0	0.237473477730665+0.00316261850084343 *i*	0.16962480943125 +0.0003584030998372 *i*
9 0	0.237473425340541+0.00316262061212312 *i*	0.16962482796911+0.0003583986622785 *i*

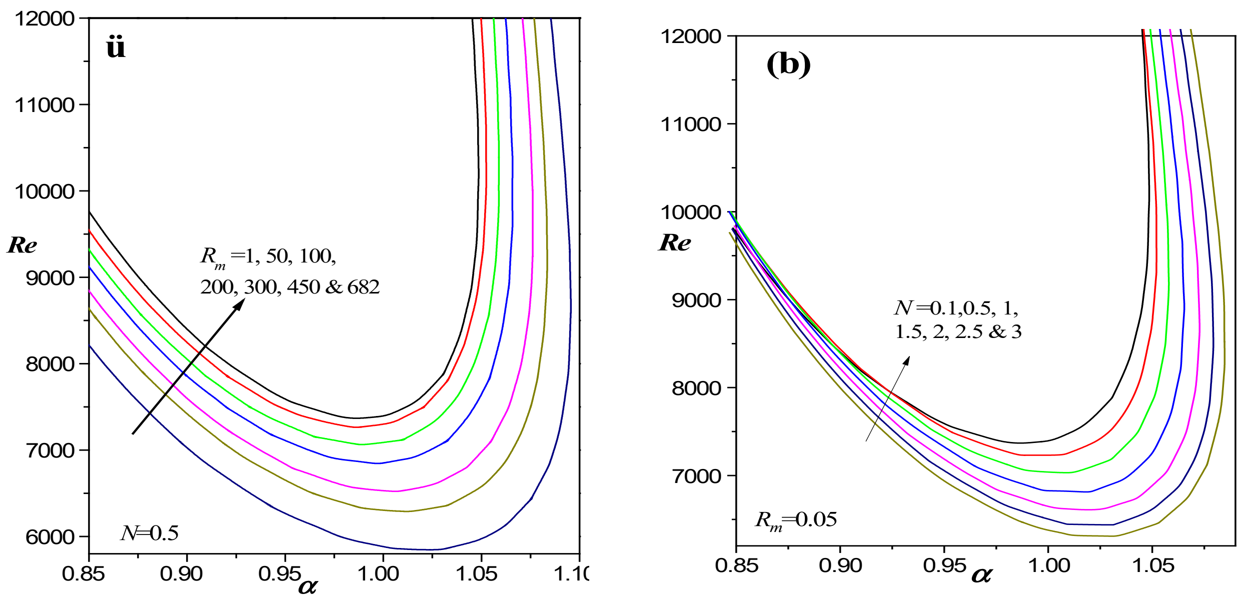

Figure 2. (a and b) Marginal stability curves.

solution of Equation (27) which is obtained by QZ algorithm. The critical parameter values of c_c, Re_c and α_c are calculated for many values of R_m.

3.0 Result and Analysis

The primary objective of present discussion is to know the effect of magnetic interaction parameter on the behavior of the fluid flow when horizontal magnetic field is applied. Using the energy method, we derived the criteria to show that the sufficient condition on the stability of the initial flow against small disturbances in terms of change

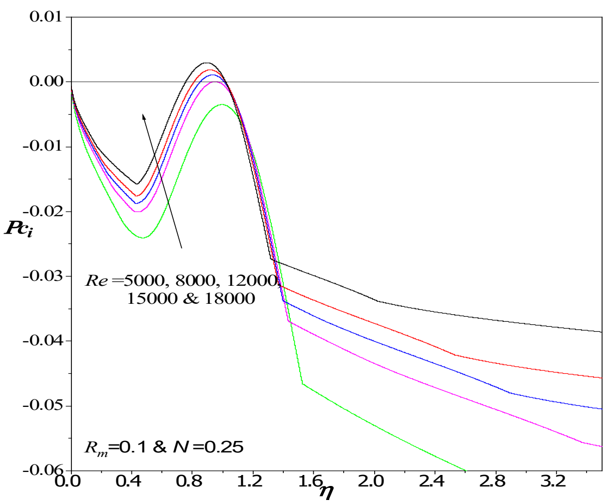

Figure 3. Variation of rate of growth of the disturbances ( ) versus the wave number (α) for different values of the Reynolds number for fixed magnetic Reynolds number (R_m) and magnetic interaction parameter (N).

of growth rate as well as the Reynolds number. Using Galerkin method, modified Orr-Sommerfeld equation is numerically solved using Legendre's based polynomials.

The parameters under the study are the magnetic Reynolds number *Rm*, Magnetic interaction parameter N and the Reynolds number *Re*. The numerical method is verified for different set of numerical of the parameters by varying order of Chebyshev polynomial *M* and convergence of corresponding eigen values is displayed in Table 1. It is clear from Table 1, that eigen values are compatible and till eight digits accuracy is established.

Figure 2, displays curves of stability for different values of magnetic Reynolds number*Rm*, magnetic interaction parameter *N*. The area below individual neutral curve corresponds to the stable region and the area above each neutral curve corresponds to unstable region. Figure 2(a) presents the neutral curves for different values of the magnetic Reynolds number *Rm* . Again, from this figure one can observe that the critical wave number α_c is weakly a decreasing function of *Rm*, whereas the critical Reynolds number is directly proportional to *Rm*. Hence increasing the values of *Rm* leads to resist the flow and hence the system becomes stable.

The reason is, when the values of the *Rm* is increased, the Reynolds stresses makes the system becomes stable. Figure 2(b) shows effect of *N* on the neutral stability and from this graph it is clear that the region of stability constantly increases when the parameter *N* increased considerably. As increasing conductivity parameter, decreases the instability region and hence makes the

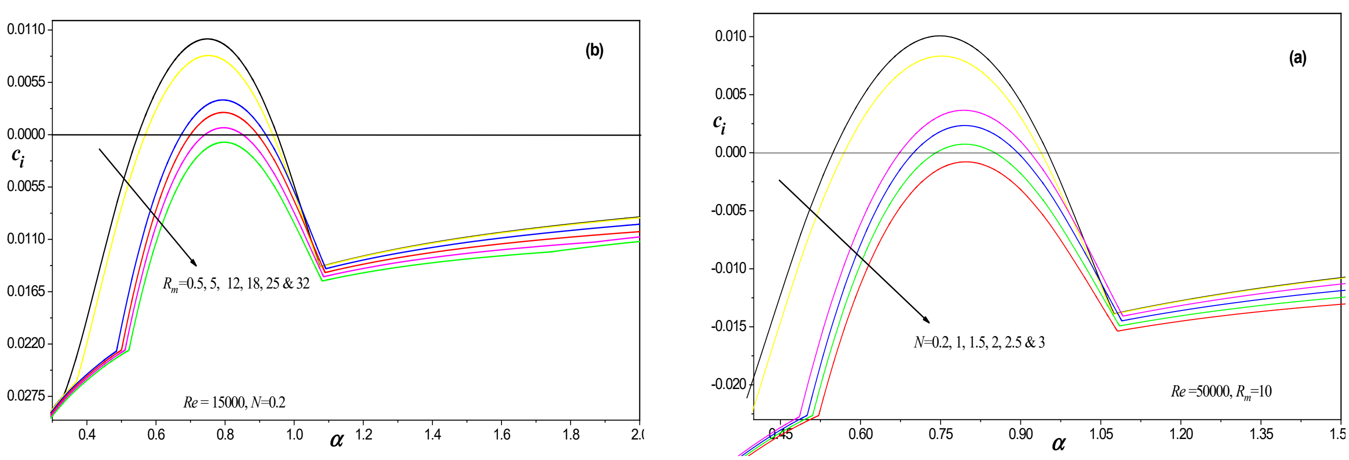

Figure 4. Variation of Growth rate versus wave number for different values of the magnetic interaction parameter when $R_m = 10$ and $Re = 50000$.

system more stable. The Maxwell's stress present in the system is responsible to make the system stable.

The effect of *N* and *Rm*, on the stability of the flow in terms of growth rate is presented in Figure 3 for different values of the Reynolds number *Re* = 8000, 10000, 12000 and 14000. From these figures it is evident that by upgrading the values of the *N* and *Rm*, weakness the growth of the small perturbations on the basic parallel flow. The inertial force is insignificant in the system suppress the growth of unstable modes and this can be observed.

The changes in the growth rate of disturbances versus the wave number for various values of the magnetic interaction parameter and the conductivity of the fluid is shown from Figure 4(a)-(b). From Figure 4(a),it is observed that, *Ci* is inversely proportional to magnetic interaction parameter, *N*, for fixed values of *Re* = 50000 and *Rm* = 10. This figure clearly visualizes that magnetic field strength dampens the rate of growth of small disturbances and this leads to more stable system. Form Figure 4(b), one can see that *Ci* is decreasing function of the conductivity parameter, m R , for fixed values of *Re* = 15000 and N = 0.2 . ItIt is clear that growth rate of disturbances is decreased when the conductivity of the fluid parameter increased from 0.5, 12, 18, 25 and 32. This is due to the reason that the Maxwell stress increases when the magnetic Reynolds number increases gradually.

4.0 Conclusion

The behavior of an electrically conducting fluid flow between parallel infinite non-conducting two plates is examined with constant longitudinal magnetic field for two dimensional disturbances. The main intention of the problem is to find the role of magnetic interaction parameter and the conductivity of the fluid on the stability of the MHD fluid. From the investigations we found that critical Reynolds number is dependent on both the parameters considered in the problem. Using the energy method, the stability criterions in terms of the growth rate as well as Reynolds umber is derived. From the study we observed, the critical Reynolds number is directly proportional to both parameters, magnetic interaction number *N* and magnetic Reynolds number *Rm*. Magnetic interaction parameter has a major role on the growth rate of the fluid flow, it has stabilizing effect on the flow. *Re* is an increasing function of the magnetic interaction parameter *N*. The critical wave speed is inversely proportional to the magnetic interaction parameter and critical wave number is also inversely proportional to magnetic interaction parameter *N*. The rate of change of disturbances becomes damped when magnetic interaction parameter is increased appropriately.

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