

# Effect of Diffusion-Thermal on Mixed Convective Casson Fluid flow in a Porous Channel

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## Abstract

The main purpose is to study analytically about the Diffusion-thermo impact on mixed convective flow of Casson fluid in a vertical channel in occurrence of porous media, uniform magnetic field and amplification. Similarity transformation is implemented to transform nonlinear coupled PDEs into ODEs. Further, obtained equations were solved using perturbation technique and studied the characters of heat, velocity and concentration of the corporeal system. The influence of non-dimensional factors such as Darcy number  $Da$ , buoyancy parameter of concentration  $N$ ,  $M^2$  Hartmann number, dufour number  $df$ , rate of chemical reaction  $\gamma$ , Schmidt number  $Sc$ , thermal buoyancy parameter  $\lambda$ , Prandtl number  $Pr$ , Casson parameter  $\beta$ , and Reynolds number  $R$  on concentration, temperature and velocity deliberated explicitly. Few important computational work reveals that the Dufour effect  $Df$  enhances the concentration, temperature and fluid flow whereas Casson fluid parameter  $\beta$  diminishes the profiles. The earlier work and present work have been compared for a particular case in the nonexistence of Dufour effect and porous media and were found to be coinciding.

**Keywords:** Casson Fluid, Dufour Effect, Perturbation Technique, Porous Media.

## 1.0 Introduction

In recent trends, non-Newtonian fluids are found in biological systems, irrigation issues, heat-storage beds, petroleum processing, textile, paper, and polymer composite industries. In particularly, Casson fluid in a channel has received more consideration of researchers due to its applications in polymer processing industries, biomechanics and Engineering. Casson<sup>1</sup> introduced the Casson fluid. In general, Dufour effect plays very important role to transfer heat. Diffusion-thermo or Dufour effect arises by concentration gradient while transferring the heat. On the other hand, when

a concentration gradient arised by isothermal fluid mixture, non-equilibrium thermodynamics forecasts the temperature gradient which develops the diffusion. In latest trends, the non-Newtonian fluids flow such as Casson in a channel has received more consideration of scholars due to its applications in polymer processing industries, biomechanics and Engineering. Casson<sup>1</sup> introduced the Casson fluid. In general, Dufour effect plays very important role to transfer heat. Diffusion-thermo or Dufour effect arises by concentration gradient while transferring the heat. On the other hand, when a concentration gradient arised by isothermal fluid mixture, non-equilibrium thermodynamics forecasts the

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temperature gradient which develops the diffusion. This singularity is known as Dufour effect according to<sup>2-6</sup>. Rastogi *et al.*<sup>7</sup> studied the Dufour effect on liquids such as Benzene and chloro-benzene system. Mason *et al.*<sup>8</sup> investigated the diffusion thermo effect on gases such as H<sub>2</sub>-Ar and H<sub>2</sub>-CO<sub>2</sub> mixtures in presence of pressure.

Rastogi *et al.*<sup>9</sup> investigated the Dufour effect on low heat generating mixtures of liquids by mixing different mixtures of benzene and estimated the Dufour co-efficient. Sara *et al.*<sup>10</sup> determined that the effect of Dufour could be measured in a cell which has well defined geometry. Richard *et al.*<sup>11</sup> considered Dufour experiments to study the heat transport in liquid mixtures. Hort *et al.*<sup>12</sup> analysed the impact of Dufour number on binary gas mixtures. Adrian studied numerically about transfer of mass and heat in existence of Dufour and Soret effects<sup>13</sup>. Adrian examined the influence of reaction of chemicals to transfer mass and heat over a porous medium in existence of Soret- Dufour number<sup>14</sup>. García-Colín *et al.* analysed analytically about the transport co-efficient based on Boltzmann equation in presence of Soret-Dufour effect<sup>15</sup>. Anwar *et al.* analyzed convection of Newtonian, Boussinesq fluid on enlarging outward by considering the Soret-Dufour number<sup>16</sup>. Hossein *et al.* theoretically analysed transfer of heat in nanofluids, in existence of Dufour impact<sup>17</sup>.

Basant *et al.* in calculated influence of Dufour effect to heat mass transfer on transient natural convective fully developed channel bounded by plates<sup>18</sup>. Nadeema *et al.* investigated MHD flow of Casson fluid on exponentially lessening sheet<sup>19</sup>. Hayat *et al.* numerically inspected about the peristaltic flow of mixed convective flow of nanofluid in existence of Soret- Dufour effect<sup>20</sup>. Animasaun *et al.* implemented HAM to study MHD Casson fluid flow on exponentially extending surface<sup>21</sup>. Srinivasacharya *et al.* numerically examined influence of Soret- Dufour number on convective transfer flow of mass and heat along the wavy surface<sup>22</sup>. Kiran *et al.* considered the Diffusion-thermo effect on unsteady natural convective incompressible, chemically reacting, radiating and viscous fluid flow on a vertical plate and stated that fluid flow with Dufour effect and temperature are significant close to the plate<sup>23</sup>.

Krupa Lakshmi *et al.* approached numerically and investigated transfer of heat and mass on boundary layer laminar flow of two-phase particulate suspension along the linearly stretching sheet in occurrence of thermal

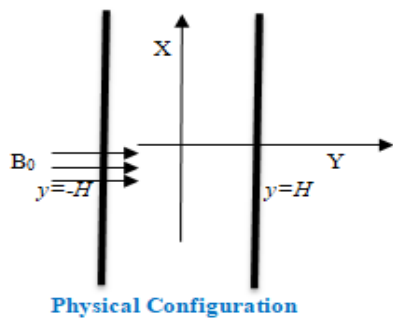
radiation, diffusion-thermo, magnetic field, chemical reaction of first order and thermal-diffusion<sup>24</sup>. Alao *et al.* considered the fluid flow over the moving vertical infinite plate under the impact of thermo-physical characters of fluid, Dufour and Soret effects<sup>25</sup>. Sravanthi inspected 2D boundary layer, steady, MHD convective viscous fluid flow over the exponentially stretching inclined porous media with Dufour-Soret number<sup>26</sup>. Mahanthesh *et al.* analysed the role of Soret-Dufour number on 3D steady flow of nanofluid on non-linear extending sheet in existence of non-linear thermal radiation<sup>27</sup>. Manideep *et al.* analysed natural convection flow on a plate in occurrence of porous media, constant temperature at walls with oscillating velocity<sup>28</sup>. Implemented the finite element method to solve the simultaneous algebraic linear equations which have been obtained from governing equations. Irfan considered Soret and Dufour number impact on irregular square cavity for mass and heat transmission<sup>29</sup>. Idowu *et al.* examined numerically using spectral relaxation method about the Soret-Dufour effects on MHD viscoelastic fluid<sup>30</sup>. Khuram Rafique *et al.* considered the Casson nanofluid boundary layer flow to study the impact of Soret and Dufour number over the inclined channel, in which Killer Box scheme have been implemented to obtain dimensionless parameters<sup>31</sup>.

Kaladhar *et al.* inspected slip parameter, chemical reaction, Soret-Dufour effect on incompressible, laminar, convective flow of Newtonian fluid in an annulus with convective slip boundary conditions<sup>32</sup>. Sohail *et al.* examined diffusion-thermo and thermo diffusion impact on electrically conducting convective viscous fluid flow over a rotating cone<sup>33</sup>. Mojeed *et al.* analysed influence of multi-slip, Dufour and Soret effect on MHD Casson flow of fluid over a slandering stretching sheet<sup>34</sup>. Idowu *et al.* deliberate thermophoresis, Soret-Dufour number effect on MHD flow on an tending plate<sup>35</sup>. Timothy *et al.* observed the effect of dissipation of Casson nanofluid motion over the plate in existence of magnetic field, Soret-Dufour effect<sup>36</sup>. Sowbhagya considered Forchheimer-Darcy model to analyse impact of onset of porous convection using cubic density relationship<sup>37</sup>. Galerkin technique is implemented to explain stability eigenvalue problem. Vijaya Kumara *et al.* studied numerically about impact of dissimilar Rayleigh numbers by considering free convection in a trapezoidal porous enclosure with heating wall at the lower wall<sup>38</sup>.

With all the above literature studies and as per our knowledge it has been noticed that the most of the work is carried out numerically to study Soret and Dufour effect on different channels or surfaces to transfer heat and mass. The main intention of the paper is to inspect analytically about the influence of Dufour number on forced and free convection flow of a Casson fluid in a vertical channel by taking an account of amplification, porous media and uniform magnetic field by implementing the method called perturbation. Further, the effects of dimensionless parameters have been discussed graphically on thickness, flow and heat of the fluid.

## 2.0 Formulation of the Problem

A steady, incompressible, laminar flow of Casson fluid in a vertical channel in occurrence of Diffusion-thermo effect, porous media, amplification and uniform magnetic field  $B_0$  to be considered as a physical configuration of the system. The physical model consists of axisymmetric flow of the fluid with one wall of the channel is placed at  $y = H$  and the other is at  $y = -H$ . The flow of the channel along  $x$ -axis and  $y$ -axis is perpendicular to it. The fluid is led at  $y = +H$  and is pulled out at  $y = -H$  with constant velocity  $V$ .



The constitutive equations for above system are expressed as follows.

Equation of Continuity:

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0, \quad (1)$$

Momentum equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + v \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\mu_p}{k_2} u - \frac{\sigma B_0^2 u}{\rho} - \left[ \frac{g[\beta_c(C - C_2) + \beta_T(T - T_2)]}{\rho} \right] \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - \frac{\mu_p}{k_2} v + v \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 v}{\partial x^2} \quad (3)$$

Equation of energy:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{D_m K_T}{C_s C_p} \frac{\partial^2 C}{\partial y^2}, \quad (4)$$

Concentration equation:

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - Ck_1, \quad (5)$$

where velocity components are  $u$  and  $v$  in  $x$  and  $y$  direction,  $B_0$  is uniform magnetic field,  $\sigma$  is electrical conductivity,  $\rho$  is fluid density,  $p$  is pressure,  $T$  is fluid temperature,  $\nu$  is kinematic viscosity,  $\beta$  is Casson fluid parameter,  $\beta_T$  is thermal expansion coefficient,  $g$  is acceleration due to gravity,  $C_p$  is the specific heat at constant pressure,  $\mu$  is dynamic viscosity,  $\beta_c$  is concentration expansion coefficient,  $D_m$  is mass diffusion,  $k_1$  is rate of chemical reaction,  $k$  is thermal conductivity, permeability of the medium is  $k_2$  and  $C$  is concentration field.

From (2) and (3), we get

$$\begin{aligned} & u \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + v \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} \frac{\partial v}{\partial y} - u \frac{\partial^2 v}{\partial x^2} - \frac{\partial v}{\partial x} \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \frac{\partial v}{\partial x} - v \frac{\partial^2 v}{\partial x \partial y} \\ & = v \left( \frac{1}{\beta} + 1 \right) \left( \frac{\partial^3 u}{\partial y^3} - \frac{\partial^3 v}{\partial x^3} \right) - \left( \frac{\sigma B_0^2}{\rho} + \frac{\mu_p}{k} \right) \frac{\partial u}{\partial y} + \frac{\mu_p}{k} \frac{\partial v}{\partial x} \\ & \pm \frac{\partial g}{\partial y} [\beta_c(C - C_2) + \beta_T(T - T_2)] \end{aligned} \quad (6)$$

### 2.1 Boundary Conditions

The Casson fluid flow in a vertical channel is axisymmetric and the following boundary conditions are considered for the physical configuration.

$$v = \frac{V}{2}, u = 0, T = T_2, C = C_2 \text{ at } y = H,$$

$$v = 0, \frac{\partial u}{\partial y} = 0, T = T_1, C = C_1, \text{ at } y = 0. \quad (7)$$

### 2.2 Non - Dimensionalisation

Equation (4), (5) and (6) are non-dimensionalized by using the below mentioned parameters to get dimensionless equations

$$x^* = \frac{x}{H}, v = aVf(y^*), y^* = \frac{y}{H}, \theta(y^*) = \frac{T - T_2}{T_1 - T_2},$$

$$\phi(y^*) = \frac{C - C_2}{C_1 - C_2}, u = -v x^* f'(y^*).$$

(8)

Equation (4), (5) and (6) subjected to (8) to get below equations,

$$\phi'' - Sc \gamma (\phi + b) - a Sc \phi' f = 0 \tag{9}$$

$$\theta'' - a f Pr \theta' + Df Pr \phi'' = 0 \tag{10}$$

$$\left(1 + \frac{1}{\beta}\right) f'' - (M^2 + Da) f'' + \lambda[\theta' + N\phi']$$

$$+ R[(2 - a)f''f' - a f'''f] = 0 \tag{11}$$

Where  $b = \frac{C_2}{C_1 - C_2}$ ,  $\lambda = \frac{Gr_x}{R^2}$  is the thermal buoyancy parameter,

$M^2 = \frac{\sigma B_0^2 H^2}{\mu}$  is Hartmann number,

$R = \frac{VH}{\nu}$  is Reynolds number,

$Gr_x = \frac{VH^4 g \beta_T (T_1 - T_2)}{\nu^3}$  is Grashof number,

$N = \frac{\beta_C (C_1 - C_2)}{\beta_T (T_1 - T_2)}$  is buoyancy parameter of concentration,

$Sc = \frac{HV}{D}$  is Schmidt number,

$Pr = \frac{\rho C_p HV}{k}$  is prandtl number,

$Da = \frac{H^2 \mu_p}{K \nu}$  is Darcy number,

$Df = \frac{K_T D_m (C_1 - C_2)}{C_s (T_1 - T_2) C_p V H}$  is Dufour effect and

chemical reaction rate is  $\gamma = k_1 H/\nu$ .

The boundary conditions after non-dimensionalization reduces to below mentioned form.

$$f(1) = \frac{1}{2}, \theta(1) = 0, f''(0) = 0, \phi(1) = 0,$$

$$\phi(0) = 1, f'(1) = 0, f(0) = 0, \theta(0) = 1, \tag{12}$$

### 2.3 Perturbation Technique

A classical perturbation technique is employed to solve (9), (10) and (11), a single perturbation of below mentioned terms are considered by neglecting the higher order terms of the series for concentration  $\phi$ , temperature  $\theta$ , velocity  $f$  with perturbation parameter 'ε' is taken for very small values.

$$\phi = \phi_0 + \epsilon * \phi_1, \tag{13}$$

$$\theta = \theta_0 + \epsilon * \theta_1, \tag{14}$$

$$f = f_0 + \epsilon * f_1, \tag{15}$$

Perturbation technique is implemented to solve (11), (10) and (9) by using (15), (14) and (13) respectively gives the below mentioned concentration, temperature and velocity equations.

$$\phi = d_2 e^{d_1 y} + d_3 e^{-d_1 y} - b + \epsilon(d_{18} e^{d_1 y} + d_{19} e^{-d_1 y}$$

$$+ d_{43} y e^{d_1 y} + d_{44} y e^{-d_1 y} + d_{45} y^2 e^{d_1 y} + d_{46} y^2 e^{-d_1 y} +$$

$$d_{36} + d_{30} e^{d_{20} y} + d_{31} e^{d_{21} y} + d_{32} e^{d_{22} y} + d_{33} e^{d_{23} y}$$

$$+ d_{34} e^{d_{24} y} + d_{35} e^{-d_{21} y} + d_{37} y^3 e^{d_1 y} + d_{40} y^3 e^{-d_1 y})$$

$$\tag{16}$$

$$\theta = d_4 + d_5 y + \epsilon(d_{61} + d_{62} y + d_{76} y^2 + d_{103} y^3 + d_{106} y^5$$

$$+ d_{107} e^{d_1 y} + d_{108} e^{-d_1 y} + d_{109} y e^{d_1 y} + d_{110} y e^{-d_1 y}$$

$$+ d_{111} y^2 e^{d_1 y} + d_{112} y^2 e^{-d_1 y} + d_{89} e^{d_{21} y} + d_{90} e^{-d_{21} y}$$

$$+ d_{91} e^{d_{20} y} + d_{92} e^{d_{21} y} + d_{93} e^{d_{22} y} + d_{94} e^{d_{23} y} + d_{95} y^3 e^{d_1 y}$$

$$+ d_{99} y^3 e^{-d_1 y} + d_{104} e^{d_{10} y} + d_{105} e^{-d_{10} y})$$

$$\tag{17}$$

$$f = d_{11} + d_{12} y + d_{13} e^{d_{10} y} + d_{14} e^{-d_{10} y} + d_{15} y^2 + d_{16} e^{d_1 y}$$

$$+ d_{17} e^{-d_1 y} + \epsilon(d_{128} + d_{129} y + d_{222} e^{d_1 y} + d_{223} e^{-d_1 y}$$

$$+ d_{224} y e^{d_1 y} + d_{225} y e^{-d_1 y} + d_{226} y^2 e^{d_1 y} + d_{227} y^2 e^{-d_1 y}$$

$$+ d_{228} y^2 + d_{229} y^4 + d_{230} y e^{d_{10} y} + d_{231} y e^{-d_{10} y}$$

$$+ d_{232} y^2 e^{d_{10} y} + d_{233} y^2 e^{-d_{10} y} + d_{170} e^{d_{21} y} + d_{171} e^{-d_{21} y}$$

$$+ d_{180} e^{d_{20} y} + d_{181} e^{-d_{20} y} + d_{182} e^{d_{21} y} + d_{183} e^{d_{22} y}$$

$$+ d_{184} e^{d_{23} y} + d_{185} e^{d_{20} y} + d_{188} y^6 + d_{194} y^3 e^{d_1 y}$$

$$+ d_{200} y^3 e^{-d_1 y} + d_{208} y^3 e^{d_{10} y} + d_{219} y^3 e^{-d_{10} y} + d_{130} e^{d_{10} y}$$

$$+ d_{131} e^{-d_{10} y})$$

$$\tag{18}$$

All the  $d_i$  ( $i=1, 2, 3, \dots$ ) terms, which are cited in the above equations are constants and these are not pointed out here due to want of space.

### 3.0 Result and Discussion

Here, to find solution of the physical system an analytical approach is tried. Firstly, similarity transformation is implemented to transform highly coupled PDE to a simultaneous ordinary differential equations. Further, perturbation method with its very small value of parameter  $\epsilon$  is used. By this technique, the equations of the physical model are converted to set analytical expressions of ODE and six equations are obtained. Computational work is carried out by Mathematica software to study the

dimensionless parameters such as Dufour effect, Schmidt number, thermal buoyancy parameter, concentration of buoyancy parameter, Reynolds number, Casson fluid parameter, Prandtl number, rate of chemical reaction, Darcy number and Hartmann number on heat, thickness and flow of fluid.

**Casson fluid parameter ( $\beta$ ) effect** Concentration, heat and fluid flow profiles decays with rise in Casson fluid parameter ( $\beta$ ) is observed in Figure 1, which is because of thickness of fluid.

**Darcy number (Da)** As permeability of porous medium increases then temperature, thickness and flow profiles enhances with very lesser values of Da is seen in Figure 2.

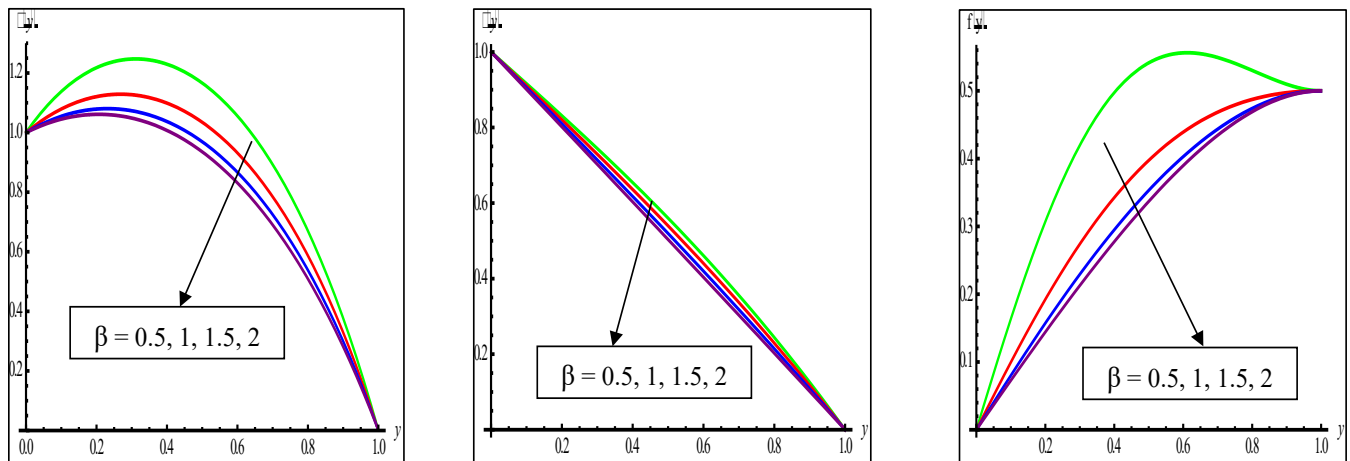


Figure 1. Concentration, temperature and velocity with Casson fluid parameter ( $\beta$ ).

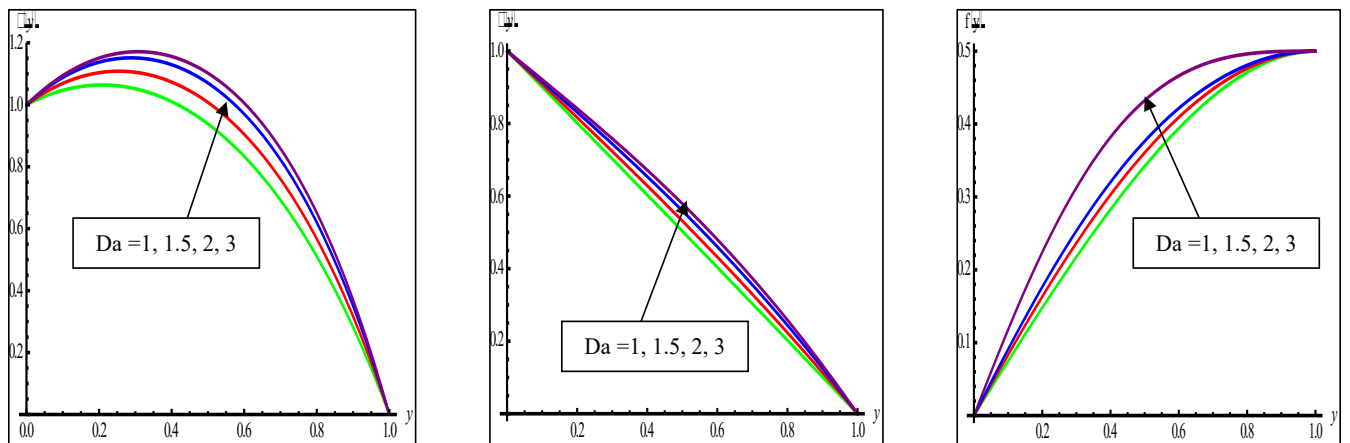


Figure 2. Concentration, temperature and velocity with Darcy number (Da).

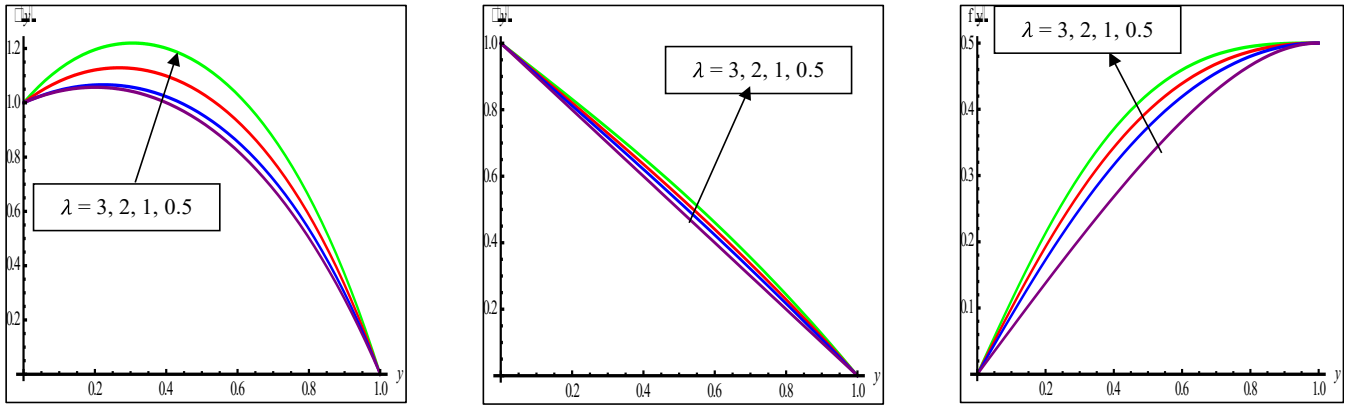


Figure 3. Concentration, temperature and velocity with Thermal buoyancy parameter ( $\lambda$ ).

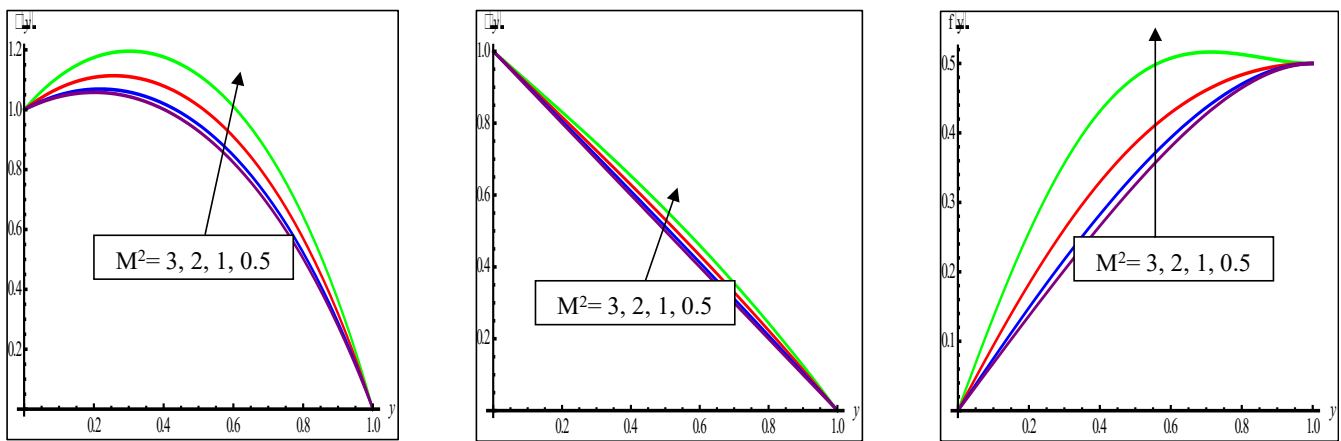


Figure 4. Concentration, temperature and velocity with Hartmann number ( $M^2$ ).

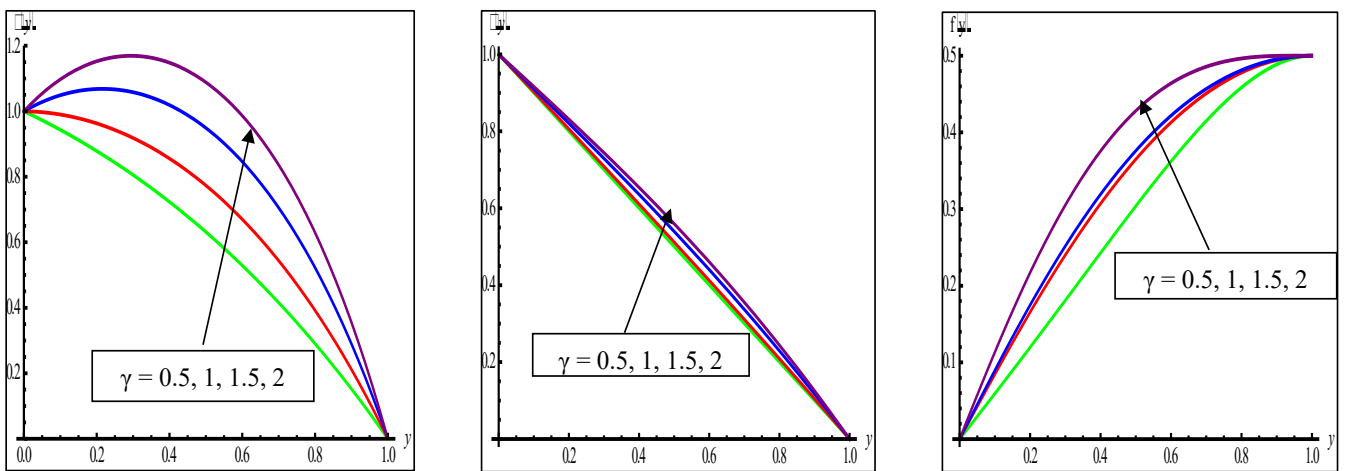
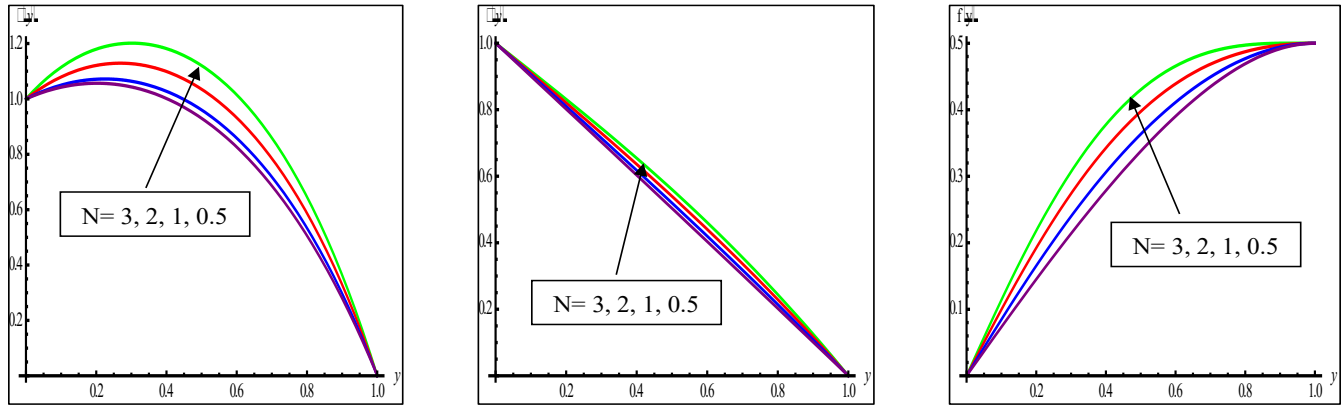


Figure 5. Concentration, temperature and velocity with Chemical reaction ( $\gamma$ ).

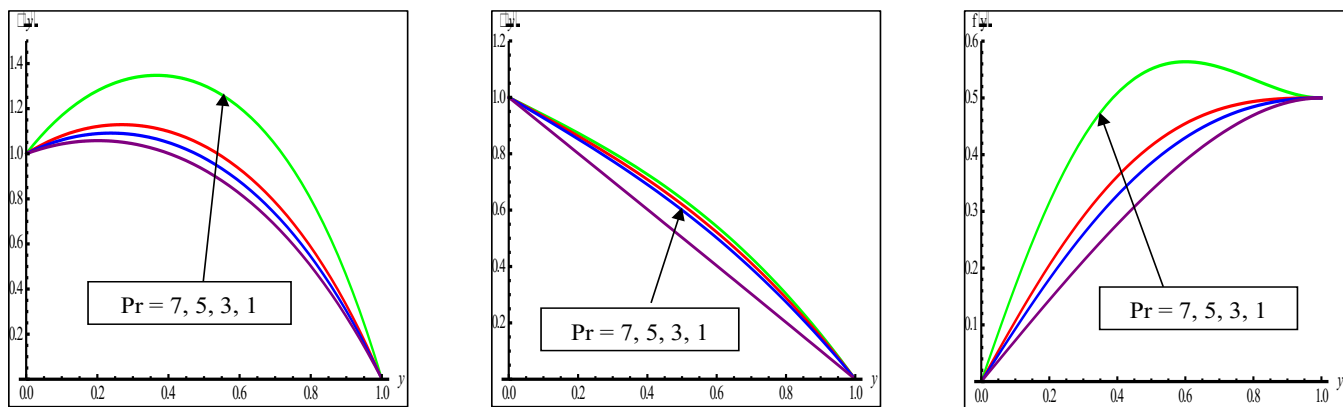
**Thermal buoyancy parameter ( $\lambda$ ) effect.** The increase in very small values of  $\lambda$  such as 0.5, 1, 2, 3 the profiles

diminishes because of dominating character of Dufour number is observed in Figure 3.

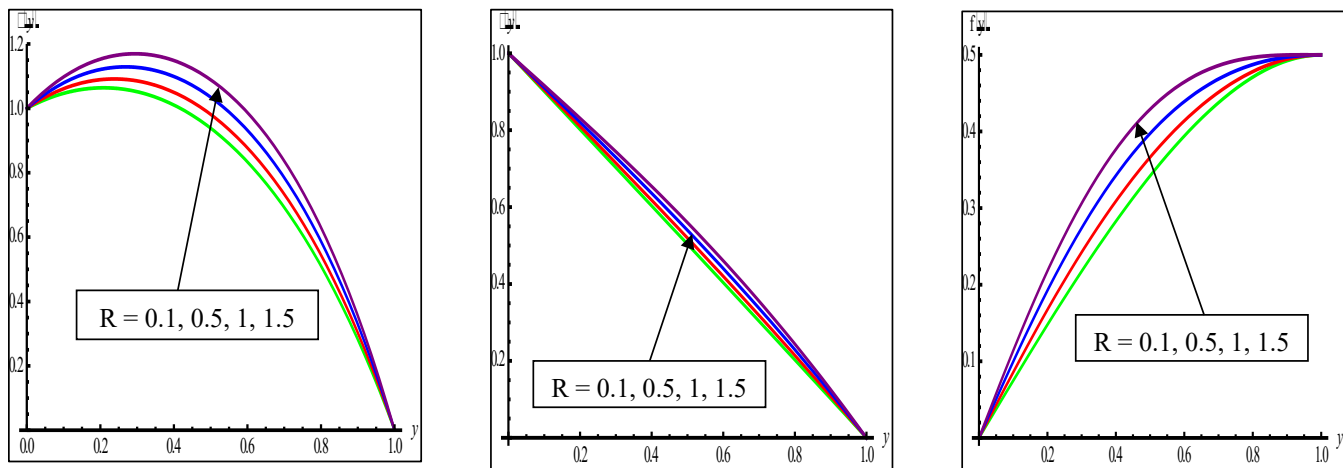




**Figure 6.** Concentration buoyancy parameter (N) on Concentration, temperature and velocity.



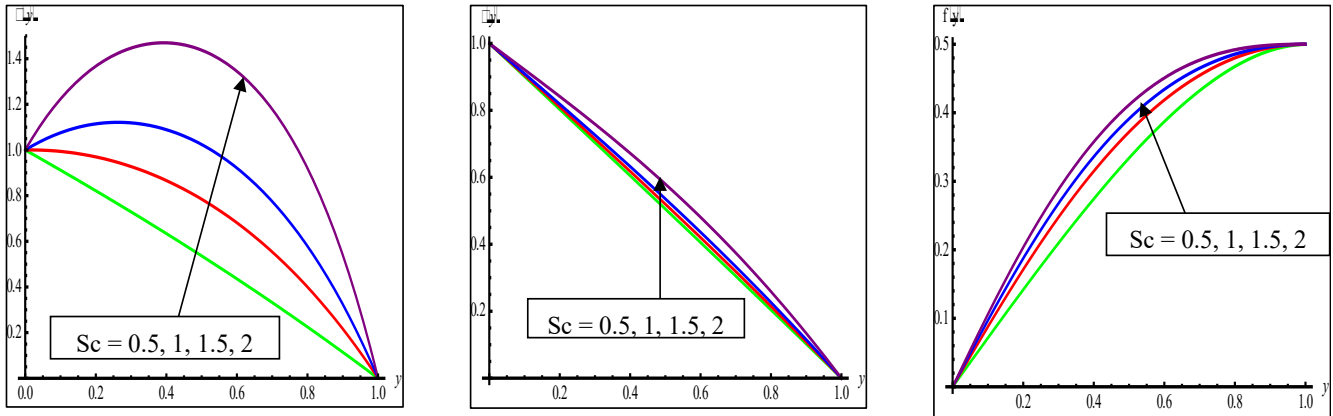
**Figure 7.** Concentration, temperature and velocity with Prandtl number (Pr).



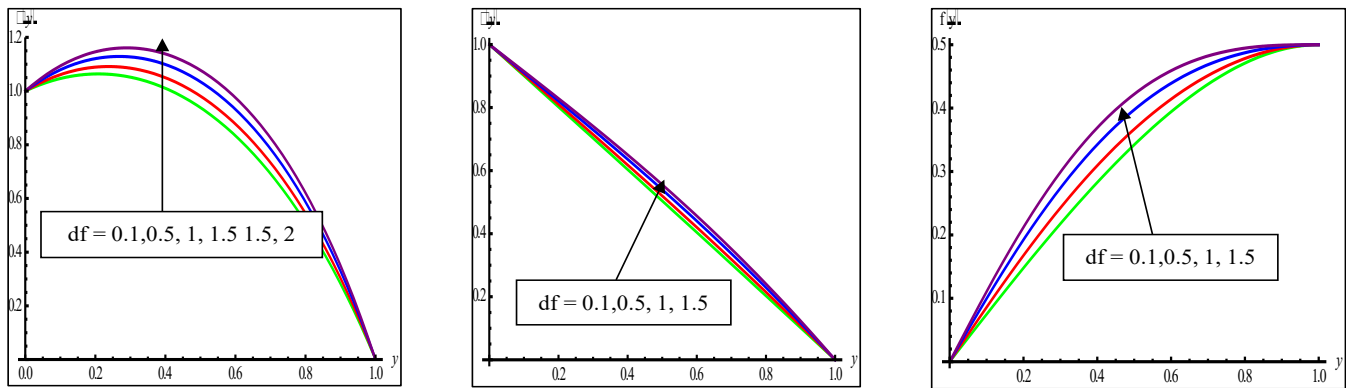
**Figure 8.** Concentration, temperature and velocity with Reynolds number (R).

**Hartmann number ( $M^2$ ) effect** Lorentz's force exists in Hartmann number which acts against the flow, So all profiles shrinks with surge in  $M^2$  is seen in Figure 4.

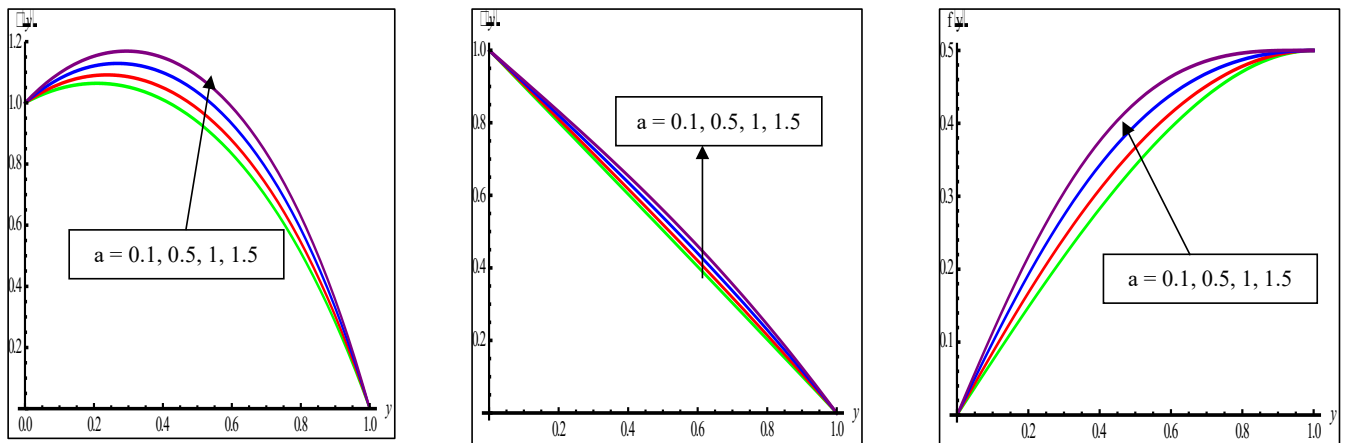
**Chemical reaction rate ( $\gamma$ ) effect** The increase in chemical reaction enhances all the profiles which is due to the exchange of molecules in the fluid is noted in Figure 5.



**Figure 9.** Schmidt's number (Sc) on Concentration, temperature and velocity.



**Figure 10.** Concentration, temperature and velocity with Dufour number (df).



**Figure 11.** Concentration, temperature and velocity with Amplification (a).

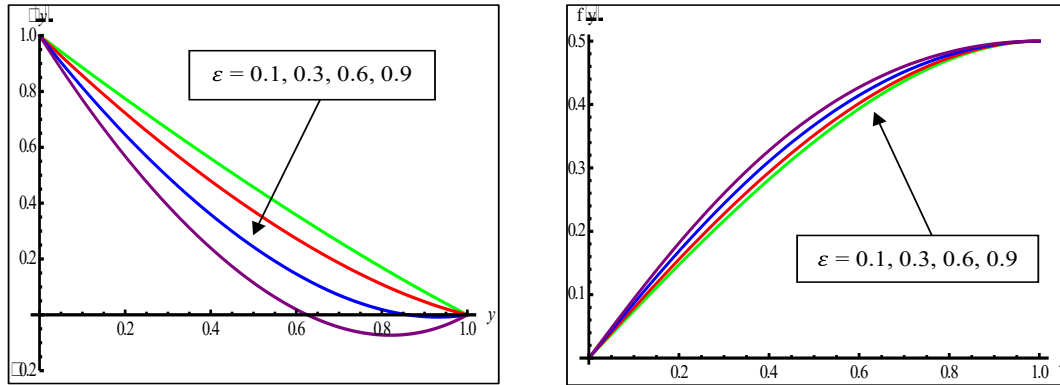
**Buoyancy Parameter of Concentration (N) effect**

This decreases the heat, thickness and fluid flow profiles at small values of N such as 0.5, 1, 2, 3 which is seen in Figure 6.

**Effect of Prandtl number (Pr)**

It is defined as viscous force to thermal force. Viscosity of the fluid increases with rise in Prandtl number which in turn shrinks all the profiles is observed in Figure 7.





**Figure 12.** Temperature and velocity with Perturbation Parameter ( $\epsilon$ ).

**Reynolds number effect (R)** It is defined as inertial force by viscous force. The surge in inertial force decreases the viscosity of the fluid which leads in growth of all the profiles is observed in Figure 8

**Schmidt's number (Sc) effect** This is defined as diffusivity of momentum to diffusivity of mass. The rise in diffusivity of momentum enhances all the profiles is seen in Figure 9

**Dufour number (Df) effect** The difference in concentration of the fluid leads to enhance the thickness of the fluid which in turn surges all the profiles with increase in Da is seen in Figure 10.

**Amplification (a) effect** Solution is carried out in presence of amplification. Concentration, fluid flow and heat enhances with growth in amplification is seen in Figure 11.

**Perturbation parameter ( $\epsilon$ ) effect** The problem is solved by using perturbation method. Figure 12 shows the rise in perturbation parameter  $\epsilon$  surges the temperature diminishes whereas velocity profiles are increases rapidly but there is no much effect of  $\epsilon$  on concentration.

## 4.0 Conclusion

Dimensionless axis from 0 to 1 is considered in horizontal channel to study behavior of concentration, temperature and velocity profiles.

- Concentration, heat and flow of fluid decreases as Casson parameter increases, this is because of very high viscosity of non-Newtonian Casson fluid.
- Temperature, concentration and flow profiles enhances with growth of Darcy number because of porosity of medium.

- Buoyancy parameter of concentration and Thermal buoyancy parameter diminishes all profiles due to the Dufour effect domination.

- The surge in Hartmann number depresses the profiles due to magnetic field.

- Prandtl number decays the profiles because of increase in viscosity of the fluid.

- Due to exchange of particles in chemical reaction enhances the temperature, velocity and concentration profiles.

- The Dufour number increases all the profiles which is due to the thickness of the fluid and difference in concentration gradient.

- Reynolds number surges temperature, velocity and concentration profiles this is due to inertial forces where it descends viscosity of fluid.

- The surge in amplification indicates enhancement of all the profiles, so it can be considered as a tool to control the profiles.

- The profiles rises with surge in Schmidt number this is because of increase in momentum diffusivity.

- Perturbation parameter ( $\epsilon$ ) increases velocity and decreases the temperature profiles with rise in perturbation parameter.

Present work with earlier work of Shilpa *et al.*<sup>37</sup> is compared which has good agreement in absence of Dufour effect.

## 5.0 Acknowledgement

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