Journal of Mines, Metals and Fuels, *71(10): 1492-1500; 2023. DOI: 10.18311/jmmf/2023/35815*

Print ISSN : 0022-2755

Journal of Mines, Metals and Fuels

Contents available at: *www.informaticsjournals.com/index.php/jmmf*

Effect of Gravity Modulation on the Onset of Heat Transfer by Buoyancy Induced Convection in Boussinesq- Stokes Suspension with Temperature

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Abstract

The current paper considers the Boussinesq-Stokes suspensions and the temperature-dependent viscosity with the influence of gravity modulation to analyze a weak non-linear stability problem of Rayleigh Benard magnetoconvection. In the study of convective instability problems, the impact of time-periodic body force also known as gravity modulation or g-gitter is essential. In the problem of gravity modulation, the gravity field has two components: one is the constant part and another an externally imposed time periodic part, which can be produced by oscillating the fluid layer. The effect of varying frequency of gravitational oscillation on convection is examined. The truncated form of the Fourier series is used in the non-linear analysis. The effects of numerous factors on the onset of convection have been discussed in this paper. The thermal Nusselt number is computed and shown for slow time periods using non-linear theory. The impacts of gravity modulation frequency and amplitude have been investigated in order to study heat transport in the system, as well as other aspects that exist in the problem.

Keywords: *Boussinesq Stokes Suspension, Gravity Modulation, Nusselt Number, Rayleigh-Benard Convection*

1.0 Introduction

The classical convection under Rayleigh-Benard conditions caused by heating underneath has long been a well-studied and an extensively researched phenomena[1,2]. Time-periodic vibration is another major component influence the natural convection in the system. It can cause fluctuating gravity, which manifests as time-periodic gravitational interruption, also known as gravity modulation ad commonly referred to as g-jitter in various literature. Few applications of time dependent gravitational force found in the areas of crystal growth, big scale atmospheric convention problems, mining industries[3], Geodetic Monitoring, Ground Penetrating Radar[4,5], floating and half floating zone convectional problems, and many more.

The very first report on gravity modulation effect had been done by Greshuni and Zhukhovitskii[6], Gresho and Sani[7]. They investigated the time-dependent buoyancy force and analyze the impact of finite-amplitude flows by modulating the fluid layer. Wadih and Roux[8] discussed the onset of convection and the cause of gravity modulation for the vertical cylinder of infinite length. Floquet theory has been adopted to analyze the

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modulation for low values of , and fixed value of the frequency ω. They, established the asymptotic relation to study the modulation for large values of ε . Biringen and Peltier[9] went on look into the cause of gravity modulation in the Rayleigh-Benard problem. Clever *et al.*[10] conducted a thorough nonlinear analysis and examined the stability criteria throughout a significantly larger parameter range.

Jin and Chen[11] used a finite difference method to study, how the natural convection is affected in the fluid when placed in a vertical slot by the gravity modulation which is sinusoidal in nature. To analyze this they used a finite difference method and found that, the fluid flow has a stronger effect compared to heat transfer rate under low-frequency gravity modulation.

The other factors which can also influence the modulations are magneto convection and variable viscosity which can be found in various astrophysical, geophysical, atmospheric convection and industrial applications. In the presence of a magnetic field, Siddheshwar and Bhadauria[12] studied the effect of temperature and gravity modulation on a weekly conducting Newtonian liquid. They performed a non-linear stability study using the Ginzburg-Landau model and analyzed the influence of these modulations and other factors on heat transfer. The thermo-convective instability for the two component Newtonian liquid subjected to the vertical magnetic field, when the gravity field varies with time in a sinusoidal manner is analyzed by Bhadauria[13]. They made a theoretical non-linear analysis and discussed the amount of heat and mass transports for various parameters. They found that the magnitude of mass and heat transportations can be regulated by varying the amplitude and frequency of gravity modulation. Zhao *et al.*[14] adopted the Darcy-Brinkman model to study the chaotic convection for the couple-stress liquid saturated porous media under gravity modulation. In order to examine the chaotic activity and measure of heat transfer for the liquid, non-linear stability analysis is performed. Keshri and Gupta *et al.*[15] premeditated the non-linear mass transport for the Newtonian fluid with applied magnetic field under both concentration and Gravity modulation. The rate of mass transfer is achieved by deriving the equation in the form of Ginzburg-Landau Model and illustrated the influence of several parameters on the mass transport. Palle Kiran[16,17] discussed the effect of Gravity modulation on the oscillatory and double diffusive convection under the influence of heat source and magnetic field. They have performed a weakly nonlinear stability analysis using finite amplitude Ginzburg-Landau model.

All the above related works mentioned above considered Newtonian fluid for the investigation of fluids. Most of the fluids considered in industries and many practical situations may contain suspended particles rather than having pure Newtonian liquids. Such fluids are usually non-Newtonian in nature and have distinct features, such as polar effect. The study related to these types of fluids attracts many researchers, engineers, and physicists due to the presence of large stabilizing/destabilizing effects on thermal convection. Stokes[18] was the first to develop the theory of couple-stress and its constitutive equations. He analyzed the consequence of couple-stresses in the fluid and found that a size effect comes in, which is not present in the absence of a couple-stress. Later many researchers diverted their study towards couple-stress fluids. Sharma and Gupta[19] discussed the Thermal instability of magnetic field and rotation for the couple-stress fluid. They inferred that the system stability is improved by rotation whereas the suspended particles cause the destabilization to the system.

Siddheshwar and Pranesh[20] studied convective stability in the couple-stress fluid under $1g$ and μg situations in the presence of applied magnetic field under different boundary conditions. Recently Kavitha *et al.*,[21-23] made nonlinear stability analysis on couple-stress fluid by considering various external factors such as magnetic field, variable viscosity, non-inertial acceleration, variable heat source/sink and so on. The effect of these parameters has been discussed using Lorenz model. The present paper aims in analyzing the effect of gravity modulation for the couple-stress fluid under the influence of uniform magnetic field and the temperature dependent viscosity.

2.0 Mathematical Formulation

The schematic flow arrangement consists of a temperature-sensitive, electrically conducting Boussinesq-Stokes suspension liquid contained between two horizontal parallel planes planes $z = 0$ and $z = d$ that are separated by a distance apart. A heat is introduced from the bottom layer which is hotter than the top one. A Cartesian frame of reference has been taken such that, the origin lies on the bottom plane and the z-axis is vertical upward.

Considering the time-periodic boundary condition the top and bottom layers are kept at different temperatures with constant gradient $\Delta T / d$. In addition to temperature gradient, a vertical magnetic field is also imposed across the fluid layer. Here the boundaries are kept isothermal (i.e., maintained at constant temperature) and stress-free. The schematic representation of the problem is shown in the Figure 1. The fundamental governing equations of motion for an electrically conducting couple-stress fluid, according to the Oberbeck-Boussinesq approximations, are as follows:

$$\rho_0\left(\frac{\partial \vec{q}'}{\partial t}+(\vec{q}.\nabla)\vec{q}\right)-\nabla p-\nabla\left(\mu_f(T)\left(\nabla\vec{q}+\nabla\vec{q}^T\right)\right)+$$

$$\rho(T)g-\mu_m^2\left(\vec{H}.\nabla\right)\vec{H}+\mu'\nabla^4\vec{q}=0 \tag{1}$$

$$\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T=\chi\left[\frac{\partial^2 T}{\partial x}+\frac{\partial^2 T}{\partial z}\right] \tag{2}$$

$$\frac{\partial \vec{H}}{\partial t}+(\vec{q}.\nabla)\vec{H}=\nu_m\nabla^2\vec{H}-\left(\vec{H}.\nabla\right)\vec{q} \tag{3}$$

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0, \tag{4}$$

$$\frac{\partial H_x}{\partial x}+\frac{\partial H_z}{\partial z}=0, \tag{5}$$

$$\rho(T)=\rho_0[1-\beta(T-T_0)], \tag{6}$$

$$\mu_f(T)=\mu_0 e^{-\delta(T-T_0)}, \tag{7}$$

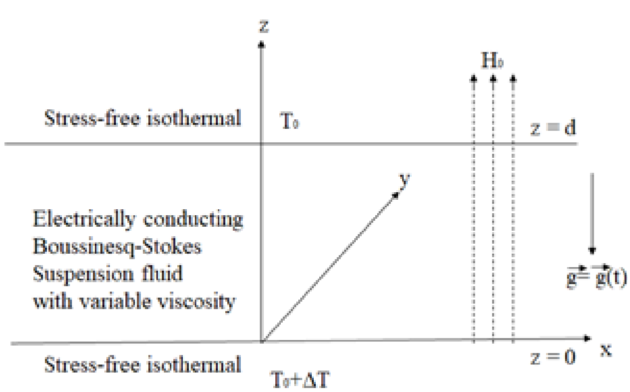

Figure 1. Rayleigh-Benard convection with Gravity modulation.

$$\vec{g}=g_0[1+\delta_1\cos(\Omega t)]\hat{k}. \tag{8}$$

The variables and constants which are used in the equations (1)-(8) are mentioned in the nomenclature part. In the present paper, we discuss the effect of gravity modulation for the Boussinesq Stokes suspension fluid by magneto-convection and variable viscosity.

The fundamental condition of the fluid is considered to be quiescent and is given by

$$\left.\begin{aligned}&(u,w)=(0,0),\ \overrightarrow{H_b}=H_0\hat{k},\mu_{f_b}\left(\frac{z}{d}\right)=\mu_0 e^{-Vf\left(\frac{z}{d}\right)},T_b=T_0+\Delta T\left(\frac{z}{d}\right)\\&\frac{\partial T_b}{\partial t}=-\kappa_T\frac{\partial^2 T_b}{\partial z^2},\frac{\partial p_b}{\partial z}=-\rho_b\vec{g},\rho_b\left(\frac{z}{d}\right)=\rho_0\left(1-\beta\Delta Tf\left(\frac{z}{d}\right)\right)\\&p_b\left(\frac{z}{d}\right)=-\int\rho_b\left(\frac{z}{d}\right)gd\left(\frac{z}{d}\right)+c_1,\ \frac{\partial p_b}{\partial z}=-\rho_b\vec{g}\end{aligned}\right\} \tag{9}$$

where, $f\left(\frac{z}{d}\right)=1-\frac{z}{d}$ and c_1 is the constant of integration.

The disturbances of finite magnitude for the basic quiescent condition are now superimposed in the following form,

$$\left.\begin{aligned}&\vec{q}=\vec{q}_b(z)+\vec{q}'(x,y,z,t),\ \rho=\rho_b(z)+\rho'(x,y,z,t),\\&T=T_b(z)+T'(x,y,z,t),\vec{H}=\vec{H}_b(z)+\vec{H}'(x,y,z,t),\\&p=p_b(z)+p'(x,y,z,t),\mu=\mu_b(z)+\mu'(x,y,z,t),\end{aligned}\right\} \tag{10}$$

when the Equation (10) is substituted for the Equations (1) - (8), the following equations are obtained:

$$\rho_0\left(\frac{\partial\vec{q}'}{\partial t}+(\vec{q}'.\nabla)\vec{q}'\right)=-\nabla p'-\rho'(T)g\hat{k}+\nabla\left(\mu_f(T)\left(\nabla\vec{q}'+\nabla\vec{q}'^T\right)\right)\cdot$$

$$-\mu_m^2\left(\vec{H}'.\nabla\right)\vec{H}'+\mu_m H_b\frac{\partial H'}{\partial z}-\mu'\nabla^4\vec{q}', \tag{11}$$

$$\frac{\partial T'}{\partial t}+(\vec{q}'.\nabla)T'+w'\frac{\partial T_b}{\partial z}-\chi\left[\frac{\partial^2 T'}{\partial x}+\frac{\partial^2 T'}{\partial z}\right]=0, \tag{12}$$

$$\frac{\partial u'}{\partial x}+\frac{\partial w'}{\partial z}=0, \tag{13}$$

$$\frac{\partial H'_x}{\partial x}+\frac{\partial H'_z}{\partial z}=0, \tag{14}$$

$$\rho'(T)=-\rho_0\beta T', \tag{15}$$

$$\frac{\partial \vec{H}'}{\partial t}+(\vec{q}'.\nabla)\vec{H}'=\left(\vec{H}'.\nabla\right)\vec{q}'+H_b\frac{\partial w'}{\partial z}+\upsilon_m\nabla^2\vec{H}' \tag{16}$$

Because, the research is confined to rolls with the coordinate as their axis, we write the following two equations for u and v and from Equation (1), assuming that all physical variables are independent of y, (Reference[1]):

$$u'=-\frac{\partial\psi'}{\partial z},w'=\frac{\partial\psi'}{\partial x} \tag{17}$$

The magnetic potential ϕ' and stream function ψ' are also introduced in the above form as,

$$H'_x=-\frac{\partial\phi'}{\partial z},\quad H'_z=\frac{\partial\phi'}{\partial x} \tag{18}$$

Elimination of pressure term in the Equation (11) is carried out by operating curl on it and the resulting system is non-dimensionalized using the scaling mentioned below,

$$(X,Z)=\left(\frac{x}{d},\frac{z}{d}\right),\ \tau=\chi/_{d^2}t,\ \psi=\psi'/_{\chi},\ \theta=T'/_{\Delta T},\ \Phi=\phi/_{dH_b}, \tag{19}$$

The dimensionless version of governing equations is derived as follows:

$$\frac{1}{Pr}\left(\frac{\partial}{\partial\tau}(\nabla^2\Psi)+\frac{\partial(\Psi,\nabla^2\Psi)}{\partial(X,Z)}\right)-R_E\frac{\partial\Theta}{\partial X}(1+\delta_1\cos\Omega\tau)+C\nabla^6\Psi+$$

$$-\mu_{f_b}\nabla^4\Psi-\frac{\partial\mu_{f_b}}{\partial Z}\frac{\partial}{\partial Z}(\nabla^2\Psi)-QPm\left(\frac{\partial(\nabla^2\Phi)}{\partial Z}+\frac{\partial(\Phi,\nabla^2\Phi)}{\partial(X,Z)}\right)=0 \tag{20}$$

$$\frac{\partial\Theta}{\partial\tau}-\nabla^2\Theta-\frac{\partial\Psi}{\partial X}\frac{\partial T_b}{\partial Z}-\frac{\partial(\Psi,\Theta)}{\partial(X,Z)}=0, \tag{21}$$

$$\frac{\partial\Phi}{\partial\tau}-Pm\nabla^2\Phi-\frac{\partial\Psi}{\partial Z}-\frac{\partial(\Psi,\Phi)}{\partial(X,Z)}=0, \tag{22}$$

where, $\nabla^2=\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Z^2}$ is the Laplacian operator, $Pr=\frac{\mu}{\rho_0\chi}$ is the Prandtl number,

$Pm=\frac{v_m}{\chi}$ is the magnetic Prandtl number,

$R_E=\frac{\beta\rho_0 g d^3\Delta T}{\mu_0\chi}$ the thermal Rayleigh number,

$C=\mu'/_{\mu d^2}$ the couple stress parameter,

$Q=\frac{\mu_m^2\sigma H_0^2 d^2}{\mu}$ is the Chandrasekhar number,

$R_I=\frac{Q^* d^2}{\chi}$ the internal Rayleigh number.

$\mu_{f_b}=\mu_0 e^{-V(1-Z)}$ and $\frac{\partial\mu_{f_b}}{\partial Z}=\mu_0 V e^{-V(1-Z)}$

both will be stated as half-range Fourier cosine series in the interval [0,1], which aids in getting the analytical formula for the thermal Rayleigh number.

To study the stationary magneto convection, we use the time variations only at the slow time scale $\tau=\epsilon^2 t$, the system of Equations (20) - (22) in the matrix form can now be written as,

$$\begin{bmatrix}\frac{\varepsilon^2}{\Pr}\frac{\partial}{\partial\tau}\nabla^2+f_1 & R(\tau)\frac{\partial}{\partial X} & -QPm\frac{\partial}{\partial Z}\nabla^2\\ -\frac{\partial}{\partial X} & \varepsilon^2\frac{\partial}{\partial\tau}-\nabla^2 & 0\\ \frac{\partial}{\partial Z} & 0 & \varepsilon^2\frac{\partial}{\partial\tau}-Pm\nabla^2\end{bmatrix}\begin{bmatrix}\Psi\\ \Theta\\ \Phi\end{bmatrix}=\begin{bmatrix}f_2\\ \frac{\partial(\Psi,\Theta)}{\partial(X,Z)}\\ \frac{\partial(\Psi,\Phi)}{\partial(X,Z)}\end{bmatrix} \tag{23}$$

where,

$$R(\tau)=R(1+\varepsilon^2\cos\Omega\tau)$$

$$f_1=C\nabla^6-\mu_{f_b}\nabla^4-\frac{\partial\mu_{f_b}}{\partial Z}\frac{\partial}{\partial Z}\nabla^2$$

$$f_2=\frac{1}{Pr}J(\Psi,\nabla^2\Psi)+QP_m J(\Phi,\nabla^2\Phi)$$

The perturbed quantities have the following boundary requirements that are adequate for stress-free and isothermal boundaries:

$$\Psi=\nabla^2\Psi=\Theta=D\Phi=0\quad\text{at}\quad Z=0,1 \tag{24}$$

2.1 Quantification of Heat Transfer using the Ginzburg-Landau Model using Minimal Representation of Fourier Series

In-order to discuss the weak non-linear stability analysis, it is required to represent the stream function, temperature distribution, magnetic stream function and external Rayleigh number using the following asymptotic expansions:

$$\Psi = \varepsilon\Psi_1 + \varepsilon^2\Psi_2 + \varepsilon^3\Psi_3....... \tag{25}$$

$$\Theta = \varepsilon\Theta_1 + \varepsilon^2\Theta_2 + \varepsilon^3\Theta_3........ \tag{26}$$

$$\Phi = \varepsilon\Phi_1 + \varepsilon^2\Phi_2 + \varepsilon^3\Phi_3........ \tag{27}$$

$$R = R_0 + \varepsilon^2 R_2 + \varepsilon^4 R_4.........., \tag{28}$$

we now substitute the Equations (25)-(28) in the Equation (23) and compare the lowest power of ε to get the following expression as:

$$\begin{bmatrix} f_1 & R\frac{\partial}{\partial X} & -PmQ\frac{\partial}{\partial Z}\nabla^2 \\ -\frac{\partial}{\partial X} & -\nabla^2 & 0 \\ \frac{\partial}{\partial Z} & 0 & -Pm\nabla^2 \end{bmatrix}\begin{bmatrix}\Psi\\ \Theta\\ \Phi\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} \tag{29}$$

A set of linear equations for the stationary phase of convection is produced by the above matrix.

The following will serve as the preceding's solution:

$$\left.\begin{aligned} \Psi_1(X,Z,\tau) &= \sin(m_c X)\sin(\pi Z)A(\tau) \\ \Theta_1(X,Z,\tau) &= \frac{m_c^{\ 2}}{\pi^2 + m_c^{\ 2}}\cos(m_c X)\sin(\pi Z)A(\tau) \\ \Phi_1(X,Z,\tau) &= \frac{\pi}{Pm(\pi^2 + m_c^{\ 2})}\sin(m_c X)\cos(\pi Z)A(\tau) \end{aligned}\right\} \tag{30}$$

Where, $\eta^2 = \pi^2 + m_c^2, m_c = \pi\alpha,$ the horizontal wave number. The following Equation (29), which yields the critical Rayleigh number expression, as follows:

$$R_{Ec} = \frac{\left(\frac{\eta^4}{2}\left((1-2\pi^2)b_2 - b_0\right) + Q\pi^2 + C\eta^6\right)\eta^2}{\pi^2\alpha}. \tag{31}$$

where,

$$b_0 = 2\int_0^1 \mu_{f_b} dZ \ \ and \ \ b_2 = 2\int_0^1 \mu_{f_b}\cos(2\pi Z)dZ$$

are the Fourier Cosine coefficients.

Now at the second order, we obtain,

$$\begin{bmatrix} f_1 & R_0\frac{\partial}{\partial X} & -PmQ\frac{\partial}{\partial Z}\nabla^2 \\ -\frac{\partial}{\partial X} & -\nabla^2 & 0 \\ \frac{\partial}{\partial Z} & 0 & -Pm\nabla^2 \end{bmatrix}\begin{bmatrix}\Psi\\ \Theta\\ \Phi\end{bmatrix} = \begin{bmatrix}M_{21}\\M_{22}\\M_{23}\end{bmatrix} \tag{32}$$

where,

$$M_{21} = -QPmJ(\Phi_1, \nabla^2\Phi_1) + \frac{1}{\Pr}J(\Psi_1, \nabla^2\Psi_1) = 0$$

$$M_{22} = \frac{\partial(\Psi_1, \Theta_1)}{\partial(\mathrm{X},\mathrm{Z})} = \left(-\frac{\pi^3\alpha^2}{2\eta^2}A^2(\tau)\right)\sin(2\pi Z)$$

$$M_{23} = \frac{\partial(\Psi_1, \Phi_1)}{\partial(X,Z)} = \left(-\frac{\pi^3\alpha^2}{2Pm\eta^2}A^2(\tau)\right)\sin(2\pi\alpha X)$$

Now, in case of the second order system the solution is,

$$\left.\begin{aligned} \Psi_2(X,Z,\tau) &= 0, \\ \Theta_2(Z,\tau) &= \left(-\frac{\pi^2\alpha^2}{8\pi\eta^2}A^2(\tau)\right)\sin(2\pi Z), \\ \Phi_2(X,\tau) &= \left(-\frac{\pi^2}{8\pi\alpha Pm^2\eta^2}A^2(\tau)\right)\sin(2\pi\alpha X). \end{aligned}\right\} \tag{33}$$

For the stationary mode of magneto convection, the horizontally averaged Nusselt number is as follows:

$$Nu(\tau) = \frac{\left[\frac{\alpha_c}{2}\left(\int_{X=0}^{2/\alpha}(1 - Z + \Theta_2)_Z\, dX\right)\right]_{Z=0}}{\left[\frac{\alpha_c}{2}\left(\int_{X=0}^{2/\alpha}(1 - Z)_Z\, dX\right)\right]_{Z=0}}, \tag{34}$$

on substituting the Equation (33) in Equation (34), the Nusselt number expression is obtained as,

$$Nu(\tau) = \frac{A^2(\tau)(\pi\alpha)^2}{(2\eta)^2} + 1. \tag{35}$$

The following matrix obtained on comparing with the third order,

$$\begin{bmatrix} f_1 & R\frac{\partial}{\partial X} & -PmQ\frac{\partial}{\partial Z}\nabla^2 \\ -\frac{\partial}{\partial X} & -\nabla^2 & 0 \\ \frac{\partial}{\partial Z} & 0 & -Pm\nabla^2 \end{bmatrix} \begin{bmatrix} \Psi \\ \Theta \\ \Phi \end{bmatrix} = \begin{bmatrix} M_{31} \\ M_{32} \\ M_{33} \end{bmatrix}, \tag{36}$$

Where,

$$\left.\begin{aligned} M_{31} &= \left(-\frac{Q\pi^4}{Pm^2\eta^2}\left(\frac{1}{4} - \frac{\pi^2\alpha^2}{\eta^2}\right)\cos(2\pi\alpha X) + \frac{dA}{d\tau}\frac{\eta^2}{\Pr} - \frac{\pi^2\alpha^2}{\eta^2}A(\tau)(-R_0\cos\Omega\tau + R_2) \right)\sin(m_c X)\sin(\pi Z) \\ M_{32} &= \left(\frac{\pi\alpha}{\eta^2}\frac{dA}{d\tau} - \frac{\pi^3\alpha^3}{4\eta^2}\cos(2\pi Z)A^3(\tau) \right)\cos(m_c X)\sin(\pi Z), \\ M_{33} &= \left(-\frac{\pi}{Pm\eta^2}\frac{dA}{d\tau} - \frac{\pi^3}{4Pm^2\eta^2}\cos(2\pi\alpha X)A^3(\tau) \right)\sin(m_c X)\sin(\pi Z). \end{aligned}\right\} \tag{37}$$

The solvability conditions have been applied for the above equations, we obtain the Ginzburg-Landau equations in the form,

$$\left(\frac{\eta^6}{\Pr} + \pi^2\alpha^2 R_0 - \frac{Q\pi^2\eta^2}{Pm}\right)\left(\frac{dA}{d\tau}\right) = \pi^2\alpha^2\eta^2 R_0 (R_2^* + (\delta_1\cos(\Omega\tau))A$$

$$-\left(\frac{\pi^4\alpha^4}{8} + \frac{Q\pi^4\eta^2}{2Pm}\left(\frac{\pi^2\alpha^2}{\eta^2} - \frac{1}{2}\right)\right)A^3. \tag{38}$$

3.0 Result and Analysis

The primary goal of this paper is to investigate the effect of gravity modulation of a fluid with Boussinesq-Stokes suspension under the influence of temperature dependent viscosity and the magnetic field. The other important factors which are also influence the modulation are electrical conductivity and magnetic permeability of the fluid which arises in the form of Pr and Pm respectively. The heat transfer of the fluid has been investigated under the impact of modulation. The effects of amplitude and frequency modulation on heat transfer are investigated using δ_1 and Ω. In the presence of modulation Malashetty and Basavaraja[24] proved that, the critical Rayleigh number for onset of convection is larger than the ones predicted by the critical stationary Rayleigh number of the non-modulation case $R_E = \frac{K^4(\eta K^2 + Q)}{\pi^2\alpha^2}$ where $\eta = CK^2$ and $K^2 = \pi^2(1+\alpha^2)$. This is because the gravity modulation component increases the R_{E_c} of Rayleigh Benard magneto convection in a couple-stress liquid without gravity modulation. In what follows the influence of amplitude and frequency of gravity modulation, Chandrasekhar number, thermorheological parameter and couple-stress parameter on Nusselt number is examined through δ_1, Ω, Q, V and C respectively

Gravity modulation has a remarkable influence on Rayleigh-Benard system by affecting the usual gravity and resulting to an effective quantity. In the current problem, modulation acts as a physical mechanism for regulating the convection. The Ginzburg-Landau model is used to study magneto convection after a weak nonlinear analysis, which ultimately leads to Bernoulli's differential equation. An ordinary differential equation is encountered which is highly non-linear in nature. The Runge-Kutta Fehlberg 45 method, a numerical procedure has been implemented by considering the initial condition as $A(0) = 5$. The heat transfer is investigated in the current problem using the Nusselt number Nu plots for slow time intervals. In the present work, the couple-stress fluid is studied. Based on Einstein's viscosity relation, the fluid with suspended particles has a higher viscosity than the pure fluid. As a result, the fluid's prandtl number is set as $Pr = 5$ and 10 (a bit higher than Newtonian fluids) and is made it fix throughout the study. Because of the limitations of non-linear analysis, the Prandtl number has barely affects the Nusselt number.

Figures (2-5) concerning the changes in the Nusselt number Nu with respect to slow time τ for different values of variable viscosity V the couple-stress parameter C and the Chadrasekhar number Q. Results are also depicts for different Prandtle number i.e $Pr = 5$ and $Pr = 10$. The fluctuation for amplitude of gravity modulation δ_1 and frequency modulation Ω are also highlighted in the Figures 2(a) and 2(b). At this step, raising the values of δ_1 enhances heat transmission in the system while maintaining the oscillation's wave length constant. The

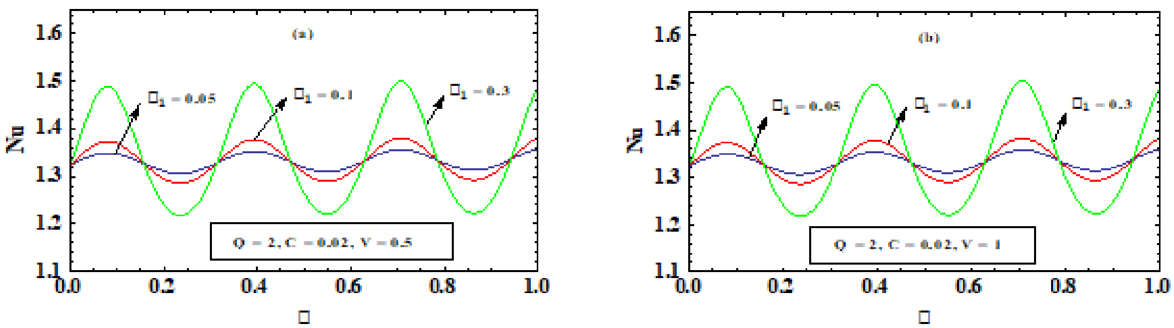

Figure 2. Nu plot with τ for $\Omega = 20$ and $Pr = 10$, $Pm = 10$.

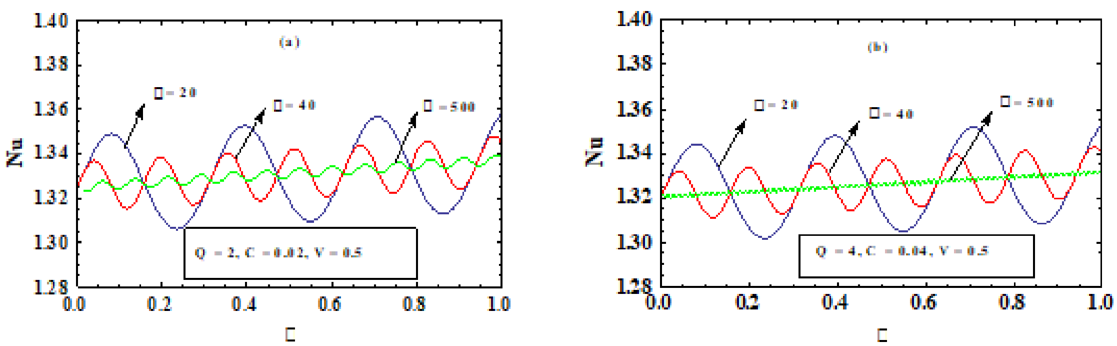

Figure 3. Nu plot with τ for $\delta_1 = 0.05$ and $Pr = 10$, $Pm = 10$.

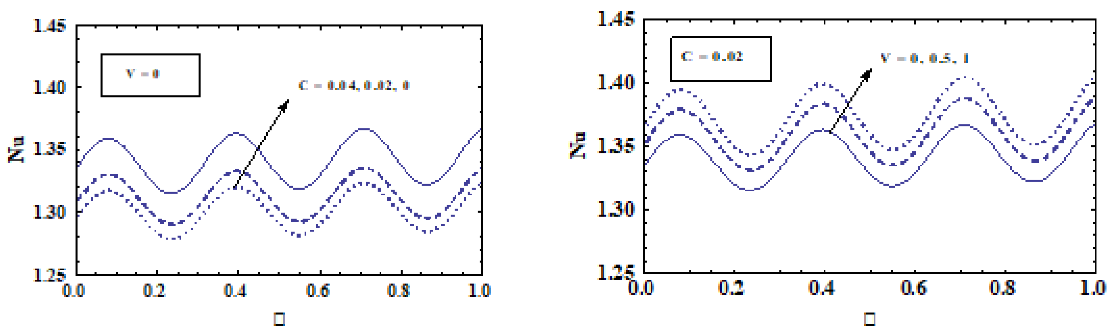

Figure 4. Nu plot with τ for $\Omega = 20$, $\delta_1 = 0.05$ and $Pr = 10$, $Pm = 10$.

figures also show a modest fluctuation in the viscosity parameter V. As the value of V grows, so does *Nu*. The individual effects of C, V and Q can be noticed in the Figure 4 and 5. Hence,

$$Nu/\delta_1 = 0.05 < Nu/\delta_1 = 0.1 < Nu/\delta_1 = 0.3,$$
$$Nu/V = 0 < Nu/V = 0.5 < Nu/V = 1,$$

The effect of frequency modulation Ω is completely opposite to the effect of δ_1, the result can be observed in the Figures 3(a) and 3(b). We can notice that, greater the values of Ω results in the less heat transfer, and hence results in the diminishing effect of *Nu*. In this case, the wavelength of the oscillation decreases as well. From the Figures 3(a) and 3(b) diminishing effects of Nusselt number can be observed for the increasing values of couple-stress parameter and the Chadrasekhar number demonstrating that the suspended particles and magnetic field decrease heat transfer in the system. Hence,

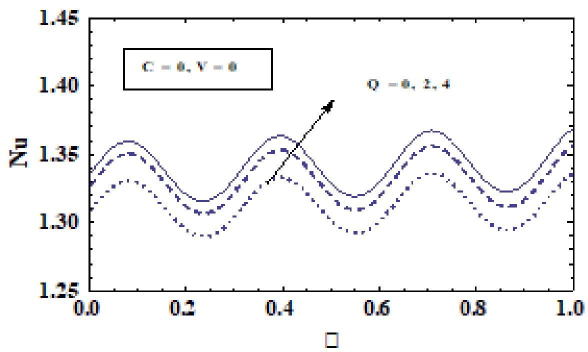

Figure 5. Nu plot with τ for Ω = 20, δ_1 = 0.05 and Pr = 10, Pm =10.

$$Nu / C = 0 > Nu / C = 0.02 > Nu / C = 0.04$$
$$Nu / Q = 0 > Nu / Q = 2 > Nu / Q = 4$$
$$Nu / \Omega = 20 > Nu / \Omega = 40 > Nu / \Omega = 500$$

4.0 Conclusion

1. The non-autonomous Ginzburg-Landau model for magneto convection in couple-stress fluid under gravity modulation demonstrates that, the Nusselt number rises with modulation amplitude while it falls with modulation frequency.
2. Increases in the magnetic field and the couple-stress parameter lead to a significant reduction in the Nusselt number.
3. The heat transfer of the system rises as the variable viscosity increases, resulting in an increase in the Nusselt number.

5.0 Acknowledgment

The first three authors are grateful to the principal and management of Ramaiah Institute of Technology, Bengaluru, whereas the fourth author is grateful to the management of R. R. Institute of Technology for the encouragement of research activities.

6.0 References

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