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# Linear Stability Analysis in Ethylene Glycol-Copper Nanoliquid Saturated Porous Medium in the Presence of Different Shaped Nanoparticles

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## Abstract

*In the present paper we carried out linear stability analysis for ethylene glycol-copper nanoliquid-saturated porous medium. The thermal properties of baseliquid, nanoparticles and porous medium are used in the calculation of properties of nanoliquid and nanoliquid saturated porous medium using phenomenological laws and mixture theory. The rigid-free isothermal boundaries are considered in the study. Analytical expression for critical Rayleigh number is presented in the paper. Dissimilar shapes of nanoparticles are examined and their effect on the onset of convection is studied in great detail. In addition the effect of various parameters namely porous parameter, aspect ratio, volume fraction are also studied and analysed graphically. It is observed that the onset of convection is advanced when nanoparticles are added to baseliquid whereas delayed in the addition of the porous medium to the nanoliquid. Unicellular convection is possible only when the aspect ratio lies in the range* $0.8 < A < 2$.

**Keywords:** *Ethylene Glycol-Copper Nanoliquid, Linear Stability Analysis, Rayleigh-Bénard Convection, Single-Phase Model*

## 1.0 Introduction

Heat transfer and cooling techniques are the very important concepts to be studied and well understood in the present decade for their enormous applications in the field of science and engineering. The concept of heat transfer and cooling are very much involved in our daily lives than we actually realize.

Techniques of cooling are important in the places where large amount of heat has to be transfered, such as metallurgic process, drives, rectifiers and converters, research, medical application, non-electrical environment, data center, automobiles, nuclear reactors, geothermal energy[1-3].

The time taken by base liquid to transfer heat and cool is more and hence maintaining the safety during the process becomes a challenging task. As a result a new kind of fluid with high thermal conductivity compared to base liquid was used for the first time by Choi and Eastman[4]. They named the fluid as nanoliquid. Nanoliquid is the combination of baseliquid and nanoparticles with size measuring from 1 to 100 nm which are uniformly distributed. Masuda *et al.*[5] have studied the properties of nanoparticles and came to a conclusion that thermal conductivities of nanoliquids are quite high even with the small addition of volume fraction of nanoparticles. Eastman *et al.*[6] showed that 0.3 volume % of 10 nm copper nanoparticle mixed with ethylene glycol made the effective thermal conductivity to increase by 40%. Das *et al.*[7] showed that 1-4 volume % of alumina nanoparticle mixed with water resulted in a 10-30% increment in the effective thermal conductivity. The result of remarkable

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enhancement in the thermal conductivity due to addition of small fraction of nanoparticles to base liquid made the present researchers to study heat transfer in nanoliquids. Generally used base liquids are water, ethylene-glycol, engine-oil, and glycerine. The metal and metal oxide nanoparticles used are silver, gold, copper, nickel, platinum, titanium, and zinc, titanium dioxide, copper-oxide, titania, and alumina.

Modelling governing equations in the presence of nanoliquids can be done using the following models like single-phase model of Khanafer *et al.*[8], two-phase model of Buongiorno[9] and modified-Buongiorno model of Siddheshwar *et al.*[10]. Innumerable authors have analysed heat transfer using one among the three models[11-13].

Convection study in nanoliquid-saturated porous medium is also important for its applications in the field of heat storage. This is due to the fact that presence of nanoparticles and porous medium as it maintains thermoregulation time in the system. Bourantas *et al.*[14] and Kasaeian *et al.*[15] in their papers have discussed applications of nanoliquids and porous media in heat transfer phenomenon.

Convection is studied in enclosures occupying nanoliquid-saturated porous medium due to their real life applications. The review of literature assures that there are two kinds of situation being studied namely:

a) Enclosures with lateral boundaries being isothermal and horizontal boundaries being adiabatic.[8,16-19]

b) Enclosures with lateral boundaries being adiabatic and horizontal boundaries being isothermal.[20-22]

The review gives a clear idea that many of the existing enclosures problems are numerically solved and has a scope to solve them analytically.

In the problem under study, we use amalgamation of nanoliquid and porous medium due to their applications. The nanoliquid and porous medium selected for the study here are ethylene glycol-copper saturated in 30% fiber glass reinforced polycarbonate. The choice of porous material is explained in detail by Nield[23] based on porosity. For densely-packed porous medium, the Darcy model is used whereas for sparsely-packed porous medium Brinkman model is used. Since there is a wide scope for budding researchers to study the enclosure problem in a porous medium filled with nanoliquid, we have considered the present problem.

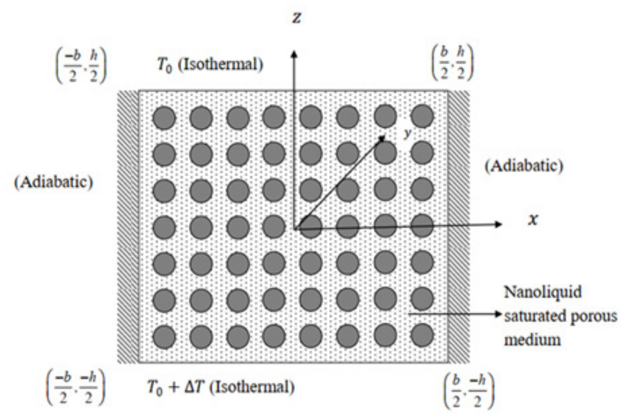


**Figure 1.** Physical flow.

In the present paper onset of convection in enclosures is studied by considering dissimilar shapes of nanoparticles, the range of values for the parameter aspect ratio, for which unicellular convection is possible is discussed. The effect of various parameters appearing in the study on the onset of convection namely porous parameter, volume fraction are also discussed and analysed graphically.

The flow configuration is represented in Figure 1. The lower boundary is assumed to be rigid free isothermal, hot compared to rigid-rigid adiabatic upper boundary with the temperature difference of ΔT.

**Assumptions:**

1. Single phase model with local thermal equilibrium assumption.

2. Ethylene glycol as base liquid, copper as nanoparticles with distinct shapes such as blades, platelets, cylinder, brick, and sphere are considered.

3. 30% fiber glass reinforced polycarbonate as porous medium is considered.

**Conservation of mass**

$$\nabla \cdot q = 0 \tag{1}$$

**Conservation of linear momentum**

$$\mu_{ne}^{'}\nabla^2 q - \frac{\mu_{nl}}{K}\vec{q} + \left[\rho_{ne} - (\rho\beta)_{ne}(T - T_0)\right]\vec{g} - \nabla p = 0. \tag{2}$$

**Conservation of energy**

$$M\alpha_{ne}\nabla^2 T - (\vec{q}.\nabla)T = 0, \tag{3}$$

where

$$M = \frac{(\rho C_p)_{ne}}{(\rho C_p)_{nl}},\ \alpha_{ne} = \frac{k_{ne}}{(\rho C_p)_{ne}}.$$

The thermophysical properties of nanoliquid, nanoliquid saturated porous medium, base liquid, nanoparticles and porous medium are calculated using phenomenological laws and mixture theory (Brinkman model[24] (1952), Hamilton crosser model[25] (1962), also using the definitions used by Siddheshwar and Veena[22].)

The dimensionless form of vorticity and heat transport equations as follows:

$$a\Lambda\nabla_A^4\Psi - a\sigma^2 A^2\nabla_A^2\Psi - a^2 Ra_{ne}A^4\frac{\partial\Theta}{\partial X} = 0, \quad (4)$$

$$-A\frac{\partial\Psi}{\partial X} + a\,M\nabla_A^2\Theta + A\frac{\partial(\Psi,\Theta)}{\partial(X,Z)} = 0 \quad (5)$$

The dimensionless parameters appearing in equations (4) and (5) are,

$$Ra_{ne} = \frac{(\rho\beta)_{ne}g\Delta T b^3}{\mu_{nl}\alpha_{ne}},\ \Lambda = \frac{\mu'_{ne}}{\mu_{nl}},\ \sigma^2 = \frac{b^2}{K}, \quad (6)$$

$$a = \frac{\alpha_{ne}}{\alpha_{bl}},\ A = \frac{h}{b}. \quad (7)$$

**Boundary condition**: Rigid-free isothermal horizontal boundaries and rigid-rigid adiabatic vertical boundaries

$$\left.\begin{array}{l} \Psi = \dfrac{\partial\Psi}{\partial Z} = \Theta = 0 \ \text{at}\ \ Z = -\dfrac{1}{2}\ \text{and}\ \dfrac{-1}{2} < X < \dfrac{1}{2} \\ \Psi = \dfrac{\partial^2\Psi}{\partial Z^2} = \Theta = 0 \ \text{at}\ \ Z = \dfrac{1}{2}\ \text{and}\ \dfrac{-1}{2} < X < \dfrac{1}{2} \\ \Psi = \dfrac{\partial\Psi}{\partial X} = \dfrac{\partial\Theta}{\partial X} = 0 \ \text{at}\ \ X = -\dfrac{1}{2},\dfrac{1}{2}\ \text{and}\ \dfrac{-1}{2} < Z < \dfrac{1}{2} \end{array}\right\}. \quad (8)$$

## Linear Stability Analysis

The stability analysis was carried out for the problem and found that the fundamental of exchange of stabilities holds good and consequently only the marginal stationary state analysis was performed. The eigen boundary value problem for the problem consists of linearized equations (4)-(5) and boundary conditions as in Eqs. (8).

The eigen functions as solutions are considered as follows:

$$\Psi(X,Z) = \Psi_0 C_{fo}(X)C_{fo}(Z) \quad (9)$$

$$\Theta(X,Z) = \Theta_0\cos(2\pi X + \pi)\sin(2\pi Z + \pi), \quad 10)$$

where $C_{fo}(X)$ and $C_{fo}(Z)$ are Chandrashekar functions[26-28] (odd function) given by

$$\left.\begin{array}{l} C_{fo}(X) = \dfrac{sinh(\mu_1 X)}{sinh(\frac{\mu_1}{2})} - \dfrac{sin(\mu_1 X)}{sin(\frac{\mu_1}{2})} \\ C_{fo}(Z) = \dfrac{sinh(\mu_1 Z)}{sinh(\frac{\mu_1}{2})} - \dfrac{sin(\mu_1 Z)}{sin(\frac{\mu_1}{2})} \end{array}\right\}, \quad (11)$$

and $\mu_1 = 7.85320$ is a root of the transcendental equation given by:

$$\tanh\left(\frac{\mu_1}{2}\right) - \tan\left(\frac{\mu_1}{2}\right) = 0 \quad (12)$$

Substituting Equations (9) and (10) in equations (4) and (5), an expression is obtained for $Ra_{nec}$, critical Rayleigh number for nanoliquid saturated porous medium in the form:

$$Ra_{nec} = \frac{M\delta_A^2}{16A^3E_2^2}\left[\frac{\Lambda E_3(1+A^4)}{A^2} - \Lambda E_4 - (1+A^2)E_1\sigma^2\right], \quad (13)$$

where

$$E_1 = \frac{1}{16}E_6E_7E_8\ ,E_2 = \frac{128\pi^3\mu_1^4}{(-16\pi^4+\mu_1^4)^2},$$

$$E_3 = \frac{\mu_2^2}{16}E_7E_8^2,\ E_4 = -\frac{\mu_2^2}{8}E_7E_9^2, \quad (14)$$

$$E_6 = \csc^4[\frac{\mu_1}{2}]csch^4[\frac{\mu_1}{2}] \quad (15)$$

$$E_7 = -2\sinh[\mu_1]\sin^2[\frac{\mu_1}{2}] + 2\sin[\mu_1]\sinh^2[\frac{\mu_1}{2}] +$$

$$\mu_1(-2+\cosh[\mu_1] \quad (16)$$

$$E_8 = 2\sin[\mu_1]\sinh^2[\frac{\mu_1}{2}] + 2\sinh[\mu_1]\sin^2[\frac{\mu_1}{2}] +$$

$$\mu_1(\cos[\mu_1] - \cosh[\mu_1]). \quad (17)$$

## 2.0 Results and Discussion

Linear stability analysis is studied by taking ethylene glycol as baseliquid, copper as nanoparticles, and 30% glass fiber reinforced polycarbonate as a porous medium. The thermophysical properties of baseliquid, nanoparticles, porous materials are collected and considered from the literature and tabulated in Table 1. We have considered the volume fraction, $\chi = 0.06$, and porosity $\phi$=0.88 in the problem. Dissimilar shapes of nanoparticles are taken into consideration and studied namely brick, sphere, platelets, blades and cylinder. Also their effects on the onset of convection are studied. Besides the effect of the various parameters appearing in the study are also discussed.

From the definition of Rayleigh number, $Ra_{nec}$ we have the following:

$$Ra_{ne} = F\ Ra_{be}.$$

From Figure (2) we observe that factor, $F$ decreases with increase in volume fraction, $\chi$. Further from same figure we are sure that the factor values takes the following order:

$$F_{blades} < F_{platelets} < F_{cylinder} < F_{brick} < F_{sphere}.$$

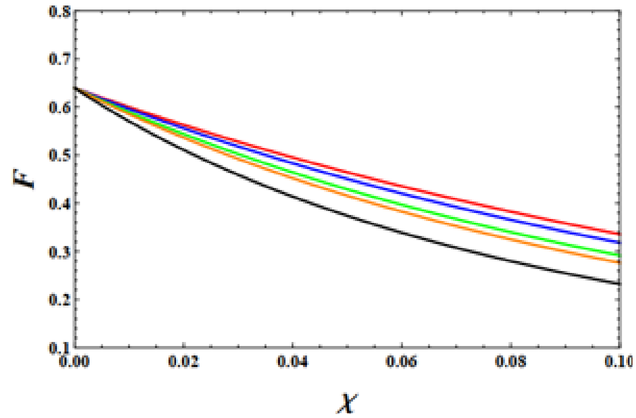


**Figure 2.** Factor, $F$ versus $\chi$. (The curves from top to bottom are for sphere, brick, cylinder, platelets and blade shaped nanoparticles).

From table (2) it is evident that

$$k_{blades} > k_{platelets} > k_{cylinder} > k_{brick} > k_{sphere}.$$

Hence we conclude that the adding nanoparticles to baseliquid leads in advancement of onset of convection. The onset of convection is faster in the case of blade shaped nanoparticles followed by the platelets, cylinder, brick and sphere shaped nanoparticles respectively among the five dissimilar shaped nanoparticles considered in the study.

Figure 3 confirms that the onset of convection advances with an increase in the aspect ratio, $A$, till $A$=2

**Table 1.** Thermophysical properties of copper, ethylene glycol, 30% glass fiber reinforced polycarbonate at 300K.

| Quantity | Copper[29] | Ethylene glycol[29] | 30% fiber glass reinforced polycarbonate[21] |
|---|---|---|---|
| $\rho$ [$kgm^{-3}$] | $\rho_{np}$=8933 | $\rho_{bl}$=1114.4 | $\rho_s$=1430 |
| $\beta$ [$k^{-1} \times 10^5$] | $\beta_{np}$=1.67 | $\beta_{bl}$=65 | $\beta_s$=3.5 |
| $C_p$ [J/kg-K] | $(C_p)_{np}$=385 | $(C_p)_{bl}$=2415 | $(C_p)_s$=1130 |
| $k$ [W/m-K] | $k_{np}$=401 | $k_{bl}$=0.252 | $k_s$=0.24 |
| $\mu$ [kg/m-s] | - | $\mu_{bl}$= 0.0157 | - |
| $\phi$ | - | - | $\phi$ =0.88 (2006) |

**Table 2.** Different shaped nanoparticles with their shape factor.

| Quantity | Shape factor[30] | Thermal conductivity | |
|---|---|---|---|
| blades | 8.6 | 0.3895 | 0.3715 |
| platelets | 5.7 | 0.3433 | 0.3309 |
| cylinder | 4.9 | 0.3305 | 0.3196 |
| brick | 3.7 | 0.3113 | 0.3028 |
| sphere | 3 | 0.3001 | 0.2929 |

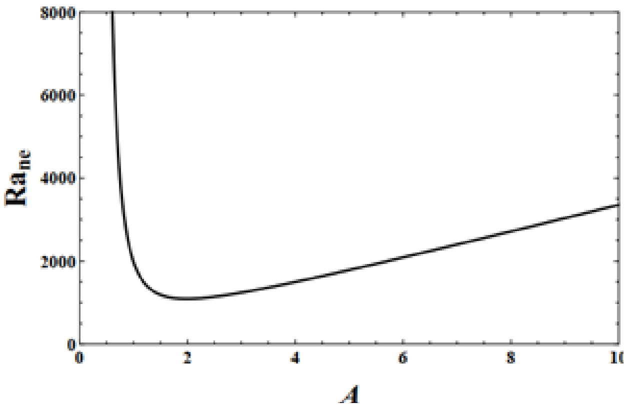


**Figure 3.** $Ra_{nec}$ versus $A$ for other fixed values of $\chi$ =0.06, $\sigma^2$=10, $\Lambda$ =1

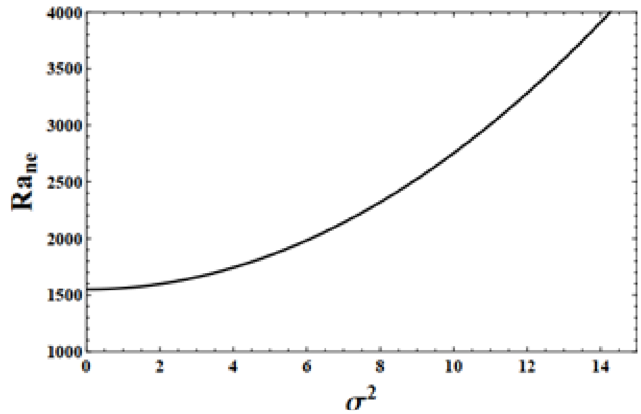


**Figure 5.** $Ra_{nec}$ versus $\sigma^2$ for other fixed values of $\chi$ =0.06, $\Lambda$=10, A =1

after that there is a sudden shoot up in the curve. This concludes the fact that unicellular convection is possible for $0.8<A<1$.

Figures 4 and 5 confirms that $Ra_{nec}$ increases with the increase in Brinkman number $\Lambda$ and $\sigma^2$ as the presence of porous medium causes disturbances to the flow pattern causing delay in the onset of convection.

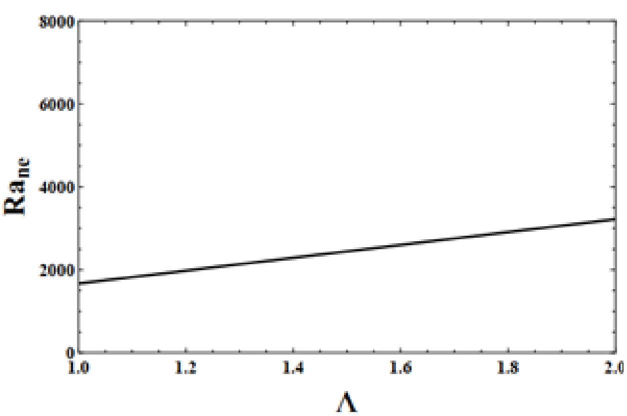


**Figure 4.** $Ra_{nec}$ versus $\Lambda$ for other fixed values of $\chi$ =0.06, $\sigma^2$=10, A =1

## 3.0 Conclusion

- Onset of convection is advanced when blade shaped nanoparticles are considered in the study.
- Unicellular convection is possible when $0.8<A<1$
- Onset of convection is advanced when nanoparticles are added to baseliquid and delayed in the addition of the porous medium to nanoliquid.


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| **Nomenclature : Latin symbols** | | $\mu$ | dynamic viscosity |
|---|---|---|---|
| $a$ | thermal diffusivity | $\mu_1$ | constant |
| $A$ | aspect ratio | $\Psi$ | non-dimensional stream function |
| $C_p$ | specific heat | $\rho$ | actual density |
| $\vec{g}$ | Acceleration $(0,0,-g)$ | $\phi$ | porosity |
| $h$ | height | $\Theta$ | non-dimensional temperature |
| $M$ | ratio of specific heat | Subscripts | |
| $p$ | pressure | $be$ | effective base liquid-saturated porous medium |
| $q$ | velocity vector | $bl$ | base liquid |
| $Ra$ | Rayleigh number | $c$ | critical |
| $T$ | temperature | $ne$ | effective nanoliquid-saturated porous medium |
| $T_0$ | temperature of lower plate | $nl$ | nanoliquid |
| **Greek Symbol** | | $np$ | nanoparticle |
| $\alpha$ | thermal diffusivity | $s$ | solid |
| $\beta$ | thermal expansion coefficient | | |
| $\chi$ | volume fraction | | |
| $\Delta T$ | Difference in temperature. | | |